Sample Final Exam (with Solutions)

Philosophy 12A
June 23, 2010

This is a sample in-class final examination. The structure of the actual in-class final will be exactly the same as this sample. The particular problems on the actual final will be different. On the actual final exam, you can expect to see problems of roughly the same difficulty as these. You will be given all natural deduction rules (but not the truth-table definitions).

1 Ten Cumulative True/False Questions

T F 1. If it is not possible for the conclusion of an argument to be false, then the argument is valid.

T F 2. All sound arguments are valid.

T F 3. Some valid arguments are unsound.

T F 4. If the conclusion of a valid argument is true, the premises must be true as well.

T F 5. The following argument is sound (absolutely): "If Prince William is unmarried, then Prince William is a bachelor. Prince William is a bachelor. Therefore, Prince William is unmarried."

T F 6. The following LSL argument is valid:

$A \vee B$
$A \rightarrow C$
$B \rightarrow C$
$\therefore C$

T F 7. The following LMPL argument is valid:

$(\forall x)(\exists y)(Fx \rightarrow Gy)$
$\therefore (\exists x)(\forall y)(Fx \rightarrow Gy)$

T F 8. The following is a legitimate application of the ∀I rule:

1	(1)	(∀x)(Fx→Gx)	Premise
2	(2)	Fb	Premise
1	(3)	Fb→Gb	1 ∀E
1,2	(4)	Gb	2,3 →E
1,2	(5)	(∀x)Gx	4 ∀I

T F 9. The following is a legitimate application of the ∃E rule:

1	(1)	(∃x)Fx	Premise
2	(2)	Ga	Premise
3	(3)	Fa	Assumption
2,3	(4)	Fa&Ga	2,3 &I
2,3	(5)	(∃x)(Fx&Gx)	4 ∃I
1,2	(6)	(∃x)(Fx&Gx)	1,3,5 ∃E

T F 10. The following LSL sentence is a *tautology*: '$(A \vee B) \leftrightarrow ((A \rightarrow B) \rightarrow B)$'.

Answers: (1) **T**, (2) **T**, (3) **T**, (4) **F**, (5) **F**, (6) **T**, (7) **F**, (8) **F**, (9) **F**, (10) **T**.

2 An LSL Semantics Problem

Do a full truth-table for the following LSL sentence: '$(A \vee B) \leftrightarrow ((A \rightarrow B) \rightarrow B)$'.

3 Completing an Incomplete LMPL (or L2PL) Natural Deduction

Complete the following natural deduction proof of the LMPL theorem $\vdash (\forall x)[(Fx \to Gx) \vee (Gx \to Fx)]$. All assumptions and premises have been filled in. Fill in any missing labels (right) and premise/assumption numbers (left). NOTE: *SI and TI might be used in this proof.*

	(1)	$Ga \vee {\sim}Ga$	
2	(2)	Ga	Assumption
	(3)	$Fa \to Ga$	
	(4)	$(Fa \to Ga) \vee (Ga \to Fa)$	
5	(5)	${\sim}Ga$	Assumption
	(6)	$Ga \to Fa$	
	(7)	$(Fa \to Ga) \vee (Ga \to Fa)$	
	(8)	$(Fa \to Ga) \vee (Ga \to Fa)$	
	(9)	$(\forall x)[(Fx \to Gx) \vee (Gx \to Fx)]$	✦

Answer:

	(1)	$Ga \vee {\sim}Ga$	TI (LEM)
2	(2)	Ga	Assumption
2	(3)	$Fa \to Ga$	2 SI (PMI)
2	(4)	$(Fa \to Ga) \vee (Ga \to Fa)$	3 $\vee$I
5	(5)	${\sim}Ga$	Assumption
5	(6)	$Ga \to Fa$	5 SI (PMI)
5	(7)	$(Fa \to Ga) \vee (Ga \to Fa)$	6 $\vee$I
	(8)	$(Fa \to Ga) \vee (Ga \to Fa)$	1, 2, 4, 5, 7 $\vee$E
	(9)	$(\forall x)[(Fx \to Gx) \vee (Gx \to Fx)]$	8 $\forall$I ✦

4 Diagnosing an Incorrect LMPL Natural Deduction

Explain what is wrong with the following *alleged* proof of the sequent $(\forall x)(\exists y)(Fx \,\&\, Gy) \vdash (\exists y)(\forall x)(Fx \,\&\, Gy)$. Is it possible to *fix* this *illegitimate* proof? If so, explain how. If not, explain why it *can't* be fixed. Try to find a counterexample LMPL interpretation. If you can't find one that contains *fewer than 3 individuals* in it, then you can assume that the argument is *valid*, and that it *can* be proven.

