

Bayesian Confirmation Theory

- Administrative: Almost everyone has either seen me or scheduled an appointment with me to discuss paper topics (please do) ...
- Bayesian Confirmation Theory
 - Basic Assumptions of Bayesian Epistemology
 - * Degrees of belief of rational agents are *probabilities*
 - * Rational degrees of belief are updated *via conditionalization*
 - * ‘Coherence’ (probabilism) is *only* constraint on rationality
 - Two Kinds of (Qualitative) Bayesian Confirmation
 - Quantitative Bayesian Confirmation — Measures of Support
 - Some Problems for Bayesian Confirmation Theory
 - * “Old evidence”, “new theories”, and Bayesian confirmation
 - Time Permitting: Some Success Stories for Bayesianism

Bayesian Epistemology I

- An epistemically rational agent's degrees of belief (or degrees of credence) are *probabilities* (in the technical sense).
- An (epistemically) rational agent learns by *conditionalizing* on their (total) evidence (up to that point in time).
- That is, at any given time t , a rational Bayesian agent a has degrees of belief that conform to a probability function $\text{Pr}_t^a(\cdot)$.
- And, if a rational Bayesian agent a learns E between t_1 and t_2 , then their degrees of belief at t_2 are given by $\text{Pr}_{t_2}^a(\cdot) = \text{Pr}_{t_1}^a(\cdot | E)$.
- Moreover, it is assumed that a Bayesian agent (scientist) has a *complete* probability distribution over *all* competing hypotheses.
- That is, a Bayesian agent will have precise probabilities – both *prior* probabilities $\text{Pr}(H)$ and *posterior* probabilities $\text{Pr}(H | E)$.

Bayesian Epistemology II

- The “priors” are really conditionalized on an agent’s *background knowledge* K (here, K plays the role of Ω in the formal theory).
- The background knowledge (or, contextual assumptions) K plays a crucial role. Altering K can change *all* probabilistic judgments.
- Bayesian epistemology does *not* require that degrees of belief “correspond” to the “objective” probabilities (if there be such!).
- All that is required is *coherence* (that is, conformity to the probability axioms, and to the rule of conditionalization).
- Here is a classic quote from De Finetti on this issue:

By denying any objective value to probability I mean to say that, however an individual evaluates the probability of a particular event, no experience can prove him right, or wrong; nor, in general, could any conceivable criterion give any objective sense to the distinction one would like to draw, here, between right and wrong.

Bayesian Confirmation I

- Historically, there have been two kinds of Bayesian confirmation:
 1. **Absolute:** E confirms H (relative to K) if $\Pr(H \mid E \ \& \ K) > \tau$, for some “threshold value” τ (*i.e.*, if $\frac{E \ \& \ K}{\therefore H}$ is “*inductively strong*”).
 2. **Incremental:** E confirms H (rel. to K) if $\Pr(H \mid E \ \& \ K) > \Pr(H \mid K)$ (*i.e.*, if E is *positively stochastically relevant* to H , given K).

Incremental confirmation has become more popular in recent years. It will be the main focus of our discussion of Bayesian confirmation.

- Absolute confirmation is just Skyrms’ account of “inductive strength”.
- Incremental confirmation is probabilistic *relevance*. In this sense, it is more like the notion of “corroborative evidence” in Skyrms’ chapter 8.
- These accounts differ dramatically (as we’ve seen in our discussion of Skyrms). Both accounts also differ greatly from *deductive* accounts.

Bayesian Confirmation II

- Some differences between the two Bayesian accounts:
 - Absolute confirmation is *insensitive to relevance* (Fred Fox, etc.).
 - Absolute confirmation is *asymmetric*, incremental is *symmetric* (here, symmetry means X confirms $Y \implies Y$ confirms X).
 - Absolute confirmation is the *only* account we've seen with the following property: E confirms $H \not\Rightarrow \sim E$ does *not* confirm H .
 - Absolute account satisfies (SCC), incremental account does not.
- Some differences between Bayesian and deductive account:
 - Incremental account satisfies *neither* (SCC) *nor* (CCC).
 - *Neither* Bayesian account satisfies the *consistency condition* (CC).
 - *Both* bayesian accounts easily admit of *degrees*.
 - *Both* Bayesian accounts make sense in *statistical* settings.

