Some Final Thoughts (for now) on Skyrms

- Administrative: in the next week or two, you should begin thinking about your topics for the first short paper (see me)
- Re-cap of Worked Probability Example from Last Time
- Re-cap of Skyrms's Account of Inductive Strength
  - Understanding Skyrms' Rejection of "$\sim q \,\&\, p$" Account
  - Virtues of Skyrms' "$\sim q$ *given* p" Account
  - Vices of Skyrms' "$\sim q$ *given* p" Account
- Moving Beyond Skyrms' Account of Inductive Strength
- Segue Into Bayesian Confirmation (Skyrms Chapter 8)
- Next Time: Salmon *&* Earman on Confirmation (Reader)

Reasoning About Probabilities: A Worked Example

- Let q = 'a card drawn at random from a standard deck is *not* a face card', and p = 'the card is a spade.' We assume Ω is the usual reference class for standard (well-shuffled) decks of playing cards.

- The four basic probabilities regarding p and q are:
  - $\Pr(p \,\&\, {\sim}q) = \alpha = \frac{\text{\# of face spades}}{\text{total \# of cards}} = \frac{3}{52}$
  - $\Pr(p \,\&\, q) = \beta = \frac{\text{\# of non-face spades}}{\text{total \# of cards}} = \frac{10}{52}$
  - $\Pr({\sim}p \,\&\, q) = \gamma = \frac{\text{\# of non-face non-spades}}{\text{total \# of cards}} = \frac{30}{52}$
  - $\Pr({\sim}p \,\&\, {\sim}q) = \delta = \frac{\text{\# of face non-spades}}{\text{total \# of cards}} = \frac{9}{52}$

$\Pr(p) = \alpha + \beta = \frac{13}{52}$	$\Pr(q) = \beta + \gamma = \frac{40}{52} = \frac{10}{13}$
$\Pr(p \mid q) = \frac{\Pr(p\&q)}{\Pr(q)} = \frac{10/52}{10/13} = \frac{13}{52}$	$\Pr(q \mid p) = \frac{\Pr(p\&q)}{\Pr(p)} = \frac{10/52}{13/52} = \frac{10}{13}$
$\Pr(p \mid q) = \Pr(p)$, $\therefore\ p \perp q$	$\Pr(q \mid p) = \Pr(q)$, $\therefore\ q \perp p$
$\Pr(q \mid p)$ is *high* (≈ 0.77). But, is $\frac{p}{\therefore\ q}$ (intuitively) a *strong argument*?	

Re-capup of Skyrms on Inductive Strength I

- Skyrms (pp. 20–21) gives two examples, *both* of which show that:

$$\Pr({\sim}q \,\&\, \mathbf{P}) \text{ is low} \not\Rightarrow \frac{\mathbf{P}}{\therefore\, q} \text{ is inductively strong.}$$

- In Skyrms' first example (p. 20), $\sim q \,\&\, \mathbf{P}$ is improbable *merely because* **P** — by itself — is improbable. Skyrms correctly points out that **P** *need not* be '*evidentially relevant*' to q in such cases.

- **Question**: Does $\mathbf{P} \perp q$ hold in Skyrms' first example? Use your answer to this question to say something about whether **P** *is* 'evidentially relevant' to q in Skyrms' *particular* example on page 20. **Hint**: "If $p \vDash q$, then $\Pr(p) \leq \Pr(q)$" is *crucial* here (*why?*).

- **New Paper Topic**: Give a compelling demonstration that:

$$\Pr({\sim}q \,\&\, \mathbf{P}) \text{ is low} \not\Leftarrow \frac{\mathbf{P}}{\therefore\, q} \text{ is inductively strong.}$$

Re-cap of Skyrms on Inductive Strength II

- Here is Skyrms' second counterexample (page 21) to the claim that:

$$\Pr({\sim}q \,\&\, \mathbf{P}) \text{ is low} \Longrightarrow \frac{\mathbf{P}}{\therefore q} \text{ is inductively strong}$$

(p) There is a man in Celeveland who is 1999.99 y.o. and in good health.

(q) $\therefore$ No man will live to be 2000 years old.

