# Studies in Bayesian Confirmation Theory

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## Some Bayesian Background I

- Orthodox Bayesianism (*i.e.*, Bayesian *epistemology*) assumes that the degrees of belief (or credence) of rational agents are (*Kolmogorov* [29]) *probabilities*.
- $\Pr_a(H \mid K)$ denotes an (rational) agent *a*'s degree of credence in $H$, given the corpus $K$ of background knowledge/evidence (called *a*'s "prior" for $H$).
- $\Pr_a(H \mid E \,\&\, K)$ denotes *a*'s degree of credence in $H$ (relative to $K$) given that (or *on a's supposition that*) $E$. This is also the agent's degree of belief in $H$ (relative to $K$) *upon learning E* (called *a*'s "posterior" for $H$, on $E$, given $K$).
  - ∗ Credences are *Kolmogorov* [21], [27], [12], *probabilities* [50], on *crisp sets* [54].
  - ∗ Agents learn (*with certainty* [26]) *via conditionalization* [32], [33].
  - ∗ "Priors" (∴ *Bayesianism itself*) are *subjective* [49], [47], [34], [30], [6].
- I will bracket all of these issues. The problem I'm discussing only gets *worse* if Bayesianism is made more sophisticated along any of these dimensions!
- For simplicity, I will assume there is a *single* rational Bayesian probability function Pr (and I'll drop the subscript "$_a$" and the background corpus "$\mid K$").





## Some Bayesian Background II

- In (contemporary) Bayesian confirmation theory, evidence $E$ *confirms* (or *supports*) a hypothesis $H$ if learning $E$ *raises the probability of* $H$.
- If learning $E$ *lowers* the probability of $H$, then $E$ *dis*confirms (or *counter*-supports) $H$, and if learning $E$ *does not change* the probability of $H$, then $E$ is confirmationally *neutral* regarding $H$. This is a Pr-*relevance* theory.
- Within (*Kolmogorov*! [10], [12]) probability theory, there are many *logically equivalent* ways of saying that $E$ confirms $H$. Here are a few:
  - ∗ $E$ confirms $H$ if $\Pr(H \mid E) > \Pr(H)$.
  - ∗ $E$ confirms $H$ if $\Pr(E \mid H) > \Pr(E \mid \neg H)$.
  - ∗ $E$ confirms $H$ if $\Pr(H \mid E) > \Pr(H \mid \neg E)$.
- By taking differences, (log-)ratios, *etc.*, of the left/right sides of these (or other equiv.) inequalities, a *plethora* of candidate *relevance measures* of *degree of confirmation* can be formed. $(\mathcal{R})$ $\mathfrak{c}(H,E) \gtreqless 0$ if $\Pr(H \mid E) \gtreqless \Pr(H)$.





## Four Popular and Representative Relevance Measures

- *Dozens* of Bayesian relevance measures have been proposed in the philosophical literature (see [31] for a survey). Here are four popular ones.[a]
  - ∗ *Difference*: $d(H,E) =_{df} \Pr(H \mid E) - \Pr(H)$
  - ∗ Log-*Ratio*: $r(H,E) =_{df} \log\left[\frac{\Pr(H \mid E)}{\Pr(H)}\right]$
  - ∗ Log-*Likelihood-Ratio*: $l(H,E) =_{df} \log\left[\frac{\Pr(E \mid H)}{\Pr(E \mid \neg H)}\right]$
  - ∗ "Normalized Difference" $d$: $s(H,E) =_{df} \Pr(H \mid E) - \Pr(H \mid \neg E) = \frac{1}{\Pr(\neg E)} \cdot d(H,E)$
- Logs are taken to ensure easy satisfaction of relevance criterion $(\mathcal{R})$. They are merely a useful convention (they're inessential, but they simplify things).
- The first part of our story concerns the *disagreement* exhibited by these measures, and its ramifications for Bayesian confirmation theory …

[a] Users of $d$ include [7], [6], and [26]. Users of $r$ include [24], [35], and [25]. Users of $l$ include [19], [48], and [15]. Users of $s$ include [27] and [5]. See [10], [13], and [15] for further references.