Bayesian Confirmation III

- The incremental account will be our main focus. [It seems to me that any account which eschews *relevance* cannot be adequate.]
- So far, the incremental theory just gives us a *qualitative* account of confirmation (analogous to the previous, *deductive* accounts).
 - E confirms H (relative to K) if $\Pr(H \mid E \ \& \ K) > \Pr(H \mid K)$
 - E disconfirms H (relative to K) if $\Pr(H \mid E \ \& \ K) < \Pr(H \mid K)$
 - E is *irrelevant to* H (rel. to K) if $\Pr(H \mid E \ \& \ K) = \Pr(H \mid K)$
- The basic idea behind the incremental account is that *confirmation is probabilistic relevance*. How can we *quantitatively* generalize this?
- Each of these says “ E incrementally confirms H relative to K ”:
 - $\Pr(H \mid E \ \& \ K) > \Pr(H \mid K)$
 - $\Pr(H \ \& \ E \mid K) > \Pr(H \mid K) \cdot \Pr(E \mid K)$
 - $\Pr(E \mid H \ \& \ K) > \Pr(E \mid \bar{H} \ \& \ K)$

Bayesian Confirmation IV

- *Prima Facie*, any of these inequalities (or any other equivalent inequality!) could be used to generate a (initially plausible) *quantitative measure of degree* of (incremental) confirmation.
- For instance, we might adopt one of the following three measures:

$$d(H, E | K) =_{df} \Pr(H | E \& K) - \Pr(H | K)$$

$$r(H, E | K) =_{df} \log \left[\frac{\Pr(H | E \& K)}{\Pr(H | K)} \right]$$

$$l(H, E | K) =_{df} \log \left[\frac{\Pr(E | H \& K)}{\Pr(E | \bar{H} \& K)} \right]$$

$$= \log \left[\frac{\Pr(H | E \& K) \cdot [1 - \Pr(H | K)]}{[1 - \Pr(H | E \& K)] \cdot \Pr(H | K)} \right].$$

- The only reason we take *logarithms* in the definitions of r and l is to ensure that (like d) r and l are (i) *positive* for *confirmation*, (ii) *zero* for *irrelevance*, and (iii) *negative* for *disconfirmation*.

Bayesian Confirmation V

- These and many other measures have been proposed and defended in the literature on (incremental) Bayesian confirmation theory.
- In my dissertation (and in several publications), I survey the many measures that have been proposed and show that a wide variety of arguments in the literature depend on which measure one chooses.
- It turns out (somewhat surprisingly) that one's choice of incremental measure of confirmation has *great* impact on one's analyses.
- This is because the truth-value of " E_1 confirms H_1 more strongly than E_2 confirms H_2 " varies greatly (and in crucial ways!) depending on which measure of incremental confirmation one uses.
- How can this be, if the measures capture the same *qualitative* notion?
- Some of the new paper topics involve exploiting this *sensitivity to choice of measure of confirmation* (e.g., Earman on old evidence).