- Assuming the reference class Ω consists of the propositions in our store of background knowledge concerning the life span of human beings, Skyrms argues (plausibly) that the following probabilistic facts obtain:
  - $\Pr(q) = \Pr(q \mid \Omega)$ is *high*. Therefore, $\Pr({\sim}q) = 1 - \Pr(q)$ is *low*.
  - Hence, $\Pr({\sim}q \,\&\, p)$ is *also* low [If $p \vDash q$, then $\Pr(p) \leq \Pr(q)$!].
  - But, this argument is ***NOT*** inductively strong, since p is evidence *against* q. Thus, $\Pr({\sim}q \,\&\, \mathbf{P})$ is low $\not\Rightarrow \frac{\mathbf{P}}{\therefore q}$ is inductively strong. *QED*.
- **Question**: Does *Skyrms'* account (*necessarily*) give the *right* answer?

Rethinking *Skyrms*' Account of Inductive Strength I

- First, let's state Skyrms' definition more precisely:
  - An argument $\dfrac{\mathbf{P}}{\therefore q}$ is inductively strong iff $\Pr({\sim}q \mid \mathbf{P})$ is low.
- I find the following *equivalent* definition more perspicuous:
  - An argument $\dfrac{\mathbf{P}}{\therefore q}$ is inductively strong iff $\Pr(q \mid \mathbf{P})$ is high.
- It should be clear why *neither* of these is equivalent to "$\Pr({\sim}q \,\&\, \mathbf{P})$ is low." This is clear from the definition of conditional probability:

$$\Pr({\sim}q \mid \mathbf{P}) = \frac{\Pr({\sim}q \,\&\, \mathbf{P})}{\Pr(p)} \neq \Pr({\sim}q \,\&\, \mathbf{P}) \quad [\text{unless } \Pr(p) = 1]$$

- The "${\sim}q \,\&\, \mathbf{P}$ is improbable" proposal does *not* properly generalize the *deductive* notion of support. This is surprising, since "${\sim}q \,\&\, \mathbf{P}$ is *improbable*" is the natural inductive weakening of "${\sim}q \,\&\, \mathbf{P}$ is *impossible*". On this score, Skyrms' account is superior ...

Rethinking *Skyrms*' Account of Inductive Strength II

- On page 22, Skyrms gives (something like) the following diagram:

$\dfrac{\mathbf{P}}{\therefore q}$ strong	$\mathbf{P}$ entails q	[$\Pr(q \textit{ given } \mathbf{P}) = 1$]
	$\mathbf{P}$ does not entail q $\mathbf{P}$ does not entail $\sim q$	[$0 < \Pr(q \textit{ given } \mathbf{P}) < 1$]
$\dfrac{\mathbf{P}}{\therefore q}$ weak	$\mathbf{P}$ entails $\sim q$	[$\Pr(q \textit{ given } \mathbf{P}) = 0$]

- We seek a measure $s(q, \mathbf{P})$ of the strength of $\dfrac{\mathbf{P}}{\therefore q}$ such that (*at least*):
  1. If $\mathbf{P} \vDash q$, then $s(q, \mathbf{P})$ is *maximal*.
  2. If $\mathbf{P} \nvDash q$ and $\mathbf{P} \nvDash \sim q$, then $s(q, \mathbf{P})$ is *intermediate*.
  3. If $\mathbf{P} \vDash \sim q$, then $s(q, \mathbf{P})$ is *minimal*.
- Skyrms' measure $s(q, \mathbf{P}) = \Pr(q \mid \mathbf{P}) = 1 - \Pr(\sim q \mid \mathbf{P})$ satisfies 1–3. Does "$1 - \Pr(\sim q \,\&\, \mathbf{P})$"? What about the "*relevance*" of $\mathbf{P}$ to q?

