ABSTRACT AND NONEXISTENT OBJECTS

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• The language of important disciplines has terms that appear to refer to abstract, nonexistent, and ‘possible-but-nonactual’ objects, and even abstract, nonexistent, and possible-but-nonactual properties and relations:
  – 0, 1, 2, …, \pi, \emptyset, \{\emptyset\}, \omega, \aleph_0, …; x < y, x \in y (math)
  – possible event \(x\) in a probability distribution \(y\), \(x\)’s center of mass, the possible planet perturbing the orbit of Pluto; the aether, phlogiston, absolute simultaneity, … (science, present and past)
  – Zeus, Sherlock Holmes, Bilbo Baggins, the monster I dreamt about last night, \(x\)’s concept of \(y\), the possible state of affairs \textit{there being a communist takeover of the U.S.} (literary criticism, psychology)

• But if we accept the scientific view about what exists, namely, the physical entities and forces postulated by our best scientific theories, then what do the above terms refer to, since they aren’t mentioned by these theories?
• Are these just “ideas in our minds” (psychologism).
  – When we say ‘2 is prime’, ‘Holmes is a detective’ and ‘George Bush is president’, there is a systematicity to our use of language. The suggestion is: in the last case, we are referring to an object in the external world, but in the first two cases, we are referring to ideas.

• But ideas are particular to individuals – whose idea of ‘2’ are we referring to when we say ‘2 is prime’, yours or mine? Instead, it seems like there is something more abstract, which you and I are both referring to.

• It seems like when we say ‘Holmes is a detective’ and ‘Pinkerton is a detective’, we are, in both cases, truly attributing the property of being a detective to an object. But how can an idea truly be a detective?
• For today, let’s temporarily put aside the worry about reconciling abstract, nonexistent, and possible objects with our best theories.

• Instead, our plan is:
  – See what the data is we are trying to explain, namely, the true sentences and valid arguments involving terms referring to these entities. We’ll focus on fictions.
  – Examine critically what some philosophers have said about the analysis of this data.
  – Produce a theory of the objects in question and use them our own analysis.
  – Return to the question of reconciling our theory with our best scientific theories.
According to the Conan Doyle novels, Holmes is a detective. According to *The Tempest*, Prospero had a daughter. According to *The Iliad*, Achilles fought Hector. According to the *The Lord of the Rings*, Sauron tried to recover the master ring from Frodo.


Teams of scientists have searched for the the Loch Ness monster, but since *it* doesn’t exist, no one will ever find *it*.

Ponce de Leon searched for the fountain of youth, and though it doesn’t exist, he believed that it existed.
• Superman = Clark Kent.
  Zeus = Jupiter.
  Pegasus is not identical to Zeus.

• Some fictional characters are interesting because they find themselves in situations in which they appear to be able to choose their identity, though it inevitably turns out that factors beyond their control, antecedent to the moment of choice, had already determined what they would do.

The ancient Greeks and Romans worshipped the same gods, though they called them by different names.

This story is fictional and any similarity between the characters and real people is unintended and coincidental.

None of the characters in this story exist.
• Valid Arguments:
  – The ancient Greeks worshipped Zeus.
    Zeus is a mythical character.
    Mythical characters don’t exist.
    Therefore, the ancient Greeks worshipped something that doesn’t exist.
  – Modern Oregonians worshipped Bhagwan Rajneesh.
    Bhagwan Rajneesh is a sex-scam artist.
    Sex-scam artists shouldn’t be trusted.
    Therefore, modern Oregonians worshipped someone who shouldn’t be trusted.
  – Ponce de Leon searched for the fountain of youth.
    Therefore, Ponce de Leon searched for something.
  – I searched for the keys in my pocket.
    I searched for something.
• Frege’s (1892) theory: Names like ‘Pegasus’ fail to denote anything whatsoever. When a name ‘n’ occurs in a sentence such as ‘n is F’ (or ‘x bears relation R to n’ or ‘n bears relation R to x’) and it fails to denote, then the sentence as a whole lacks a truth value.

• Examples: ‘Odysseus is a Greek warrior’, ‘Holmes is a detective’, ‘Frodo is a hobbit’ are all truth-valueless, on Frege’s view. This is something we could live with, given that there is some kind of disanalogy to sentences like ‘Bush is president’, etc.

• Counterexamples: ‘Augustus Caesar worshipped Jupiter’, ‘Sherlock Holmes is more famous than Alan Pinkerton’ are both true, not truth-valueless! They are (historical) facts.
• Russell’s theory (1905): Names like ‘Pegasus’ abbreviate definite descriptions, such as ‘the winged horse captured by Bellerophon’. Definite descriptions are analyzed away in terms of existence and uniqueness claims.

