Inductive Probability *&* Inductive Support

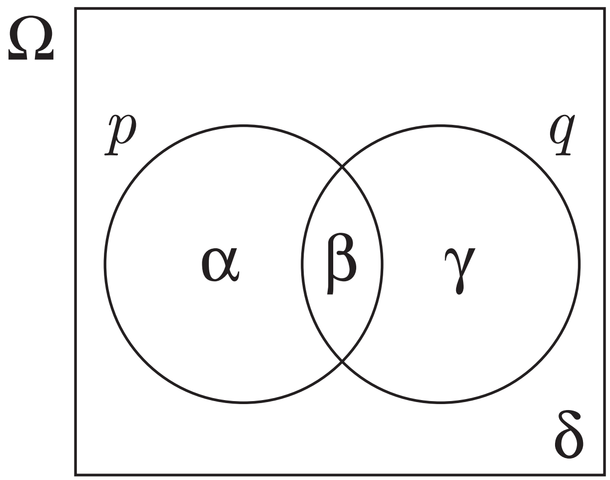
- Administrative: new office hours, new students (cards)?, pictures?, please consult website (or see me) for syllabus, etc.
- Review of basics of probability theory (and Venn diagrams)
- An example to illustrate probability reasoning w/Venn diagrams
- What *is* inductive probability?
- Skyrms' account of inductive strength — scrutinized
- Applications of inductive probability (*segue* to confirmation)

The Probability Calculus (Review)

- We can think of the inductive ("logical") probability of a claim p as (roughly) the proportion of possible worlds in which p is true.
- This leads naturally to thinking of inductive probabilities as (relative) *areas* of "claim regions" in Venn diagrams.
- In a Venn diagram, the outer "box" (Ω) represents the universe of discourse, or the *reference class.* The probability of Ω is 1 because Ω contains *all* of the possible worlds in the reference class for $\Pr(\cdot)$.
- Thinking of $\Pr(\cdot)$ in this way yields *exactly* the concept of probability that Skyrms discusses in chapter 6 (*i.e.*, his 6 rules).
- As an exercise, you should try to *prove* that the Venn diagram model of probability satisfies all of Skyrms' 6 rules in chapter 6.

Venn Diagrams & The Probability Calculus

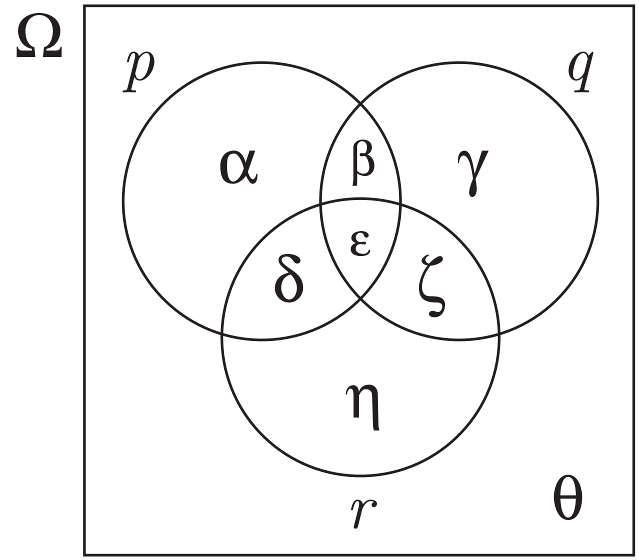
2-Claim Venn Diagram



$2^2 = 4$ "basic" propositions:

$$\Pr(p\&{\sim}q) = \alpha$$
$$\Pr(p\&q) = \beta$$
$$\Pr({\sim}p\&q) = \gamma$$
$$\Pr({\sim}p\&{\sim}q) = \delta$$

3-Claim Venn Diagram



$2^3 = 8$ "basic" propositions:

$$\Pr(p\&{\sim}q\&{\sim}r) = \alpha \qquad \Pr(p\&q\&{\sim}r) = \beta$$
$$\Pr({\sim}p\&q\&{\sim}r) = \gamma \qquad \Pr(p\&{\sim}q\&r) = \delta$$
$$\Pr(p\&q\&r) = \varepsilon \qquad \Pr({\sim}p\&q\&r) = \zeta$$
$$\Pr({\sim}p\&{\sim}q\&r) = \eta \qquad \Pr({\sim}p\&{\sim}q\&{\sim}r) = \theta$$