## Disagreement Between Alternative Relevance Measures

- What kind of disagreement between relevance measures is important?
- Mere *numerical* (or *conventional* or *syntactical*) differences between measures are not important, since they need not effect *ordinal* judgments of what is more/less well confirmed than what (by what).
- *Ordinal* differences are crucial, since they can effect the cogency of many arguments surrounding Bayesian confirmation theory.
- For instance, it is part of Bayesian lore that the observation of a black raven ($E_1$) confirms the hypothesis ($H$) that all ravens are black *more strongly than* the observation of a white shoe ($E_2$) does (given "actual corpus" $K$).
- But, given the standard background assumptions ($K$) in Bayesian accounts of Hempel's ravens paradox, this conclusion [$\mathfrak{c}(H, E_1) > \mathfrak{c}(H, E_2)$] follows only for *some* measures of confirmation $\mathfrak{c}$ (and *not* others).
- Such arguments are said to be *sensitive to choice of measure* [13].





## The Problem of Measure Sensitivity

- A detailed study of the literature shows that *virtually every argument* involving quantitative Bayesian confirmation theory is sensitive to choice of measure [13]! Here are my favorites (I'll briefly discuss three of these):
  - Horwich [24] *et al.* on Hempel's Ravens Paradox
  - Horwich [24] *et al.* on the Confirmational Value of Varied Evidence
  - The Popper-Miller Argument *Against* Bayesianism [41], [16]
  - Rosenkrantz [46] and Earman [6] on the Problem of "Irrelevant Conjunction"
  - Eells [7] and Sober [52] on Goodman's "Grue" Paradox
  - Earman [6] on the problem of (quantitative) old evidence
- There are many other important measure-sensitive arguments [3], [1], [2].
- One needn't gerrymander or comb the historical literature for Bayesian relevance measures which fail to undergird these arguments.
- Each of these arguments is valid with respect to *only some* of $d$, $r$, $l$, and $s$.





## Horwich *et al.* on Ravens & Variety of Evidence

- Almost all Bayesian accounts of both the Ravens Paradox and the value of "varied" evidence (*i.e.*, why more "varied" evidence $E_1$ is more confirmationally powerful then less "varied" evidence $E_2$) presuppose:

(1) If $\Pr(H \mid E_1) > \Pr(H \mid E_2)$, then $\mathfrak{c}(H, E_1) > \mathfrak{c}(H, E_2)$.

- The "normalized" difference measure $s$ *violates* (1).[a]
- Typically, the advocates of such arguments have used either $d$ or $r$ in their arguments (as it turns out, $d$, $r$, and $l$ *all* satisfy (1)).
- None of these authors seems to provide (*independent*) reasons to prefer their measures over $s$ (or other measures which violate (1)).
- In my [14] and [15], I propose a novel Bayesian explication of the confirmational value of *independent* evidence, based on $l$.

[a] So do Carnap's [4, §67] relevance measure $\mathfrak{r}(H, E) = \Pr(H \,\&\, E) - \Pr(H) \cdot \Pr(E)$, Mortimer's [36] measure $\Pr(E \mid H) - \Pr(E)$, and Nozick's [37] measure $\Pr(E \mid H) - \Pr(E \mid \neg H)$.





## The Popper-Miller Argument *Against* Bayesianism

- It isn't just arguments/accounts *within* Bayesian confirmation theory that are sensitive to choice of measure. Some well-known *criticisms* of Bayesianism also rest on measure sensitive arguments.
- Most famously, Popper and Miller ([41], [16]) use the following property of the difference measure $d$ to argue *against* Bayesianism (generally):

(2) $$d(H, E) = d(H \vee E, E) + d(H \vee \neg E, E).$$

- As it turns out, neither the log-ratio measure $r$ [42], nor the log-likelihood-ratio measure $l$ [20] satisfies property (2).
- ∴ The Popper-Miller argument is *sensitive to choice of measure*.
- In the absence of reasons to think that $d$ is a more accurate (and charitable) reconstruction of Bayesian confirmation theory than either $r$ or $l$, the Popper-Miller argument remains (at best) *enthymematic*.

