Symposium:

ON THE ONTOLOGICAL SIGNIFICANCE OF THE LÖWENHEIM-SKOLEM THEOREM

GEORGE D. W. BERRY AND JOHN R. MYHILL

II. JOHN R. MYHILL

I share with the previous speaker the conviction that the Löwenheim-Skolem theorem has no direct philosophical implications. This phrase should be clarified. What is implied is a proposition and to say there are philosophical implications implies that there are philosophical propositions. This runs counter to the idea that philosophy is an activity rather than a doctrine, an idea to which with reservations I subscribe. However part if not all of this activity consists in the assertion of propositions, which are not however philosophical propositions in themselves, but become philosophical in virtue of being asserted in the course of philosophical activity. Hence no proposition has philosophical implications in the strict sense, but perhaps every proposition may with propriety be asserted in the course of philosophical activity. Almost any proposition may I suppose initiate philosophical activity, and I take the invitation to contribute the present paper as a request to perform a philosophical activity initiated (after those introductory remarks) by the assertion of the Löwenheim-Skolem theorem. The assertions made by me subsequently to this assertion I shall call indirect implications of the Löwenheim-Skolem theorem, using the word 'implication' in its colloquial rather than its technical sense. My initial remark that the theorem has no direct philosophical implications is therefore a direct consequence of my view that philosophy is an activity rather than a doctrine.

I do not maintain that philosophy is wholly or primarily an activity of clarification. In particular I cannot see that clarification is the principal goal of ethics, though it might be an important instrument in achieving that goal. None the less clarification is part of philosophy, or at least the clarification of certain issues is. Much of the activity which I will perform in this paper will be clarificatory, that is, it will be devoted to stating in non-technical terms what the Löwenheim-Skolem theorem is. Why is this a philosophical activity? Would a clarification of say the binomial theorem be philosophical? Clearly not; more exactly, it seems highly dubious that the assertion of the binomial theorem could profitably initiate a philosophical discourse, except perhaps by way of illustration of some general aspect of mathematics for which purpose a good many other theorems would have served equally well. The reason why the Löwenheim-Skolem theorem seems a fruitful proposition with which to begin a philosophical discourse, while the binomial theorem does not, is that we are inclined to ask "What does the Löwenheim-Skolem theorem really mean?" while we are not inclined to ask "What does the binomial theorem really mean?"

I take such questions seriously. A question is an expression of intellectual anxiety and an answer is an attempt at resolution of that anxiety. I distinguish formal from informal questions, and within the latter I distinguish subjective and objective. A formal question carries with it the form of its answer, that is, the social context is such that the criterion of acceptability for the answer is known and agreed upon by both questioner and answerer in abstraction from the answer itself. The purest kind of formal question is the question of the truth or falsity of a mathematical theorem within a known system. For the criteria of being a proof or not being a proof within that system are capable of exact specification and are in the ideal case specifically agreed upon by questioner and answerer. Questions in the empirical sciences are a less pure kind of formal question, since the criteria of confirmation are less exactly specifiable than those of mathematical proof.

An informal question is one the form of whose answer is not known either by questioner or answerer in abstraction from the answer itself. The dictum that the meaning of a proposition is the method of its verification does not apply to propositions which answer informal questions, for part of the meaning of such a question is to question what the form of its answer would be. Thus part of the meaning of the question "How shall I face the prospect of my death?" is "What form of answer (psychoanalytic,

theological, semantical) would resolve the anxiety expressed by the question 'How shall I face the prospect of my death?'?" More simply; if and in proportion as a question is formal, the questioner is prepared to state precisely what kind of evidence would convince him of the truth of any proposed answer. A formal question asks for the matter of its answer but provides the form; an informal question asks for both.