Bayesian Confirmation VI

- The absolute account of confirmation has (potential) problems:
 - The problem of the threshold value τ (what should τ be?)
 - The problem of insensitivity to *relevance* (Fred Fox, Skyrms)
- The incremental account of confirmation has (potential) problems:
 - The problem of measure sensitivity (which I have made famous!)
 - The problem of old evidence (also a problem for absolute — *why?*)
- The problem of old evidence has caused a great stir in the literature. One of the new paper topics concerns this famous problem.
- If a Bayesian agent is always supposed to make judgments according to their “most recent” or “most well-informed” probability function Pr , then Pr will *include* all of their evidence E up to that point. But, then $\text{Pr}(H | E) = \text{Pr}(H)$, and so *their “old” evidence is no longer evidence!*

Bayesian Confirmation VII

- There are *many* proposed resolutions of the old-evidence problem:
 - **Historical.** If the question is “Does E confirm H relative to our *current* (actual) K ?”, then we need to look at $\Pr(H \mid E \ \& \ K')$ *vs* $\Pr(H \mid K')$, where K' is “the K we had just prior to learning E .”
 - **Counterfactual.** Similar to historical, but K' is “the K we *would have had*, had we *not* learned E ” and $\Pr(H \mid E \ \& \ K')$ is “the probability we would have assigned H had we *then* learned E ”.
 - **Logical Relation.** Rational Bayesian agents need *not* be logically omniscient, and so an agent may learn $H \vdash E$. By assuming various things about how probabilities get assigned to “ $H \vdash E$ ”, Garber *et al* show that agents can have $\Pr(H \mid H \vdash E) > \Pr(H)$, even if E is “old evidence”. Earman (chapter 5) has a great discussion on this.
- The logical relation approach (unlike the others) can also cope with the problem of *new theories* (H *not even pondered* before E learned).

Bayesian Confirmation VIII

- Like everybody else, I have an opinion (in formation!) about these thorny problems of old evidence and new theories.
- My idea (today!): Let's look at how Bayesian statisticians *actually make* inferences in cases of “old evidence” and/or “new theories”.
- In their encyclopedic text *Bayesian Theory*, the prominent Bayesian Statisticians Bernardo & Smith give us a hint (their emphasis):

One further point about the terms prior and posterior is worth emphasizing. *They are not necessarily to be interpreted in a chronological sense*, with the assumption that ‘prior’ beliefs are specified first and then later modified into ‘posterior’ beliefs. . . . It is true that the natural order of assessment does coincide with the ‘chronological’ order in a number of applications, but . . . this is a pragmatic issue and not a requirement of the theory.
- How are statistical inferences actually made? What do they establish? I think careful attention to these questions may yield a new (and superior) resolution of the old evidence and new theory problems.

Bayesian Confirmation IX

- Aren't statistical inferences really of the form "*E* favors H_1 over $\{H_2, \dots, H_n\}$, relative to an experimental design (and context) K "?
- If so, then why can't such claims be true in a *timeless* way? And, why should they be *undermined* if E is "currently known" or if a new hypothesis H^* is "currently out there, but as yet undiscovered"?
- This idea is *neither* "historical" *nor* "counterfactual." *Why?*
- **Analogy:** When we say that an argument $\frac{p}{\therefore q}$ is *inductively strong*, we must be clear that we mean *inductively strong* — *relative to background K* . If K contains information that *undermines* or *defeats* the inference from p to q , then — relative to K — this is *not* a strong inference.
- What this means is that inductive strength is *indexical* (or *contextual*). But, why (as B & S might ask) must that render it *chronological*?
- Can you apply this idea to the "old evidence" / "new theory" problem?

Bayesian Confirmation X

- It is sometimes claimed (see Earman page 64, sort of tongue-in-cheek) that Bayesian confirmation is able to “winnow a valid kernel of” previous (deductive) accounts of confirmation “from their chaff.”
- Earman’s cavalcade of “success stories” of Bayesian confirmation is fascinating. This list includes:
 - How Bayesian confirmation nicely generalizes H-D
 - How BC handles the paradoxes of instance confirmation (raven, etc.)
 - How BC handles “evidential variety” or “diversity”
 - How BC handles the Quine-Duhem problem
 - How BC handles “grue” like paradoxes
- But, BC also faces its own peculiar challenges, like the problems of subjectivity (especially, for priors), old evidence/new theories, Popper-Miller, measure-sensitivity, zero priors, and various others.