## Tabular Summary of Some Measure-Sensitive Arguments

| | Valid *wrt* relevance measure: | | | |
|---|---|---|---|---|
| Argument | *d*? | *r*? | *l*? | *s*? |
| Rosenkrantz on Irrelevant Conjunction | Yes | No | No | Yes |
| Earman on Irrelevant Conjunction | Yes | No | Yes | Yes |
| Eells on the Grue Paradox | Yes | No | No | Yes |
| Sober on the Grue Paradox | Yes | No | Yes | Yes |
| Horwich *et al.* on Ravens & Variety | Yes | Yes | Yes | No |
| Popper-Miller's *Critique* of Bayesianism | Yes | No | No | Yes |
| Earman's Old Evidence *Critique* of Bayesianism | Yes | Yes | No | No |





## Some Attempts to Resolve the Measure-Sensitivity Problem

- There do exist a few general arguments in the literature which aim to rule-out all but a small class of ordinally equivalent measures (*e.g.*, Milne [35], Good [18], Carnap [4], Kemeny & Oppenheim [28], and Heckerman [22]).
- Others have given "piecemeal" arguments which attack a *particular* class of measures, but fail to rule-out other competing measures (*e.g.*, Rosenkrantz [46], Earman [6], Gillies [16], Eells, and Sober [52]).
- In my dissertation [15], I provide a thorough survey of both kinds of arguments, and I show that none of them is completely satisfactory.
- Most notably, I have seen (in the literature[a]) *no* compelling reasons to prefer the difference measure *d* over either *l* or *s*.
- Until such reasons are provided, the arguments of Gillies, Rosenkrantz, Eells, Horwich *et al.* will remain *enthymematic*.

[a] Recent joint work of Eells & Fitelson in [8] and [9] has filled this gap in the literature.





## Narrowing The Field I: Symmetries and Asymmetries in Evidential Support[a]

- Consider the following two propositions concerning a card $c$, drawn at random from a standard deck of playing cards (classical model $\mathcal{M}$):

  $E$: $c$ is the ace of spades. $H$: $c$ is *some* spade.

- I take it as intuitively clear and uncontroversial that:
  1. The degree to which $E$ supports $H \neq$ the degree to which $H$ supports $E$, since $E \models H$, but $H \not\models E$. Intuitively, we have $\mathfrak{c}(H,E) \gg \mathfrak{c}(E,H)$.
  2. The degree to which $E$ confirms $H \neq$ the degree to which $\neg E$ *dis*confirms $H$, since $E \models H$, but $\neg E \not\models \neg H$. Intuitively, $\mathfrak{c}(H,E) \gg -\mathfrak{c}(H,\neg E)$.
- Therefore, *no adequate relevance measure of support* $\mathfrak{c}$ *should be such that either* $\mathfrak{c}(H,E) = \mathfrak{c}(E,H)$ *or* $\mathfrak{c}(H,E) = -\mathfrak{c}(H,\neg E)$ (for all $E$ and $H$ and $K$).
- Note: for all $H$, $E$ (and $K$), $r(H,E) = r(E,H)$ and $s(H,E) = -s(H,\neg E)$. *Both* $d$ and $l$ satisfy *both* of these (a)symmetry desiderata.

[a] This slide is drawn from recent joint work of Eells & Fitelson [9].