- Circles represent sets of possible worlds in which claims are true.
- Ω is set of possible worlds with respect to which Pr is defined.
- Pr(Ω) = 1 (total area of the reference class 'box' Ω is 1)

Conditional Probabilities

- To calculate $\Pr(p \textit{ given } q)$, we treat q *as if* it were the "new" reference class. That is, we "conditionalize" the function $\Pr(\cdot)$ on q.
- That is, to calculate $\Pr(p \textit{ given } q)$, we ask ourselves the following question: "What is the proportion of q-worlds that are p-worlds?"
- Looking at our Venn diagram, we can see that the proportion of q-worlds that are p-worlds is given by the following *ratio:*

$$\frac{\text{`area' of } p \,\&\, q\text{-worlds}}{\text{`area' of } q\text{-worlds}} = \frac{\beta}{\beta + \gamma}$$

- This leads to our definition of $\Pr(p \textit{ given } q)$ (Skyrms' Def. 12):

$$\Pr(p \textit{ given } q) =_{df} \frac{\Pr(p \,\&\, q)}{\Pr(q)}$$

- NOTE: on this def., $\Pr(p \textit{ given } q)$ is *undefined* if $\Pr(q) = 0$.

Probabilistic (Stochastic) Independence

- Probabilistic (*a.k.a.*, stochastic) independence is a relation between claims or propositions. We abbreviate this relation using the symbol $\perp$. The relation $p \perp q$ is defined (by Skyrms) as follows:
  - $p \perp q$ iff $\Pr(p \textit{ given } q) = \Pr(p)$.[a]
- With Skyrms' caveat (p. 121, see footnote), this is equivalent to:
  - $p \perp q$ iff $\Pr(p \,\&\, q) = \Pr(p) \cdot \Pr(q)$ [use def. of $\Pr(p \textit{ given } q)$]
- The intuition behind this definition is (roughly) that *conditionalizing on q has no effect on the probability of p*.
- In this sense, if $p \perp q$, then q is *irrelevant* to p (and *vice versa*, because $\perp$ is a *symmetric* relation! Can you prove this?).
- The $\perp$ relation captures a kind of (ir)relevance, which is *crucial* for our discussions of induction, confirmation, and explanation.

---

[a] What if $\Pr(q) = 0$? Skyrms, page 121, says $p \perp q$ in this case! See paper topics.

Reasoning About Probabilities: An Example

- Let q be the proposition that a card drawn at random from a standard deck is not a face card, and p = 'the card is a ♠.'
- Here, Ω is the usual reference class for standard (well-shuffled) decks of playing cards (52 cards, each equiprobable, *etc.*).
- What are the following four (basic) probabilities?
  - $\Pr(p \,\&\, {\sim}q)$ (*i.e.*, α in our p-q Venn diagram)
  - $\Pr(p \,\&\, q)$ (*i.e.*, β in our p-q Venn diagram)
  - $\Pr({\sim}p \,\&\, q)$ (*i.e.*, γ in our p-q Venn diagram)
  - $\Pr({\sim}p \,\&\, {\sim}q)$ (*i.e.*, δ in our p-q Venn diagram)
- From these, we can calculate ***ANY*** probability involving p and q.
- Are p and q independent? What are Pr(p *given* q), Pr(q *given* p), Pr(p), and Pr(q)? Is the argument $\frac{p}{\therefore\, q}$ *strong* (in Skyrms' sense)?

What *is* (Inductive) Probability? I

- Skyrms (pp. 26–28) seems skeptical about the prospects for an objective account of inductive probability and inductive logic.
- He laments that "There are no universally accepted rules for constructing inductively strong arguments; no general agreement on a way of measuring the inductive strength of arguments; no precise, uncontroversial definition of inductive probability."
- Naively, we might try thinking of inductive probability as a quantitative generalization (or measure) of deductive (logical) necessity (or modality). But, this leads to the following problem(s):
- Can we discover (*a priori*?) what the "*logical probabilities*" are? If Ω is the set of logical truths, then it is not clear what the values of $\Pr(\cdot)$ should be (except for the logical truths and logical falsehoods, the probabilities of which are 'given' by pure deductive intuition).