Any question, therefore, which asks after the general features of formalism, must be itself informal, for if the form of the answer were known to the questioner, he would already presuppose or regard as unquestioned a certain form as appropriate to answering his question, and so in questioning the nature of formalism would already be operating in the framework of a formalism which was unquestioned. Hence metamathematics must be in the final analysis informal; for the process of discussing formalisms by means of other formalisms must either terminate in a formalism which is not discussed, or be informal. But in the first case we would be doing mathematics and not in the strict sense metamathematics. Anyone familiar with the writings of Hilbert can provide illustrations for himself of this phenomenon. I shall try to show that anxiety concerning the Löwenheim-Skolem theorem originates partly in a desire to consider formal objects outside of the formalism in which they are imbedded, and so presents in a specifically acute way the informality characteristic of metamathematical anxiety. The opinions that this anxiety results from a confusion of different formalisms or from a confusion between internal and external questions, or from a self-contradictory desire that an object be at once formal and non-formal, I dismiss because of my contention that anxiety can frequently be resolved by verbal answer even when it does not provide the form of that answer. A question does not have to be precise in order to express a genuine anxiety and thus be a genuine question.

I distinguish within informal questions between objective and subjective, according as there is or is not agreement as to the effectiveness of the answer in resolving the anxiety which prompted the question. Hence there is no way of knowing whether an informal question is objective or subjective until the question is answered. Even in that case, there is usually the possibility that a subjectively satisfying answer may later be replaced by an objectively satisfying one. Evidently the distinction between subjective and objective informal questions is relative, dependent on social conditions. A question which is informal, but close to objectivity is "What is the meaning of the definite article?" At the other extreme are questions expressive of neurotic anxiety which can be resolved only by special treatment in each case.

Philosophical activity is the activity of resolving anxiety expressed by objective informal questions. Because what is at one time informal may later become formal, the philosophical area shrinks progressively, yielding place to science. Because few if any informal questions are entirely objective, there is diversity of philosophical systems. I repeat that the anxiety expressed in feeling the Löwenheim-Skolem theorem as a paradox results in part from the desire to grasp a formal object apart from its setting in a formal system; thus this anxiety concerns formalism in general and so can be resolved only informally. On the other hand, I am optimistic enough to hope that at least part of my comments will provoke agreement; hence the clarificatory part of them and a certain amount of the motivational analysis may claim to be philosophical in the sense I have explained.

I shall now state the theorem roughly and exhibit the proximate grounds for anxiety concerning its 'real meaning.' The theory states that every formal system expressed in the first order functional calculus has a denumerable model. In particular the general theory of sets as axiomatized e.g. by von Neumann and Gödel has a denumerable model; yet this theory was designed in part in order to formalize in a consistent and rigorous manner the arguments of Cantor's theory of infinite cardinals, one of the main results of which is that the continuum is more than denumerably infinite. One can state the perplexity arising from this circumstance in various ways; e.g. that the attempt to formalize the notion of a non-denumerable infinity is forever doomed to failure, and that this is an essential and unanticipated limitation of formalism. This perplexity we shall resolve incidentally later. Simpler to handle now and nearer the spirit of the foregoing discussion is the following statement: the continuum is according to formalized set-theory non-denumerable; i.e. its non-denumerability is a thesis of that theory. This thesis however asserts that no one-to-one correlation between the continuum and the integers exists; for in this way is nondenumerability defined. On the other hand, since formalized set-theory possesses a denumerable model, it