• Example: ‘Pegasus can fly’ is analyzed as ‘The winged horse captured by Bellerophon can fly’, and that in turn becomes analyzed as ‘There exists a unique winged horse captured by Bellerophon and it can fly’. Let ‘W’ abbreviate ‘winged horse captured by Bellerophon’:

\[ \exists x [Wx \& \forall y (Wy \rightarrow y = x) \& Fx] \]

Russell’s theory predicts this latter is false, which is something we can live with. But,
• Counterexample to Russell’s Theory: ‘Augustus Caesar worshipped Jupiter’, which is true, is analyzed as ‘Augustus Caesar worshipped the most powerful Roman god’, which in turn, becomes analyzed as: There exists a unique most powerful Roman god and Caesar worshipped it’:

\[ \exists x[Mx \& \forall y(My \rightarrow y = x) \& Wcx] \]

This is false, contrary to historical fact.

• Does Quine have a theory that analyzes these sentences?
• Meinong’s (1905) naive theory of objects: for any group of properties, there is an object which has (instantiates, exemplifies) those properties.

\[ \exists x \forall F (Fx \equiv \varphi). \]

- There is an object which instantiates the properties that Zeus has in the myth.

\[ \exists x \forall F (Fx \equiv \text{In the myth, } F_z). \]

- There is an object which instantiates the properties that Sherlock Holmes has in the Conan Doyle novels.

\[ \exists x \forall F (Fx \equiv \text{In the Conan Doyle novels, } F_h). \]

- There is a round square.

\[ \exists x \forall F (Fx \equiv F = R \lor F = S) \]

• Objects \( x \) and \( y \) are identical whenever they have exactly the same properties.

\[ x = y \equiv \forall F (Fx \equiv Fy) \]
Clearly, such objects could be used to give an account of the truth conditions and validity of our data.

Russell’s famous objections apply however: Meinong’s theory asserts (contrary to fact) there is an existing golden mountain, and asserts (contrary to the laws of geometry) that there is a round square, and asserts (contrary to the laws of logic) that there is a non-square square.

- $\exists x \forall F (Fx \equiv F = E! \lor F = G \lor F = M)$, but
  Fact: $\neg \exists x (E!x \& Gx \& Mx)$

- $\exists x \forall F (Fx \equiv F = R \lor F = S)$ but,
  Geometrical Law: $\forall x (Rx \rightarrow \neg Sx)$

- $\exists x \forall F (Fx \equiv F = S \lor F = \bar{S})$ but,
  Logical Law: $\forall x (\bar{S}x \equiv \neg Sx)$
• Parsons’ (1980) solution to the Russell objections: (1) distinguish nuclear and extranuclear properties, (2) define objects only relative to groups of nuclear properties, (3) restrict laws of geometry to possible objects, and (4) assert that negations of properties are not genuine complements, (5) stipulate that existence is extranuclear and not a nuclear property.

• On Parsons’ theory, the quantifier ‘there is’ (‘∃’) is distinguished from the existence predicate $E!$ – the former doesn’t imply existence. One can consistently assert that there are objects which don’t exist ($\exists x \neg E! x$). Parsons’ argues that in natural language, we distinguish between ‘there is’ and ‘there exists’, as in ‘there are fictional characters (e.g., Iago) which we loathe even though they don’t exist’. (What would Quine say about this?)
• On Parsons’ theory, you don’t get an existing golden mountain, since the property of existence is extranuclear and can’t be used to define a new object.

• On Parsons’ theory, the round square is asserted to be an ‘impossible’ object, and so doesn’t fall within the reconfigured geometrical law: for all possible objects \( x \), if \( x \) is round, \( x \) fails to be square (i.e., \( \forall x (R x \rightarrow \neg S x) \)).

• On Parsons’ theory, the negation of the property of being square (\( \bar{S} \)) works properly only for existing objects: \( \forall x (E! x \rightarrow (\bar{S} x \equiv \neg S x)) \). By asserting that the non-square square doesn’t exist, you avoid the contradiction.
• E. Mally’s (1912) solution to the Russell objections: distinguish between the properties that ‘determine’ an abstract object \( x \) and the properties that \( x \) satisfies (or instantiates, or exemplifies). Write ‘\( xF \)’ to say \( F \) determines \( x \) (or \( x \) encodes \( F \)) and ‘\( Fx \)’ to say \( x \) satisfies (or instantiates, or exemplifies) \( F \).