What *is* (Inductive) Probability? II

- We do seem to have pretty strong (*a priori*?) intuitions about what kinds of propositions are logically *impossible* (or *necessary*).
- But, when we move to *quantitative* judgments of "logical *probability*," our intuitions seem to be much more shaky.
- There are further subtleties. Claims that are *impossible* are impossible *given any other claim(s)*. That is: if p is impossible, then p is impossible *given* q — for *any* q. Not so for *improbability*!
- For, no matter how low $\Pr(p \text{ given } \Omega)$ is, $\Pr(p \text{ given } \Omega \,\&\, q)$ can be *arbitrarily high*, for appropriate choice of q (*e.g.*, $q = p$).
- That is, judgments about (im)probabilities will depend very sensitively on what we take to be part of the "background" (or the "reference class"). (Im)rpobability seems *indexical* or *contextual* in a way that (im)possibility is not. This makes things more difficult.

Back to Skyrms on Inductive Strength

- With Pr(p *given* q) and $p \perp q$ under our belts, we can now return (intelligently) to Skyrms' discussion of inductive strength.
- First, we can now state Skyrms' definition more precisely:
  - An argument $\frac{\mathbf{P}}{\therefore q}$ is inductively strong if Pr($\sim q$ *given* $\mathbf{P}$) is low.
- It should be clear why this is *not* equivalent to "Pr($\sim q \,\&\, \mathbf{P}$) is low". The first paper topics require a careful reconstruction of the first example Skyrms uses (page 20) to illustrate this non-equivalence.
- **Hints**: In Skyrms' first example, $\sim q \,\&\, \mathbf{P}$ is improbable *merely because* $\mathbf{P}$ is improbable. He claims that $\mathbf{P}$ need not be 'evidentially relevant' in such cases. Thus, he argues, the argument from $\mathbf{P}$ to q need not be strong. Does $\mathbf{P} \perp q$ hold in his example? The fact that "If $p \vDash q$, then $\Pr(p) \leq \Pr(q)$" is *crucial* here (*why?*).

Skyrms' Second Example: A Formal Reconstruction

- Skyrms' second counterexample (page 21) to the "$\sim q \,\&\, \mathbf{P}$ is improbable" account of inductive strength is as follows:

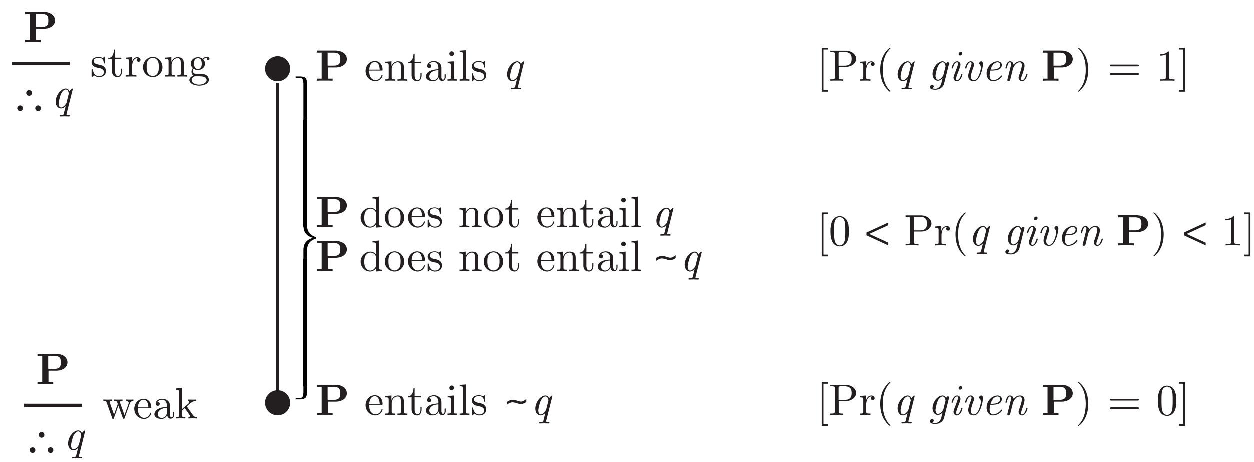
  (p) There is a man in Celeveland who is 1999.99 y.o. and in good health.

  (q) $\therefore$ No man will live to be 2000 years old.

- Assuming the reference class Ω consists of the propositions in our store of background knowledge concerning the life span of human beings, Skyrms argues (plausibly) that the following probabilistic facts obtain:
  - $\Pr(q) = \Pr(q \textit{ given } \Omega)$ is *high*. Therefore, $\Pr(\sim q) = 1 - \Pr(q)$ is *low*.
  - Hence, $\Pr(\sim q \,\&\, p)$ is *also* low [If $p \vDash q$, then $\Pr(p) \leq \Pr(q)$!].
  - Thus, the conjunction $\sim q \,\&\, p$ is *improbable*.
  - But, this argument is ***NOT*** strong, since p is strong evidence *against* q. We have a *counterexample* to the "$\sim q \,\&\, p$ is improbable" account.
- Does *Skyrms'* account (necessarily) give the *right* answer here?