possesses a model in the integers; and so there is a one-to-one correlation between all the sets of formalized set-theory and the integers, namely the correlation which correlates each set with the integer representing it in the model. A fortiori there is a correlation between those particular sets in the formal system which constitute the continuum and a subset of the integers. It appears to follow that the continuum dealt with in formal set-theory is denumerable, hence not a true continuum; moreover, that the thesis of formal set-theory to the effect that (this) 'continuum' is non-denumerable actually asserts merely that it is not capable of enumeration by any correlation appearing amongst the objects of that theory, i.e. appearing as a value of its variables. For the thesis asserts "there is no R which is a one-to-one correlation between (this) continuum and the integers"; and since there does appear in the light of the Löwenheim-Skolem theorem to be such a correlation, we seem forced to conclude that this correlation is not amongst the range of values of the variable R, i.e. not amongst the objects of formal set-theory. Hence we infer that we cannot adequately express the notion of indenumerability and of the continuum within formalized set-theory, in the sense that all we can assert is the absence of a correlation *within the set-theory itself* between the continuum and the integers, whereas to do justice to our intuitive idea of a non-denumerable continuum we would wish to assert the absence of *any correlation whatsoever.* Here 'any correlation whatsoever' is an informal notion, for as soon as it is formalized we have once more only those correlations which one represented in a particular formal system, and the whole argument could be repeated concerning this system. Hence we suspect the existence of a non-formalizable notion, and that on a very low level of mathematics. The 'paradox' thus concerns the inadequacy of formalism to its supposed informally conceived object, and is therefore, in line with our previous discussion, a paradigm of the eventually informal and philosophical character of metamathematical anxiety. Naturally this whole argument is unprecisely formulated and probably contains outright fallacies; this is unimportant if it has served its purpose of directing attention to the proximate grounds of anxiety surrounding the Löwenheim-Skolem theorem. The exhibition of the fallacies would in any case not resolve the anxiety.

As a first step to this resolution we state the theorem again with pedantic lucidity; we use this to pass to a discussion of the relation between formalism and its object. The theorem deals in its usually stated form only with systems framed in the notation of the first-order functional calculus: recent extensions to higher-order calculi by Henkin offer no essential further problem from our present point of view. We now explain what the first-order functional calculus is.

By an atomic matrix will be meant a property of or a relation between a finite number of variables, e.g. 'x is a woman', 'x hates y', 'x takes z from y'. Here the variables 'x', 'y', 'z' are without meaning, hence the matrix as a whole is without meaning. The English words 'woman', 'hates', 'takes', etc. however retain their ordinary meanings. The variables are not to be regarded as names or abbreviations. We can abbreviate the English words denoting properties or relations by so-called predicate letters; thus 'x is a woman' might be 'Wx', 'x hates y' might be 'Hxy' and so forth. As abbreviations of English words these letters have a meaning: we repeat that the variables, as mere place-holders for meanings, have in themselves no meaning, and so the entire matrix has no meaning.

From atomic matrices we form other matrices by the following two operations. (1) *Quantification.* The variables occurring in atomic matrices, along with certain other variables to be explained presently, are called free (i.e. meaningless) variables. We consider a certain domain of objects U which we call the universe of discourse; it may be any non-empty domain whatever. In order to specify the interpretation of a system in which quantifiers are used we have to state not only the interpretation of the predicate letters (e.g. that 'W' means woman, 'H' means hate and so forth) but also the universe of discourse U. Now take any matrix containing a free variable (it may or may not be atomic). Take for example the matrix 'Wx' read 'x is a woman'. We take the free variable and place it in parentheses before the matrix, thus (x) Wx. The variable then becomes so-called bound or meaningful, and the meaning is that when the variable x in the matrix is interpreted as any element whatever of U, the result is true. Thus (x)Wx means that every element of U is a woman. Suppose now that the matrix contains other free variables besides x and that we prefix the quantifier (x). For example, consider the matrix (x) Hxy. This 'means' that every element of U hates y; but y is still

meaningless and free, whereas the x has become meaningful and bound. Hxy needs either interpretation or quantification of both its variables to make it meaningful; (x)Hxy only needs interpretation or quantification of the y. If we further quantify the y we get (y) (x) Hxy which means in view of the interpretation of the quantifiers and of H that every element of U hates every element of U (including incidentally itself; distinct variables do not have to refer to distinct elements of U).