## Narrowing The Field II: Our Relevance Measures *as Generalizations of Entailment*

- $$l(H,E) = \begin{cases} +\infty & \text{if } E \models H,\ \Pr(E) > 0,\ \Pr(H) \in (0,1) \\ 0 & \text{if } E \perp\!\!\!\perp H,\ \Pr(E) > 0,\ \Pr(H) \in (0,1) \\ -\infty & \text{if } E \models \neg H,\ \Pr(E) > 0,\ \Pr(H) \in (0,1) \end{cases}$$
- $$d(H,E) = \begin{cases} \Pr(\neg H) & \text{if } E \models H,\ \Pr(E) > 0 \\ 0 & \text{if } E \perp\!\!\!\perp H,\ \Pr(E) > 0 \\ -\Pr(H) & \text{if } E \models \neg H,\ \Pr(E) > 0 \end{cases}$$
- $$r(H,E) = \begin{cases} \log\left[\frac{1}{\Pr(H)}\right] & \text{if } E \models H,\ \Pr(E) > 0,\ \Pr(H) > 0 \\ 0 & \text{if } E \perp\!\!\!\perp H,\ \Pr(E) > 0,\ \Pr(H) > 0 \\ -\infty & \text{if } E \models \neg H,\ \Pr(E) > 0,\ \Pr(H) > 0 \end{cases}$$
- $$s(H,E) = \begin{cases} \Pr(\neg H \mid \neg E) & \text{if } E \models H,\ \Pr(E) \in (0,1) \\ 0 & \text{if } E \perp\!\!\!\perp H,\ \Pr(E) \in (0,1) \\ -\Pr(H \mid \neg E) & \text{if } E \models \neg H,\ \Pr(E) \in (0,1) \end{cases}$$

## Narrowing The Field III: Independent Evidence I

- Wittgenstein [53] alludes to a man who is doubtful about the reliability of a story he reads in the newspaper, so he buys another copy of the same issue of the same newspaper to double check.
- To fix our ideas, let's assume that the story in the NYT reports that ($H$) the Yankees won the world series. Let $E_n$ be the evidence obtained by reading the $n^{th}$ copy of the same issue of the NYT.
- Intuitively, the degree to which the conjunction $E_1$ & $E_2$ confirms $H$ is no greater than the degree to which $E_1$ *alone* confirms $H$.
- Also, it seems intuitive that an *independent* report $E'$ (say, one heard on a NPR broadcast) *would* corroborate the NYT story.
- So, it seems intuitive that the degree to which $E_1$ & $E'$ confirms $H$ is greater than the degree to which $E_1$ *alone* confirms $H$.





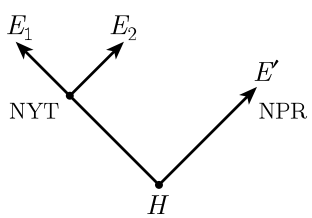
## Narrowing The Field III: Independent Evidence II

- How can we explain the epistemic difference between these two examples? Intuitively, a NYT report ($E$) and a NPR report ($E'$) are *independent* in a way that two NYT reports ($E_1$, $E_2$) are not.
- It is *not* that the NYT report and the NPR report are Pr-independent *unconditionally*, since (far more often than not) the two reports will tend to agree. So, what kind of independence is at work here?
- As Sober [51] explains, the relevant probabilistic fact is that $E$ and $E'$ are independent — *given* $H$ (or $\neg H$). That is, if we know the truth-value of $H$, then the dependence (correlation) between $E$ and $E'$ disappears.
- $H$ *explains* the correlation between $E$ and $E'$. This is because $E$ *and* $E'$ *are joint effects of the common cause* $H$. As Reichenbach [43] taught us: *common causes screen-off their joint effects from each other*.





## Narrowing The Field III: Independent Evidence III



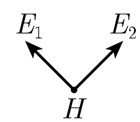

- $E_1$ and $E_2$ are *de*pendent — *even if we know the truth-value of* $H$ (perhaps if we knew the state of the NYT printing press just prior to publication, then *this* would render $E_1$ and $E_2$ independent).
- But, $E_1$ and $E'$ are *independent* — once we know the truth-value of $H$. When this happens, we say that $H$ *screens-off* $E_1$ from $E'$ or that $E_1$ and $E'$ are *conditionally independent, given* $H$ (or $\neg H$).
- This kind of structure is a "Bayesian Network" [38], [22], [23], [2].