What Do We *Want* From a Measure of Inductive Strength?

- On page 22, Skyrms gives (something like) the following diagram:



- We seek a measure $s(q, \mathbf{P})$ of the strength of $\dfrac{\mathbf{P}}{\therefore q}$ such that (*at least*):
  1. If $\mathbf{P} \vDash q$, then $s(q, \mathbf{P})$ is *maximal*.
  2. If $\mathbf{P} \nvDash q$ and $\mathbf{P} \nvDash \sim q$, then $s(q, \mathbf{P})$ is *intermediate*.
  3. If $\mathbf{P} \vDash \sim q$, then $s(q, \mathbf{P})$ is *minimal*.
- Skyrms' measure $s(q, \mathbf{P}) = \Pr(q \textit{ given } \mathbf{P}) = 1 - \Pr(\sim q \textit{ given } \mathbf{P})$ satisfies 1–3. Does $1 - \Pr(\sim q \,\&\, \mathbf{P})$? What about "*relevance*" of $\mathbf{P}$ to q?

Independence, Relevance, and Inductive Strength

- Measures satisfying properties 1–3 on the previous slide have the virtue of capturing *deductive* relations as *special* (or *limiting*) *cases.*
- In this sense, $\Pr(q \textit{ given } \mathbf{P})$ is *more* sensitive than $\Pr({\sim}q \,\&\, \mathbf{P})$ to 'evidential relations' (*e.g.*, *deductive* ones) between $\mathbf{P}$ and q.
- But, what about the relation of *probabilistic relevance* (*i.e.*, $\not\perp$)?
- Skyrms' complaint about the "${\sim}q \,\&\, \mathbf{P}$ is improbable" account of inductive strength is that it does not adequately gauge the '*evidential relevance*' of $\mathbf{P}$ to q (not even the *deductive* relevance).
- However, even $\Pr(q \textit{ given } \mathbf{P})$ does *not* adequately gauge the *probabilistic* (*a.k.a.*, *stochastic*) relevance relation between $\mathbf{P}$ and q.
- Example: $p =$ "Fred Fox has been (properly) taking birth control pills for 2 years," $q =$ "Fred Fox is not pregnant." Is the argument from p to q a strong one (intuitively)? Is $\Pr({\sim}q \textit{ given } p)$ low?

‘Relevance’ in the *Deductive* Support Relation

- Skyrms’ complaint about the “$\sim q \,\&\, \mathbf{P}$ is improbable” account of inductive strength is (roughly) that $\sim q \,\&\, \mathbf{P}$ can be improbable *even if (intuitively)* $\mathbf{P}$ *has “nothing to do with”* q.
- Put another way, Skyrms’ complaint seems to be that $\sim q \,\&\, \mathbf{P}$ can be improbable *merely because* $\mathbf{P}$ (or $\sim q$) *by itself is improbable* — regardless of the *relationship* (or lack thereof) between $\mathbf{P}$ and q.
- Some philosophers of logic have had similar complaints about the “$\sim q \,\&\, \mathbf{P}$ is impossible” account of (classical) *deductive* support.
- Such philosophers point out the (intuitive) “irrelevance” of the premises and conclusions in the following *valid* argument forms:

$$\frac{p \,\&\, \sim p}{\therefore\, q} \qquad \frac{p}{\therefore\, q \vee \sim q}$$

- Why not move to something like “$\sim q$ *given* $\mathbf{P}$ is impossible”?

Skyrms' Chapter 8: Applications (*segue* to confirmation)

- In chapter 8, Skyrms starts talking about applications of inductive logic to philosophy of science (basically, to "confirmation").
- How does Skyrms suggest (page 152) we should capture Popper's relation of "corroboration" — using inductive probability?
- How does Skyrms unpack the comparative relation: "p is better evidence for q than r is for s" in chapter 8?
- Are these concepts (*i.e.*, "corroborative evidence" and "better evidence") already implicit in his definition of inductive strength?
- If not, might this be a *weakness* of his account of inductive strength? Can we give problematic *examples* here (Fred Fox)?
- Can you think of alternative ways to define inductive strength that might overcome these weaknesses (*i.e.*, that might capture all of these notions under the single umbrella of "inductive strength")?