If free variables occur in a matrix it is meaningless pending interpretation or quantification of those variables. As soon as and not before all the variables have been bound it becomes a meaningful assertion, true or false provided that the interpretation of the quantifiers (i.e. the universe of discourse) and that of the letters 'H', 'W', etc., have been specified, as we assume they have.

I must apologize for boring you with this somewhat pedantic explanation of the first-order functional calculus. I claim, however, that it is necessary for my purpose to do this, and that even those who are familiar with the technique of proof in that calculus may perhaps profit from the semantical remarks made in the course of this exposition. The aim of the exposition is to clarify the notion of 'model', which plays such an important role in the Löwenheim-Skolem theorem. Confused ideas about 'non-standard' models are rife today, and the lay reader is not entirely to blame. If this paper explains what models and non-standard models are, it will perhaps forestall confusion in philosophical circles.

(2) The other means by which complex matrices are built up from simpler and ultimately from atomic matrices is *truth-functional composition.* Given two matrices like 'x hates z' and '(y) (y is a woman)' we can form their conjunction, disjunction, implication, and so on: and given a single matrix we can form its negation. The variables which are free and meaningless in an element of a truth-functional compound are still free and meaningless, awaiting quantification, in the compound itself; similarly for the bound variables. Thus in 'x hates z or (y) (y is a woman)' x and z are free while y is bound; the matrix is meaningless until the x and z become bound also.

A matrix in which all variables are bound is the only meaningful kind of matrix. We shall speak of it as a (formal) sentence. Thus (supposing U specified e.g. as the class of people) the matrix 'not (x) (x hates x)' is equivalent to the English sentence 'not every person hates himself,' and this is true. Formal properties of relations and properties (in this case the non-reflexivity of hate) are expressed by matrices without free variables, otherwise called (formal) sentences or closed matrices.

By a system framed within the first order functional calculus is meant a set of sentences (not necessarily finite or even axiomatizable, though it will aid understanding to concentrate for the nonce on the case of axiomatizability). We give an example of a system:

1. (x) (y) (if x hates y, then y does not hate x).
2. (x) (y) (z) (if x hates y and y hates z, then x hates z).

These two statements together (we could have conjoined them into a single statement) assert (falsely if U means all people) that U is partially ordered by hatred. Now let us make the notational change of writing Hxy for 'x hates y'. We get

1′. (x) (y) (if Hxy, then not Hyx).
2′. (x) (y) (z) (if Hxy and Hyz, then Hxz).

Let us now consider the system consisting of these two sentences in abstraction from our interpretation of U as all people and H as hatred. We now have an uninterpreted formal system; it 'means' that the (unspecified) relation H is a partial ordering of U. Pending specification of H and U, it has no real meaning; it is *true of* some choices of H and U, *false of* others. That is, some relations are partial orderings of some classes, some not.

Hence we define an uninterpreted formal system framed within the first order functional calculus as a set of sentences exactly like those of an interpreted formal system except that U is unspecified and so are the predicate letters (as H in the present instance). To any such uninterpreted system we may assign a vastly infinite number of interpretations. An interpretation consists of a specification of U (i.e. a [partial] interpretation of the quantifiers) together with an interpretation of each predicate letter appearing. Thus a possible interpretation of the uninterpreted formal system (1′, 2′) is: U (i.e. the range of the variables) is people, H is hatred. This interpretation makes false each of 1′, 2′ and a fortiori their conjunction. On the other hand if we specify U as integers and H as 'less than' we get a true interpretation; we get the truism that the relation 'less than' partially orders the integers.

A true interpretation of a system is called a model of that system. Thus the relation 'less than' and the universe of discourse consisting of the integers forms a model of the uninterpreted system (1′, 2′) and in a derivative sense a model (this time a *re*interpretation) of the interpreted system (1, 2). A system is called syntactically inconsistent if a contradiction is formally derivable from it; semantically inconsistent if it has no model. It is a fundamental theorem of (classical) metamathematics that these two notions and hence the corresponding notions of consistency are equivalent. I take this opportunity of criticizing a remark of the previous speaker that for the conceptualist the Löwenheim-Skolem theorem was a trivial consequence of his conceptualism. There are four propositions to be considered:

A. Every syntactically consistent system possesses a model.
B. Every system which possesses a model possesses a denumerable model.
C. Every model is denumerable.
D. Every syntactically consistent system possesses a denumerable model.