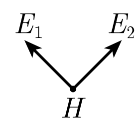


## Narrowing The Field III: Independent Evidence IV

- If two pieces of (confirmatory) evidence $E_1$ and $E_2$ are independent regarding a hypothesis $H$, then the conjunction $E_1$ & $E_2$ should confirm $H$ more strongly than either conjunct does severally:

If  then $\mathfrak{c}(H,\ E_1 \,\&\, E_2) > \mathfrak{c}(H,\ E_i)$.

- More precisely, (as C.S. Peirce [39] suggests) the degree of support provided by the conjunction $E_1$ & $E_2$ should simply be the *sum* of the several degrees of support provided by each conjunct:

If  then $\mathfrak{c}(H,\ E_1 \,\&\, E_2) = \mathfrak{c}(H,\ E_1) + \mathfrak{c}(H,\ E_2)$.

## Narrowing The Field III: Independent Evidence V

- Measures $d$, $r$, and $l$ satisfy the first of these desiderata ($s$ does *not*!) [15]. *Only* measure $l$ satisfies the second (additivity) desideratum [14], [22].
- This provides a novel way of adjudicating between $d$ and $l$ (indeed, this adjudicates between $l$ and anything else, for that matter).
- These ideas about independent evidence can also serve to ground a novel (partial) explication of the confirmational value of "varied" evidence.
- If "varied" evidence are *independent*, then they will provide a stronger confirmational boost than "narrow" or *de*pendent evidence will provide — as measured by the log-likelihood-ratio $l$ [14].
- According to $l$ (but *not* according to $d$), strong independent confirmational boosts can be provided — even to hypotheses with high priors (*e.g.*, Newton's gravitational theory $H$, planetary data $E_1$, and cometary data $E_2$, in the year 1758 when Halley's comet returned).





## Extra Slide #1: Degree of *Belief* *vs* Degree of *Support*

"But wait a minute! $l$ has *un*intuitive behavior. If $\Pr(H)$ is very high, then $H$'s probability *cannot* be raised very much, and so (*intuitively*) $H$ *cannot* be confirmed very strongly. But, $l$ *denies* this (whereas $d$ gets this 'right')!"

- If the question is "How strongly should we *believe* $H$, given the supposition that $E$ is true," then the answer will be "very strongly" if $\Pr(H)$ is high (and $E$ does not disconfirm $H$). This is *not* the issue.
- I'm asking "How strong is the *relation of support* between $E$ and $H$" (assuming $E$ does not disconfirm $H$). And, this does *not* always depend on the prior probability of $H$ — contrary to what $d$ presupposes.
- When $E \vDash H$ (*relevantly*!), the degree to which $E$ supports $H$ is *maximally strong*, and does *not* depend on $\Pr(H)$. But, $d(H, E) = 1 - \Pr(H)$ here.
- Also, I think that in the case of *independent* evidence, the degree of support does *not* depend on $\Pr(H)$. Moreover, if we talk about *odds* rather than Pr, this "edge effect" disappears (it is an *artifact* of [0,1] Pr-scale).





## Extra Slide #2: Some Details on Our $l$ and the Limiting (Deductive) Case

- I claimed that, when $E \vDash H$, $l(H, E)$ is maximal ($+\infty$), and does not depend on the prior probability of $H$ [*if* $\Pr(E) > 0$, $\Pr(H) \in (0, 1)$].
- But, if $E \vDash H$, then $\Pr(E \mid \neg H) = 0$. Shouldn't we say that $l(H, E)$ is *undefined* in such cases, since it has a zero denominator?
- There are two ways to handle this. First, one could maintain that, *in the limit as* $\Pr(E \,\&\, \neg H)$ *approaches zero*, $l(H, E)$ diverges ($+\infty$). So, $l(H, E)$ is maximal and doesn't depend on $\Pr(H)$ in such cases.
- Or, more satisfyingly, one could use the alternative measure [28]:

$$l'(H, E) = \frac{\Pr(E \mid H) - \Pr(E \mid \neg H)}{\Pr(E \mid H) + \Pr(E \mid \neg H)}$$

It is easy to show that ($i$) $l'$ *is ordinally equivalent to*[a] $l$, and ($ii$) $l'$ takes on the values $+1/-1$ in cases where $E$ (*relevantly*) entails/refutes $H$.

