Accuracy & Coherence III

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- In this talk, I will explain why *only one* of Miller's two types of language-dependence-of-verisimilitude problems is a (potential) threat to the sorts of accuracy-dominance approaches to coherence that I've been discussing.
- The first (Qualitative) LD problem involves dependence on the choice of underlying *logical/formal language* in our representations of the doxastic spaces of epistemic agents.
- Consider an agent S with a language $\mathscr{L}_{HRW}$ having 3 atomic sentences (H, R, W). There will be 256 propositions in S's doxastic algebra $\mathcal{B}$ (as usual, we assume omniscience here).
- Suppose S is such that $D_S(H)$, $B_S(R)$, and $B_S(W)$. And, suppose that the *actual* world $w_@$ is such that $H \,\&\, R \,\&\, W$.
- S's judgment (sub)set $\mathbf{B}_1 = \{D_S(H), B_S(R), B_S(W)\}$ over H, R, and W seems to be "*closer to the truth at* $w_@$" than the set $\mathbf{B}_2 = \{D_S(H), D_S(R), D_S(W)\}$, since $\mathcal{B}_1$ has 1 incorrect judgment (at $w_@$), while $\mathcal{B}_2$ contains 3 incorrect judgments.

- This assessment of the "relative verisimilitudes of $\mathbf{B}_1$ *vs* $\mathbf{B}_2$ *at* $w_@$" depends on our choice of sentential linguistic representation. This is Miller's first LD problem [7, Ch. 11].
- To see this LD problem vividly, consider an alternative representation of S's doxastic space, which uses $\mathscr{L}_{HMA}$.
- $\mathscr{L}_{HMA}$ has atoms H and M, A, where M, A are such that:
  - $M \dashv\vDash H \equiv R$.
  - $A \dashv\vDash H \equiv W$.
- In $\mathscr{L}_{HMA}$, $\mathbf{B}_1$ becomes $\{D_S(H), D_S(M), D_S(A)\}$, and $\mathbf{B}_2$ becomes $\{D_S(H), B_S(M), B_S(A)\}$. And, in $w_@$, we have $H \,\&\, M \,\&\, A$. Thus, using $\mathscr{L}_{HMA}$ leads to a *reversal* of the assessment of "relative verisimilitudes of $\mathbf{B}_1$ *vs* $\mathbf{B}_2$ *at* $w_@$."
- This *seems* to pose a problem for our accuracy-dominance norm for qualitative judgments. But, in fact, it does *not*.
- In order to see why this LD problem is *merely* an *apparent* one, we must return to our definitions from Lecture I.

- Here are the two salient definitions:
  - $\mathfrak{B}$ is the *full* set of S's qualitative judgments *over the entire algebra of propositions* $\mathcal{B}$. The *innaccuracy* of $\mathfrak{B}$ at a world w is given by the number of incorrect judgments in $\mathfrak{B}$ at w.
  - One (*full*) qualitative judgment set $\mathfrak{B}'$ *accuracy-dominates* another $\mathfrak{B}$ iff (i) $\mathfrak{B}'$ has *strictly fewer* incorrect judgments at *some* possible worlds, and (ii) $\mathfrak{B}'$ contains *at most as many* incorrect judgments as $\mathfrak{B}$ at *every* possible world.
- The crucial point here is that *our* assessments of "relative verisimilitude" are *only* made with respect to *full* judgment sets $\mathfrak{B}/\mathfrak{B}'$ *over S's entire algebra of propositions* $\mathcal{B}$.
- Miller's first LD problem trades on the fact that "the # of incorrect judgments in $\mathbf{B}$ at w" can *vary*, depending on our *linguistic representation* of S — for *proper subsets* $\mathbf{B} \subset \mathfrak{B}$.

☞ *This* language dependence *disappears* when we *restrict our comparisons to full judgment sets* $\mathfrak{B}/\mathfrak{B}'$. "The # of incorrect judgments in $\mathfrak{B}$ at w" is a *language invariant* quantity.