The previous speaker maintained correctly that for the conceptualist the Löwenheim-Skolem theorem B was a trivial consequence of the conceptualistic dogma C. D is also a form of the theorem, which from the classical standpoint (which is expressed in A) is equivalent to B. But unless the conceptualist accepts A, he cannot get D out of B. But the only grounds that he can have for accepting A are the grounds on which we usually believe the Löwenheim-Skolem theorem itself. Hence only an uninteresting form of the Löwenheim-Skolem theorem is a trivial consequence of conceptualism; no reason has been offered by the speaker why the conceptualist should accept the interesting form D. But this is a digression.

The Löwenheim-Skolem theorem may now be stated precisely as follows. Let 'system' here and henceforth mean interpreted or uninterpreted formal system framed within the first order functional calculus. *If a system has a model it has a denumerable model, hence* (for a platonist) *if a system is syntactically consistent it has a denumerable model.* Uneasiness concerning this result may now be expressed more precisely than before: it seems that we cannot ensure by means of any formal system that any of the sets, or the fields of any of the relations, referred to in the (interpreted) system are non-denumerable. For there will always exist relations satisfying all the theorems of such systems, whose fields are included in a denumerable universe of discourse U. Hence there is an elementary mathematical notion which escapes formalization within the first order functional calculus. (Notice that the sense of 'escapes formalization' is here much more far-reaching than that in which, according to Gödel's theorem, the arithmetic of natural numbers escapes formalization. For here we place no restriction on the system from the point of view of axiomatizability or recursive enumerability.)

In order to understand the reasons why this is felt to be paradoxical, it will be illuminating to reflect on just what properties of relations *are* formalizable within the first order functional calculus. First we define this phrase more precisely.

A property of relations is said to be formalizable within the first order functional calculus if there exists a consistent system framed in that calculus and a predicate letter, say 'R', appearing in that system, such that in every model of that system the relation assigned as interpretation to 'R' has the given property. Thus the property of being a partial ordering of its field is formalizable in the first-order functional calculus while the property of having a non-denumerable field is not.

The problem of characterizing in a simple fashion the set of all properties of relations which are in this sense formalizable within the first order functional calculus is one which the literature barely touches upon in its generality. For our present purposes we concentrate upon those properties of relations which involve only the cardinality of their fields. As an example of a system restricting this cardinality consider the system having as its sole member the following sentence:

Not (y) (x) not (Hxy and Gx and not Gy)

This says that H relates a G to a non-G; hence its field must contain at least two members. Similarly we can put any desired finite lower bound on the cardinality of the field of a relation. Also by means with which you must be familar, we can compel H to have a field (at least denumerably) infinite. Finally, we can trivially by means of the axiom:

(x) (y) not Hxy

assure that H has an empty field.

The following conditions upon the cardinality of the field of a relation are thus expressible within the first order functional calculus:

A. The condition of being void.

B. For each finite integer n, the condition of having at least n members.

C. The condition of being infinite.

It can be shown that no conditions other than these can be imposed by systems formalized within the first order functional calculus upon the cardinality of the field of a relation. In particular we cannot impose in this manner the restriction of having an indenumerably infinite field, nor the condition of having a field of *at most* n elements for any positive integer n.

The same holds for restrictions on the cardinality of the universe U, except that for trivial technical reasons we cannot compel it to be void. For every consistent system S there exists a number n less than or equal to A_o (the number sometimes designated by the first letter of the Hebrew alphabet with an inferior 'o') such that S has models in all sizes greater than or equal to n. Thus we may place any desired finite or denumerably infinite *lower* bound on the size of U by means of an axiomatization in the first order functional calculus, but we cannot impose any upper bound nor a non-denumerable lower one.