[a] This is because $l'$ is a monotone increasing function of $l$ [*viz.*, $l' = \tanh(l/2)$]; see [19].





## Extra Slide #3: Mere Syntactical Differences Between Measures

- Most existing Bayesian measures of support (*not* $s$!) can be written as some combination of (arithmetic) functions of the posterior and prior probabilities of $H$ [*i.e.*, some function of $\Pr(H \mid E)$ and $\Pr(H)$].
- For instance, the likelihood ratio can be written as follows:

$$\frac{\Pr(E \mid H)}{\Pr(E \mid \neg H)} = \frac{\Pr(H \mid E) \cdot [1 - \Pr(H)]}{[1 - \Pr(H \mid E)] \cdot \Pr(H)}$$

- One should not put too much weight on *mere syntactic differences* between measures. It's their *ordinal structure* that matters ...
- For instance, one should not conclude that a measure doesn't *depend on* $\Pr(H)$, simply because *one syntactic formulation* of the measure doesn't happen to contain the string of symbols "$\Pr(H)$".

$$\frac{\Pr(H \mid E)}{\Pr(H)} = \frac{\Pr(E \mid H)}{\Pr(E)}$$

## Extra Slide #4: The Problem of "Irrelevant Conjunction"

- According to deductive accounts of confirmation (*e.g.*, Hempel's H-D account), $E$ confirms $H$ (roughly) iff $H \vDash E$.
- Such accounts of confirmation have the following consequence:

(3) If $E$ confirms $H$, then $E$ confirms $H \,\&\, X$, for *any* $X$.

- While (3) is *not* a consequence of Bayesian confirmation, the following *is*:

(4) If $H \vDash E$, then $E$ confirms $H \,\&\, X$, for *any* $X$.

- Bayesians try to mitigate the effects of (4), by arguing that:

(5) If $H \vDash E$, then $\mathfrak{c}(H \,\&\, X, E) < \mathfrak{c}(H, E)$, for *any* $X$.

- Such arguments have two problems: (*i*) they are sensitive to choice of measure $\mathfrak{c}$, and (*ii*) they make no appeal to the *irrelevance* of $X$.
- In [11], I give a new-and-improved Bayesian account.





## Extra Slide #5: Rosenkrantz on "Irrelevant Conjunction"

- Rosenkrantz [46] provides a Bayesian resolution of the problem of Irrelevant Conjunction (*a.k.a.*, the Tacking Problem) which trades on the following property of the difference measure:

(6) If $H \vDash E$, then $d(H \,\&\, X, E) = \Pr(X \mid H) \cdot d(H, E)$.

- Neither $r$ nor $l$ satisfies property (6) [11].
- Rosenkrantz does provide some (pretty good) reasons to reject $r$. However, he [45] explicitly admits that he knows of "no compelling considerations that adjudicate between" $d$ and $l$.
- So, it is (at best) unclear how one might consistently complete Rosenkrantz's enthymematic treatment of the tacking problem.
- What's worse, as I will explain later, I think there are good reasons to *favor l over d* as a measure of support.





## Extra Slide #6: Earman on "Irrelevant Conjunction"

- Earman [6] gives a more robust resolution of the tacking problem which requires only the following logically weaker cousin of (6):

(6′) If $H \vDash E$, then $d(H \,\&\, X, E) < d(H, E)$.

- $r$ violates even this weaker condition, but $l$ *satisfies* (6′) [11].
- In this sense, Earman's account is *less* sensitive to choice of measure (*i.e.*, more robust) than Rosenkrantz's is.
- Earman's account can be bolstered by providing compelling independent reasons to favor $d$ (or $l$) over $r$ (*e.g.*, see below).
- Unfortunately, even the bolstered version of Earman's account is inadequate. I provide a new and improved Bayesian resolution of the problem of irrelevant conjunction in [11].