- Suppose we have two *numerical* quantities ϕ and ψ. These might be, for instance, the velocities (in some common units) of two objects, at some time (or some other physical property, like temperature, of two objects at a time).
- Suppose further that we have two sets of (false) predictions concerning the values of ϕ and ψ, which are entailed by two (false) competing hypotheses H_1 and H_2.
- Finally, let's use "T" to denote *the truth* about the values of ϕ and ψ (or, if you prefer, the true hypothesis about their values) — in our standard units. And, let H_1, H_2, and T be:

	ϕ	ψ	α	β
H_1	0.150	1.225	0.925	2.000
H_2	0.100	1.000	0.800	1.700
T	0.000	1.000	1.000	2.000

- It seems clear that the predictions of H_2 are "closer to the truth T about ϕ and ψ" than the predictions of H_1 are.

- However, as Popper [5, Appendix 2] showed (using a recipe invented by David Miller [6]), there exist quantities α and β (as in the table) satisfying both of the following conditions.
  1. α and β are symmetrically inter-definable with respect to ϕ and ψ in the following (linear) way:
  $$\alpha = \psi - 2\phi \qquad \beta = 2\psi - 3\phi$$
  $$\phi = \beta - 2\alpha \qquad \psi = 2\beta - 3\alpha$$
  2. The values for α and β entailed by H_2 are strictly "farther from the truth T about α and β" than those entailed by H_1.
- As Miller [6] explains (see [7, Chapter 11] for a recent survey), there is a much more general result in the vicinity.
- For *any* pair of false theories H_1 and H_2 about ϕ and ψ, *many* relations of "closer to the truth" can be *reversed* by looking at what the estimates provided by H_1 and H_2 for ϕ and ψ entail about quantities α and β, which are given by:
$$\alpha = \mathsf{a}\psi + \mathsf{b}\phi \qquad \beta = \mathsf{c}\psi + \mathsf{d}\phi$$
$$\phi = \mathsf{a}\beta + \mathsf{b}\alpha \qquad \psi = \mathsf{c}\beta + \mathsf{d}\alpha$$
- This is Miller's 2^{nd} (Quantitative) LDP. It *is* a (potential) threat.

- According to Joyce [4], if we view credences as "estimates" of (suitable) "numerical representations of truth-values" of propositions, then we can give an argument for probabilism that is based on the "accuracy" of these "estimates".
- Consider a very simple, logically omniscient, opinionated agent S who has only one atomic sentence P in his language.
- All that matters concerning S's *coherence* is whether S's credences b in P and $\sim P$ sum to one (and are non-negative).
- Following Joyce, let's associate the truth-value **T** (at each world w) with the number 1 and the truth-value **F** with 0. [This is our "numerical representation of the truth-values" of P, $\sim P$.]
- Let ϕ be the *numerical* value we have associated with P's truth-value, and let ψ be the *numerical* value we have associated with $\sim P$'s truth-value (of course, ϕ and ψ will vary in the obvious ways across the two salient possible worlds).
- Next, measuring the "accuracy" of Joycean "estimates" (b).

- Suppose S has a numerical credence function $b : \mathcal{B} \mapsto \mathbb{R}$ (these b functions are *opinionated*, of course, and so we're ignoring suspension of judgment here, once again).
- The *inaccuracy* of $b(p)$ at world w will be b's "distance (d) from the number associated with p's truth-value" at w.
- **Example.** Suppose S has just two (contingent) propositions $\{P, \sim P\}$ in their doxastic space. Then, there are two salient possible worlds (w_1 in which P is **T**, and w_2 in which P is **F**). And, the *overall inaccuracy* of b at w [$I(b, w)$] is given by:
  - $I(b, w_1) = d(b(P), 1) + d(b(\sim P), 0)$.
  - $I(b, w_2) = d(b(P), 0) + d(b(\sim P), 1)$.
- Various measures (d) of "distance from 0/1-truth-value" have been proposed/defended in the historical literature.
- The most popular choice (for giving an accuracy-dominance justification of probabilism) has been the *squared-difference* measure of "distance from 0/1-truth-value", which is:
  - $\mathfrak{s}(x, y) = (x - y)^2$.