Rethinking *Skyrms*' Account of Inductive Strength III

- Measures satisfying properties 1–3 on the previous slide have the virtue of capturing *deductive* relations as *limiting cases*.
- In this sense, $\Pr(q \mid \mathbf{P})$ is *more* sensitive than $\Pr({\sim}q \,\&\, \mathbf{P})$ to 'evidential relations' (at least, *deductive* ones) between $\mathbf{P}$ and q.
- But, what about the relation of *probabilistic relevance* (*i.e.*, $\not\perp$)?
- As we have seen, even $\Pr(q \mid \mathbf{P})$ does *not* adequately gauge the *probabilistic* (*a.k.a.*, *stochastic*) relevance relation between $\mathbf{P}$ and q.
- Perhaps we should think of Skyrms' proposed measure of the degree to which p supports q — $\Pr(q \mid p)$ — as *merely* a measure of the "degree to which p *deductively* supports q". This makes sense, given the way we define/interpret conditional probability, no?
- **Another Example**: $p =$ "Fred Fox has been (properly) taking birth control pills for 2 years," $q =$ "Fred Fox is not pregnant." Is the argument from p to q a strong one (intuitively)? Is $\Pr({\sim}q \mid p)$ low?

'Relevance' in the *Deductive* Support Relation

- Skyrms' complaint about the "$\sim q \,\&\, \mathbf{P}$ is improbable" account of inductive strength is (roughly) that $\sim q \,\&\, \mathbf{P}$ can be improbable *even if (intuitively)* $\mathbf{P}$ *has "nothing to do with"* q.
- Some philosophers of logic have had similar complaints about the "$\sim q \,\&\, \mathbf{P}$ is *impossible*" account of (classical) *deductive* support.
- Such 'relevant' logicians point out the (intuitive) "irrelevance" of the premises and conclusions in the following *valid* arguments:

$$\frac{p \,\&\, \sim p}{\therefore\, q} \qquad\qquad \frac{p}{\therefore\, q \vee \sim q}$$

- How does Skyrms' measure of strength judge these arguments?
- Perhaps we want *more* from a measure of *inductive* strength than *merely* a gauge "partial entailment" . . . perhaps we also want sensitivity to *other (inductive!) kinds* of *evidential relevance* . . .

Relevance Measures of Inductive Support

- We seek a measure $s(q, p)$ of the degree to which p supports q such that:
  1. s captures the *deductive* relations as *limiting cases* (previous slide),
  2. s is *also* sensitive to *probabilistic relevance*.
- What (2) says is that we want a measure $s(q, p)$ which is *positive* if p *raises* the probability of q, *negative* if p *lowers* the probability of q, and *zero* if $p \perp q$. That is, we want $s(q, p)$ to be a *relevance measure*:

$$s(q,p) \begin{cases} > 0 & \text{if } \Pr(q \mid p) > \Pr(q), \\ < 0 & \text{if } \Pr(q \mid p) < \Pr(q), \\ = 0 & \text{if } \Pr(q \mid p) = \Pr(q) \quad [\textit{i.e.}, \text{ if } p \perp q]. \end{cases}$$

- We know that Skyrms' measure $c(q, p) = \Pr(q \mid p)$ satisfies (1) but *not* (2).
- **Exercises**: Show that $d(q, p) = \Pr(q \mid p) - \Pr(q)$ satisfies (2) but *not* (1). Show that $l(q, p) = \frac{\Pr(p|q) - \Pr(p|{\sim}q)}{\Pr(p|q) + \Pr(p|{\sim}q)}$ satisfies *both* (1) *and* (2)! How does $l(q, p)$ judge the "irrelevant" deductive arguments on the previous slide?

Skyrms' Chapter 8: Applications (*segue* to confirmation)

- In chapter 8, Skyrms starts talking about applications of inductive logic to philosophy of science (basically, to "confirmation").
- How does Skyrms suggest (page 152) we should capture Popper's relation of "corroboration" — using inductive probability?
- How does Skyrms unpack the comparative relation: "p is better evidence for q than r is for s" in chapter 8?
- Are these concepts (*i.e.*, "corroborative evidence" and "better evidence") already implicit in his definition of inductive strength?
- If not, might this be a *weakness* of his account of inductive strength? Can we give problematic *examples* here (Fred Fox)?
- Can you think of alternative ways to define inductive strength that might overcome these weaknesses (*i.e.*, that might capture all of these notions under the single umbrella of "inductive strength")?