1	(1)	$(\forall x)(\exists y)(Fx \,\&\, Gy)$	Premise
1	(2)	$(\exists y)(Fa \,\&\, Gy)$	1 $\forall$E
3	(3)	$Fa \,\&\, Gb$	Assumption
3	(4)	$(\forall x)(Fx \,\&\, Gb)$	3 $\forall$I
3	(5)	$(\exists y)(\forall x)(Fx \,\&\, Gy)$	4 $\exists$I
1	(6)	$(\exists y)(\forall x)(Fx \,\&\, Gy)$	2, 3, 5 $\exists$E

Answer: The problem with this proof occurs at step (4). The constant 'a' occurs in line (3) upon which line (4) depends. This violates the restriction on t in the definition of the $\forall$I rule. The sequent *is* valid, however, (if you try to find a counterexample interpretation, you'll quickly be forced to have *more than 2* individuals in your domain). I proved this LMPL sequent in 12 steps in my lecture notes.

5 Working with a Given LMPL (or L2PL) Interpretation

Consider the following LMPL interpretation:

$(\mathcal{I})$

	F	G	H	J
a	+	+	+	−
b	+	−	−	−

Explain why $\mathcal{I}$ shows that the following claim is true:

$$(\forall x)(Fx \to Gx) \to (\forall x)(Hx \to Jx) \nvDash (\exists x)(Fx \,\&\, Gx) \to (\forall x)(Hx \to Jx)$$

Answer: The premise '$(\forall x)(Fx \to Gx) \to (\forall x)(Hx \to Jx)$' is true, because its antecedent '$(\forall x)(Fx \to Gx)$' is false, since its instance '$Fb \to Gb$' is false on $\mathcal{I}$ [$b \in \text{Ext}(F)$, $b \notin \text{Ext}(G)$]. The conclusion '$(\exists x)(Fx \,\&\, Gx) \to (\forall x)(Hx \to Jx)$' is false. Its antecedent '$(\exists x)(Fx \,\&\, Gx)$' is true, since its instance '$Fa \,\&\, Ga$' is true on $\mathcal{I}$ [$a \in \text{Ext}(F)$, $a \in \text{Ext}(G)$]. But, its consequent '$(\forall x)(Hx \to Jx)$' is false, because its instance '$Ha \to Ja$' is false on $\mathcal{I}$ [$a \in \text{Ext}(H)$, $a \notin \text{Ext}(J)$]. So, $\mathcal{I}$ shows that this is an invalid argument.

6 Constructing a Counterexample LMPL Interpretation

Construct an LMPL interpretation which shows that the following claim is true (make sure to *explain why* your interpretation is a counterexample to the validity of the argument):

$$(\forall x)(\exists y)(Gy \to Fx) \nvDash (\forall x)[(\exists y)Gy \to Fx]$$

Answer: Try

	F	G
a	+	+
b	−	−

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7 An LSL Natural Deduction

Give a natural deduction proof of the following LSL sequent. You *may* use SI and TI (you *don't have to*).

$$S \to (R \vee P), P \to ({\sim}R \to Q) \vdash S \to (Q \vee R)$$

Answer: Here's a 13-step proof using SI

1	(1)	S→(R∨P)	Premise
2	(2)	P→(~R→Q)	Premise
3	(3)	S	Assumption
1,3	(4)	R∨P	1,3 →E
5	(5)	R	Ass
5	(6)	Q∨R	5 ∨I
7	(7)	P	Assumption
2,7	(8)	~R→Q	2,7 →E
2,7	(9)	~~R∨Q	8 SI (IMP)
2,7	(10)	R∨Q	9 SI (SDN)
2,7	(11)	Q∨R	10 SI (Com)
1,2,3	(12)	Q∨R	4,5,6,7,11 ∨E
1,2	(13)	S→(Q∨R)	3,12 →I

8 An LMPL Natural Deduction

Give a natural deduction proof of the following LMPL argument. You *may* use SI and TI (*don't have to*).

$$(\forall x)[Lx \to ((\forall y)(Py \to Vy) \to Mx)]$$
$$(\exists z)(Pz \,\&\, Vz) \to (\forall y)(Py \to Vy)$$
$$\therefore (\exists x)Lx \to [(\exists z)(Pz \,\&\, Vz) \to (\exists y)My]$$

Answer: Here's a 13-step proof (w/out SI):

1	(1)	(∀x)(Lx→((∀y)(Py→Vy)→Mx))	Premise
2	(2)	(∃z)(Pz&Vz)→(∀y)(Py→Vy)	Premise
3	(3)	(∃x)Lx	Assumption
4	(4)	(∃z)(Pz&Vz)	Assumption
5	(5)	La	Assumption
1	(6)	La→((∀y)(Py→Vy)→Ma)	1 ∀E
1,5	(7)	(∀y)(Py→Vy)→Ma	6,5 →E
2,4	(8)	(∀y)(Py→Vy)	2,4 →E
1,2,4,5	(9)	Ma	7,8 →E
1,2,4,5	(10)	(∃y)My	9 ∃I
1,2,3,4	(11)	(∃y)My	3,5,10 ∃E
1,2,3	(12)	(∃z)(Pz&Vz)→(∃y)My	4,11 →I
1,2	(13)	(∃x)Lx→((∃z)(Pz&Vz)→(∃y)My)	3,12 →I