- The distance measure $\mathfrak{s}$ gives rise to a measure of *overall* inaccuracy ($I_{\mathfrak{s}}$), which is known as the Brier Score. In our toy example, the Brier Scores of b in worlds w_1 and w_2 are:
  - $I_{\mathfrak{s}}(b, w_1) = \mathfrak{s}(b(P), 1) + \mathfrak{s}(b(\sim P), 0) = (b(P) - 1)^2 + b(\sim P)^2$.
  - $I_{\mathfrak{s}}(b, w_2) = \mathfrak{s}(b(P), 0) + \mathfrak{s}(b(\sim P), 1) = b(P)^2 + (b(\sim P) - 1)^2$.
- If one adopts the Brier Score as one's measure of b's inaccuracy, then one can give an accuracy-dominance argument for the axioms of the probability calculus.
- de Finetti [1] was the first to prove such a *Brier*-dominance theorem. Joyce [4, 3] interprets this as *accuracy*-dominance.
  - **Theorem** (de Finetti). b is *non*-probabilistic *if and only if* there exists a *probabilistic* credence function b' such that (a) *b' has a strictly lower Brier Score than b at some worlds*, and (b) *b' never has a greater Brier Score than b at any world.*
- One can use other underlying measures of distance d here and still preserve a de Finetti-style Theorem. Miller's second LD problem is a potential threat to *any* of these approaches.

- The easiest way to see the (potential) threat posed by Miller's 2^{nd} LD problem is to consider a simple numerical example of a toy agent S of the type we've been discussing.

	ϕ	ψ
b	$\frac{1}{2}$	$\frac{1}{4}$

- Here, S's "estimates" (b) of ϕ and ψ do not sum to one. As a result, there exist alternative "estimates" b' of ϕ/ψ that Brier-dominate b in both of the salient possible worlds.

	ϕ	ψ
b	$\frac{1}{2}$	$\frac{1}{4}$
b'	$\frac{5}{8}$	$\frac{3}{8}$
w_1	0	1
w_2	1	0

- b' is the *Euclidean-closest* (to b) set of "estimates" of ϕ and ψ that Brier-dominate b — with respect to ϕ/ψ "estimation".

- Does Joycean "numerical estimation" face a (*prima facie*) problem analogous to Miller's second LD problem? Yes!

	ϕ	ψ	α	β
b	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{9}{16}$	$\frac{3}{16}$
b'	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{1}{4}$
w_1	0	1	$\frac{7}{16}$	$\frac{9}{16}$
w_2	1	0	$\frac{9}{16}$	$\frac{7}{16}$

- Here, we have numerical quantities α and β, such that:

  (i) α/β are symmetrically inter-definable w.r.t ϕ/ψ, *via* ($f^\star$):

$$\alpha = \frac{1}{2}\phi + \frac{1}{2}\psi + \frac{1}{16}\left(\frac{\phi+\psi}{\phi-\psi}\right) \qquad \beta = \frac{1}{2}\phi + \frac{1}{2}\psi - \frac{1}{16}\left(\frac{\phi+\psi}{\phi-\psi}\right)$$

$$\phi = \frac{1}{2}\alpha + \frac{1}{2}\beta + \frac{1}{16}\left(\frac{\alpha+\beta}{\alpha-\beta}\right) \qquad \psi = \frac{1}{2}\alpha + \frac{1}{2}\beta - \frac{1}{16}\left(\frac{\alpha+\beta}{\alpha-\beta}\right)$$

  (ii) The "estimates" of α/β entailed (*via* $f^\star$) by b *Brier dominate* the "estimates" of α/β entailed by b' (*via* $f^\star$).

☞ So, we have a Miller-style *reversal* of Brier-domination here!

- Here is a *more general theorem* about our toy agent S.

  Theorem. For *any* coherent b' that Brier-dominates S's credence function b with respect to ϕ and ψ, there exist quantities α and β that are symmetrically inter-definable with respect to ϕ and ψ, *via* the transformation $f^\star$ above, such that b *Brier-dominates* b' with respect to α and β.

- It is also noteworthy that the *true* values of α and β "behave like truth-values", in the sense that (a) the true value of α (β) in w_1 (w_2) is identical to the true value of β (α) in w_2 (w_1), and (b) the true values of α and β always *sum to one*.
- Indeed, this transformation $f^\star$ is guaranteed to *preserve coherence* of *all* dominating b''s, *and* the "truth-vectors".
- It is not a coincidence that $f^\star$ is *non*-linear. It can be shown that *no linear f* can play the role that $f^\star$ plays here. There are several reasons for this (some of which I'll mention below).
- Next, two possible responses to this (*prima facie*) threat.