• The existing golden mountain is an object \( x \) which is determined by, i.e., encodes, the properties of existence, goldenness, and mountainhood (i.e., \( xE! \& xG \& xM \)), but it doesn’t instantiate these properties. It is consistent with the claim \( \neg \exists x(E!x \& Gx \& Mx) \).

• The round square is an object \( y \) which is determined by (encodes) roundness and squareness (i.e., \( yR \& yS \)). It is consistent with the unrestricted law: \( \forall x(Rx \rightarrow \neg Sx) \).

• On Mally’s view, the non-square square is an object \( z \) which encodes squareness and non-squareness (i.e., \( zS \& z\bar{S} \)), and it is consistent with the unrestricted law: \( \forall x(\bar{S}x \equiv \neg Sx) \).
• Mally’s theory is formalized and applied in the cited works by Zalta.

• $x$ is an ordinary object iff it is possible that $x$ is concrete.
  
  $O!x \equiv \Diamond E!x$

• $x$ is an abstract object iff it is not possible that $x$ is concrete.
  
  $A!x \equiv \neg \Diamond E!x$

• Ordinary objects don’t encode properties.
  
  $O!x \rightarrow \neg \exists FxF$

• $x$ and $y$ are identical iff either (a) they are both ordinary and they exemplify the same properties, or (b) they are both abstract and they encode the same properties.
  
  $x = y \equiv [O!x \& O!y \& \forall F(Fx \equiv Fy)] \lor [A!x \& A!y \& \forall F(xF \equiv yF)]$

• For any condition on properties, there is an abstract object that encodes just the properties meeting the condition.
  
  $\exists x(A!x \& \forall F(xF \equiv \varphi))$
<table>
<thead>
<tr>
<th>Abstract Object</th>
<th>Theoretical Description</th>
<th>Formal Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>The null set of ZF</td>
<td>... in Zermelo-Fraenkel set theory, $\emptyset$ has $F$</td>
<td>$ZF \models F\emptyset$)</td>
</tr>
<tr>
<td>$\kappa$ of math theory $T$</td>
<td>... in theory $T$, $\kappa$ has $F$</td>
<td>$T \models F\kappa$)</td>
</tr>
<tr>
<td>The aether of 19th century physics</td>
<td>... in 19th century physics, the aether has $F$</td>
<td>$XIX \models F$(the aether))</td>
</tr>
<tr>
<td>Sherlock Holmes</td>
<td>... according to the Conan Doyle novels, Holmes has $F$</td>
<td>$CD \models Fh$)</td>
</tr>
<tr>
<td>The actual world</td>
<td>... $F$ is a property of the form being such that $p$ (where $p$ is a true proposition)</td>
<td>$\exists p(p &amp; F=[\lambda y p]))$</td>
</tr>
<tr>
<td>The Form of $G$</td>
<td>... $F$ is the property $G$</td>
<td>$F=G))$</td>
</tr>
<tr>
<td>The Leibnizian concept of Alexander</td>
<td>... Alexander exemplifies $F$</td>
<td>$Fa))$</td>
</tr>
<tr>
<td>The natural number 0</td>
<td>... $F$ is unexemplified by ordinary objects</td>
<td>$\neg \exists uFu))$</td>
</tr>
<tr>
<td>The truth value of proposition $p$</td>
<td>... $F$ is a property of the form being such that $q$ (where $q$ is materially equivalent to $p$)</td>
<td>$\exists q(q \equiv p &amp; F=[\lambda y p]))$</td>
</tr>
<tr>
<td>The set of $Gs$</td>
<td>... $F$ is materially equivalent to $G$</td>
<td>$\forall y(Fy \equiv Gy))$</td>
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</table>
• How do we naturalize this theory? It appears to assert the existence of an infinite number of abstract objects. How do we reconcile it with our best physical science?

• One method of reconciliation: argue it is required for our very understanding of science (Linsky & Zalta, 1995). The idea: the theory of abstract objects is required for our understanding of mathematical language, and understanding of mathematical language is required for our understanding of science. So, ‘naturalism’ is properly formulated as the view: accept only what science requires or what is required for our understanding of science.

• A second method of reconciliation: argue that the formal theory simply systematizes our linguistic practices. We utter words and sentences in systematic ways, and these linguistic practices, when viewed from the bottom-up, constitute very general, large-scale patterns in the natural world. The theory simply systematizes and quantifies over those large-scale patterns—abstract objects aren’t mysterious.
References


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