It will bear on our problem if we now consider how we would "naturally" impose an upper bound on the size of U. Clearly the sentence:

4. Not (x) (y) not (z) (z is x or z is y)

where 'is' is taken in the sense of identity imposes the upper bound 2 on the size of U. Yet is not this couched within the first order functional calculus? For surely we could just as easily write it:

4′. Not (x) (y) and (z) (Izx or Izy)

But in this form, *where we do not insist on any particular interpretation of I,* we no longer impose any upper bound whatever on the size of U. It is only when we interpret I as identity that the intended effect is achieved. For instance 4′ has an infinite model, if we take U for example as the set of integers and I as congruence *modulo 2*.

We therefore see that if certain interpretations are specified in advance, we can say more about an unspecified relation H than we can if no interpretation is specified in advance. For instance if

4′ appears in some formal system along with other sentences containing 'H', and if I is interpreted as identity, then neither U nor a fortiori the field of H can contain more than two elements; but in the light of our previous discussion this restriction could not have been put upon H if the interpretation of I had been left free.

We are thus led to the conception of a *standard model*, which we define as follows. Let there be given a system S formalized within the first-order functional calculus, and let there be given preassigned interpretations to certain of the predicate letters appearing therein, and possibly also to U. Then by an (arbitrary) model will be meant any interpretation of U and of the predicate letters appearing in S which makes all the sentences in S come out true, regardless of whether or not the interpretations coincide with the preassigned ones. Those models which accord with the preassigned interpretation are called standard (relative to that preassigned interpretation); the others non-standard. Thus we can say that, relative to the interpretation of I as identity, no standard model of a system containing 4′ has more than two elements in its U, but that non-standard models of arbitrarily great cardinality exist.

We can now deal formally with the so-called 'Skolem paradox'. Let the interpretation 'x is a member of y' be given to the Greek letter epsilon. Then by the Löwenheim-Skolem theorem, denumerable models for set-theory exist; that is, there exist relations having all the formal properties assigned to class-membership by the axioms of (any consistent) set-theory, and also having denumerable fields. But *none of these relations is class-membership;* for class-membership certainly has a vastly non-denumerable field. Hence all the Skolem-Löwenheim models of set-theory are non-standard, relative to the given interpretation. Indeed there is evidently only one standard model of set-theory, because the predicate-letter epsilon, the only one which appears, is already preassigned on interpretation.

The insight we get into the relation between formalism and its object is therefore as follows: Formalism can determine its object with varying degrees of specificity. The more determination we bring to the interpretation of the formalism, the more formalism will determine its object. Thus any finite relation can be determined to within an isomorphism by a system in which a

symbol for identity occurs, provided this symbol is antecedently given a standard interpretation: but if the interpretation of I is left free, the other relations of the system are determined only to within a 'homomorphism'.

This remark sounds trivial, however, if we take the limiting case that *all* the symbols have been preassigned on interpretation, as is in particular the case with the set-theoretic situation which prompted the original anxiety. For then it seems to say only that if we fix the interpretation of epsilon, the set-theory has but one model, namely the one which interprets epsilon in the way we fixed it, and U as the field of epsilon thus interpreted; and that this model is indenumerable. But then it seems after all that the indenumerability arose in the (informal, external) interpretation and not in the formalism itself; so that after all there is a good sense in which formalism is inadequate to express indenumerability.