- **Response #1: Naturalness**. The first response is to argue that the ϕ/ψ "numerical representation of the truth-values of $P/{\sim}P$" is somehow "more natural" than the α/β "numerical representation of the truth-values of $P/{\sim}P$".
- I'm not sure how such a "naturalness argument" would go.
- After all, the truth-values of $P/{\sim}P$ are *disanalogous* to *numerical* physical quantities like velocity or temperature.
- In the case of temperature, for instance, the *numerical level* of description is (arguably) the most fundamental/scientific.
- But, in the case of truth-values, their theoretical role seems to be given *fundamentally* at the level of their *algebraic* and *meta-logical* (*viz.*, *logico-structural*) properties.
- The "numerical properties" of the truth-values (*if there be such*) do not seem to be theoretically fundamental. So, it's not clear to me how probative "naturalness" is here.
- I think there is a more promising line of response. . .

- **Response #2: Disanalogies between "Estimation" & Prediction**.
- The second line of response (which I prefer) is to argue that there are crucial disanalogies between "estimation" (in Joyce's sense) and prediction (in Miller's sense).
- Let ⌜$\mathcal{E}_S(x,y) = \langle p,q\rangle$⌝ express the claim that ⌜S is committed to the values $\langle p,q\rangle$ as their "estimates" of the quantities $\langle x,y\rangle$⌝. Our "reversal argument" presupposes the following (as applied to our toy, numerical S, above):
  - (†) If $\mathcal{E}_S(\phi,\psi) = \langle p,q\rangle$, then $\mathcal{E}_S(\alpha,\beta) = f^{\star}(p,q)$, where $f^{\star}$ is the symmetric inter-translation function that maps values of $\langle \phi,\psi\rangle$ to/from values of $\langle \alpha,\beta\rangle$ in our Theorem.
- Ultimately, what Joyce needs to argue is that (†) is *false*.
- In order to do this, Joyce needs to tell us more about what "estimation" is. Ideally, he needs to give us a *theory* of $\mathcal{E}_S$.
- Unfortunately, what Joyce *explicitly says* about $\mathcal{E}_S$ is insufficient to explain why (†) should come out false.

- The only *explicit* commitments Joyce has re $\mathcal{E}_S$ seem to be:
  1. Estimates are *not guesses*. Joyce [4, 587] explicitly distinguishes estimation and guessing.
     - This doesn't help me assess (†), so I won't discuss it.
  2. Estimates are *not expectations*. Joyce [4, 587–8] explicitly *disavows* thinking of estimates as expectations. Indeed, Joyce thinks it would be *question-begging* to think of "estimation" as expectation (*e.g.*, expectation *builds-in* too much probabilistic structure from the outset).
     - If estimates *were* expectations, then this would entail that (†) is *false*, since this would rule-out *all non-linear* transformation functions. [A reason to *like* expectation?]
  3. Estimates are *not assertions that* the values of the parameters *are such-and-so*. This is clear, since it's *not* a good idea to assert things that you know (*a priori*) *must be false*. And, this happens whenever you offer "estimates" of "numerical correlates of truth-values" that are *non-extreme*.
     - If estimates *were* assertions, then this would entail that (†) is *true* — assuming a truth/closure norm for assertions.

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[1] B. de Finetti, *The Theory of Probability*, Wiley, 1974.

[2] ______, *Foresight: Its Logical Laws, Its Subjective Sources*, in H. Kyburg and H. Smokler (*eds.*), *Studies in Subjective Probability*, Wiley, 1964.

[3] J. Joyce, *Accuracy and Coherence: Prospects for an Alethic Epistemology of Partial Belief*, in F. Huber and C. Schmidt-Petri (*eds.*), *Degrees of Belief*, 2009.

[4] ______, *A Nonpragmatic Vindication of Probabilism*, *Philosophy of Science*, 1998.

[5] K. Popper, *Objective Knowledge: An Evolutionary Approach*, 2nd edition, 1979.

[6] D. Miller, *The Accuracy of Predictions*, *Synthese*, 1975.

[7] D. Miller, *Out Of Error: Further Essays on Critical Rationalism*, Ashgate, 2006.