## Extra Slide #7: Goodman's "Grue" Paradox

- Goodman presents an example involving the following two hypotheses ($H$ and $H'$) and observation report ($E$):

  $H$: All emeralds are green.
  $H'$: All emeralds are grue.
  $E$: All emeralds that have been observed are green ($\therefore$ grue).

- Where, the predicate "grue" is defined as follows:

  $x$ is grue if and only if either (*i*) $x$ has been observed and $x$ is green, or (*ii*) $x$ has not been observed and $x$ is blue.

- Bayesian answers to Goodman's "new riddle of induction" have aimed to establish that $H$ is better supported by $E$ than $H'$ is. That is, Bayesians have tried to show that $\mathfrak{c}(H, E) > \mathfrak{c}(H', E)$.
- As we have seen, at least two Bayesian accounts along these lines (those of Eells and Sober) are sensitive to choice of measure $\mathfrak{c}$.

**Extra Slide #8: Eells on Goodman's "Grue" Paradox**

- Eells [7] offers a Bayesian account of the Grue paradox (*a.k.a.*, Goodman's "new riddle of induction") which trades on the following property of the difference measure [where $\beta$, $\delta$ are: $\beta =_{df} \Pr(H_1 \,\&\, E) - \Pr(H_2 \,\&\, E)$, and $\delta =_{df} \Pr(H_1 \,\&\, \neg E) - \Pr(H_2 \,\&\, \neg E)$]:

(7) If $\beta > \delta$ and $\Pr(E) < \frac{1}{2}$, then $d(H_1, E) > d(H_2, E)$.

- Neither $r$ nor $l$ satisfies property (7).
- Eells does provide reasons (as reported in a paper by Sober, see below) to prefer the difference measure $d$ over the log-ratio measure $r$, but he does not supply reasons to prefer $d$ over $l$.
- Pending such reasons, Eells's argument remains *enthymematic*.
- Moreover, I will later provide reasons to favor $l$ over $d$.





**Extra Slide #9: Sober on Goodman's "Grue" Paradox**

- Sober [52] describes a more robust Bayesian account of the Grue paradox which exploits the following weaker property of $d$:

(7′) If $H_1$, $H_2$ entail $E$ and $\Pr(H_1) > \Pr(H_2)$, then $d(H_1, E) > d(H_2, E)$.

- $r$ violates even this weaker condition, but $l$ *satisfies* (7′).
- In this sense, Sober's resolution of Goodman's "Grue" paradox is *less* sensitive to choice of measure (*i.e.*, more robust) than Eells's is.
- And, like Eells, Sober does provide *some* reasons to prefer $d$ to $r$.
- However, as I explain in my [13] and [15], these reasons (which are borrowed from Eells) are not very good reasons to prefer $d$ to $r$.
- Like Earman's account of "Irrelevant Conjunction," Sober's account of "Grue" can be bolstered by providing compelling independent reasons to favor $d$ (or $l$) over $r$ (*e.g.*, see below).





**Extra Slide #10: Earman on the Quantitative Problem of Old Evidence**

- Earman [6, pp. 120–121] argues that quantitative Bayesian confirmation theory, together with the "radical probabilism" of Jeffrey [26] does not suffice to avoid Glymour's problem of old evidence [17, pp. 63–69].
- His argument presupposes that Bayesians use $d$ to measure degree of confirmation, and it rests on the following fact about $d$:

(8) If $H \vDash E$, then $\Pr(E) \approx 1 \Rightarrow d(H, E) \approx 0$.

- This argument has two flaws. First, (8) does hold for $d$ and $r$, but it does *not* hold for $l$ or $s$ (contrary to what Earman suggests [6, p. 243, note 8]). Second, this argument only applies to the case of *deductive evidence* ($H \vDash E$).
- As it turns out, we can avoid Earman's objections, by using our $l$ instead of $d$:

(9) *Even if* $H \vDash E$ *and* $\Pr(E) \approx 1$, $l(H, E)$ can be *arbitrarily large*.

- As Joyce [27] and Christensen [5] point out, $s$ *also* satisfies (9).





# References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