This difficulty can I think be resolved by distinguishing a private and a public aspect of formalism. Formalism in its private aspect is a computational device for avoiding 'raw thought'—we operate with symbols which keep their shape rather than with ideas which fly away from us. All real mathematics is made with ideas, but the formalism is always ready in case we grow afraid of the shifting vastness of our creations. Ultimately formalism in its private aspect is an expression of fear. But fear can lend us wings and armor, and formalism can penetrate where intuition falters, leading her to places where she can again come into her own. The Skolem 'paradox' thus proclaims our need never to forget completely our intuitions. We could shift to a formalism indistinguishable from set-theory and it could be something other than set-theory. It only remains set-theory as long as the intuition of membership has not slipped away from us. It could be formally the same and have a grotesquely different meaning. The astonishing thing is perhaps less the Skolem "paradox" that formalism apart from prior interpretation does not completely determine its object, than the fact that an uninterpreted formalism can determine its object at all. At least, even if our intuition of membership perishes entirely, we can rely on set-theory not to turn overnight into the theory of some finite group, even though we cannot guarantee that it will not turn into a theory about some complicated arithmetical relation.

If it did it would look the same in print, though the motivation would puzzle the reader. This brings us to the public aspect of formalism. In those cases (e.g. the setting of a lower bound on the cardinality of the universe) where formalism adequately delimits its object, we may consider that communication has been established. (But really even this is a matter of degree; for we presuppose a standard interpretation of truth-functional connectives even in arbitrary models, and perhaps a good deal more.) But (ignoring this point) there seem to be no *formal* means of assuring that our conception of membership any more than our perception of a particular sense-quality is the same as another person's. For no finite or even infinite number of formal assertions agreed on by us both could be evidence that his set-theory was not in my sense denumerable. Of course it would not be denumerable in his sense, but I would not know if he meant by 'denumerable' what I meant by 'denumerable' unless I knew that he meant the same as I meant by 'membership'. The second philosophical lesson of the Löwenheim-Skolem theorem is that the formal communication of mathematics presupposes an informal community of understanding.

A formalism constrains and limits its objects in certain ways. If we preassign an interpretation to some of these objects, we thereby restrict the interpretation of the rest. A formalism which is *completely* uninterpreted (i.e. even in regard to truth-functional connectives, the part of the meaning of quantifiers which is independent of the specification of U, and the juxtaposition of symbols) imposes *no* restriction on its object. If we fix these three interpretations, there is still room for some latitude. In particular, a formalism interpreted only in these respects cannot force the interpretation of any of its predicate-letters as a relation with a non-denumerable field. Such is the burden of the Löwenheim-Skolem theorem.

The constraint exerted by a formalism on its objects is therefore a resultant of the formalism itself and the preassigned interpretation of its symbols. The theorem places before us in a striking fashion the role of the second factor.

Symposium:

ARE RELIGIOUS DOGMAS COGNITIVE AND MEANINGFUL?

RAPHAEL DEMOS AND C. J. DUCASSE

I. RAPHAEL DEMOS

Although in this paper I am solely concerned with the cognitive elements of religion, I do not of course assume that cognition is all that is important in religion. In this paper, by religion I will mean chiefly the Christian religion; this is the one I know best by far and it is the one in whose truth I believe. I will first discuss religious belief and then I will explore religious meaning. My study of religious cognition will also involve extended digressions into general epistemology.

We may tentatively distinguish the following systems of belief: common sense, science, animism, religion and philosophy. In this scheme common sense stands to science as animism to religion, the first member of the pair representing a relatively undeveloped version of the second. Later on, I will make a similar distinction of philosophical levels.

It is generally taken for granted that the appeal to faith is a uniquely distinguishing feature of religious belief. Certainly religious thinkers do not hesitate to declare that faith is a valid source of belief in religion; thus, the author of the Epistle to the Hebrews speaks of faith as the 'evidence' of things unseen. In this passage faith means belief not resting on the evidence of the senses; but the word has for me a wider meaning; namely, as belief which rests on no evidence whatever, whether empirical or a priori. Of the other systems of belief it is popularly assumed that scientific beliefs rest on empirical evidence exclusively; and that so do those of common sense, although with a lesser degree of firmness. It is also believed that philosophers at any rate *intend*