# The Problem of Irrelevant Conjunction — Revisited

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- Three (formal) *qualitative* concepts of **confirmation**:
  - **Hypothetico–Deductive confirmation.**
    - $E$ confirms$_h$ $H$ if $H$ entails $E$.
  - **Confirmation as Firmness.**
    - $E$ confirms$_f$ $H$ iff $\Pr(H \mid E) > t$.
  - **Confirmation as Increase in Firmness.**
    - $E$ confirms$_i$ $H$ iff $\Pr(H \mid E) > \Pr(H)$.
- Two (formal) *comparative* confirmation relations:
  - **Comparative Firmness**.
    - $E$ confirms$_f$ $H_1$ more strongly than $E$ confirms$_f$ $H_2$ iff $\Pr(H_1 \mid E) > \Pr(H_2 \mid E)$.
  - **Comparative Increase in Firmness**.
    - $E$ confirms$_i$ $H_1$ more strongly than $E$ confirms$_i$ $H_2$ iff $\mathfrak{c}(H_1, E) > \mathfrak{c}(H_2, E)$. [where $\mathfrak{c}$ is some *relevance measure*]
- Two *informal* **evidential support** concepts:
  - $E$ **supports$_1$** $H$ iff $E$ is (positively) evidentially relevant to $H$.
  - $E$ **supports$_2$** $H$ iff $E$ warrants belief/acceptance of $H$.





(1) If $E$ confirms $H$, then $E$ confirms $H \,\&\, X$, for *any* $X$.

- Clark Glymour [5] raises two worries in connection with (1):

  (1a) [Confirmation$_h$ has property (1).] But we cannot admit, generally, that $E$ will lend plausibility to an arbitrary $X$. One might ... deny ... the special consequence condition. But ... sometimes ... confirmation does ... follow entailment.

  (1b) As evidence accumulates, we may come to accept [$p$] ... and when we accept [$p$] we commit ourselves to accepting all of its logical consequences. So, if [$E$] could bring us to accept ... $H$, and whatever confirms $H$ confirms $H \,\&\, X$ ... then ... [$E$] ... ought, presumably, to bring us to accept $X$.
- Both of these worries have to do with confirmation/support provided by $E$ “rubbing off” onto an *irrelevant conjunct* $X$.
- (1b) involves explications of support$_2$, which imply (1).
  - We don’t think (1b) is probative. *Nobody* thinks confirms$_h$ is a good explication of support$_2$. We’ll focus on support$_1$.
- To that end, let’s take a closer look at Glymour’s (1a).





- The Special Consequence Condition is:

  (SCC) If $E$ confirms $H \,\&\, X$, then $E$ confirms $X$.
- If we combine (1) & (SCC), we get an (absurd) consequence: *if E confirms **any** hypothesis, it confirms **every** proposition.*
- So, any theory that entails (1) — *e.g.*, confirms$_h$ — must *not* entail (SCC) — *on pain of triviality*. HD confirmation theory does *not* entail (SCC). But, Glymour wants something more.
- Glymour wants an explication of support$_1$ that avoids triviality — but not by a *mere* rejection of (SCC). In (1a), he is demanding a *principled* (and *explanatory*) rejection of (SCC).
- Next, we’ll examine two confirmation$_i$-based approaches to “the (1)-problem” — due to Earman and Rosenkrantz.
- After critiquing those approaches, I will discuss some alternative confirmation$_i$-based approaches that I prefer.
- Finally, I’ll return to Glymour’s (1a), and (time permitting) some recent objections due to Maher and Crupi *et. al.*

- Before getting into confirms$_i$-based approaches to "the problem of irrelevant conjunction" [*i.e.*, "the (1) problem"], we must ask whether there *is* such a problem *for confirms$_i$*.
- First, note that the confirms$_i$-analogue of (1) is *false*[1] — *i.e.*:

  (2) $E$ confirms$_i$ H $\not\Rightarrow$ $E$ confirms$_i$ $H \,\&\, X$.

- Thus, confirms$_i$ does *not* suffer from a *strictly analogous* problem of irrelevant conjunction. However, we *do* have:

  (3) If $H$ *entails* $E$, then $E$ confirms$_i$ $H \,\&\, X$ (for arbitrary $X$).

- So, in the *special (deductive) case* where $H$ *entails* $E$ (*i.e.*, where $E$ confirms$_h$ $H$), confirmation$_i$-theory *does* entail (1).
- Contemporary confirms$_i$-theorists have had various things to say about (3). I will discuss two prominent approaches.
- Then, I'll explain why I don't think these approaches are very satisfying. And, I'll discuss some alternatives.

[1] Of course, the confirms$_f$-analogue of (1) is *also* false, but I won't go there.





- Earman [3] points out that — for *many* $\mathfrak{c}$'s — we have:

  (3.1) If $H$ entails $E$, then $\mathfrak{c}(H \,\&\, X, E) < \mathfrak{c}(H, E)$.

- What (3.1) says is that, while "irrelevant conjunctions" $H\&X$'s will be confirmed$_i$ by *deductive* evidence for $H$, such conjunctions will be confirmed *less strongly* than $H$ is.
- Closer scrutiny of Earman's approach reveals:
  (a) The "irrelevance" of $X$ is *irrelevant* to the decrease in $\mathfrak{c}_i$. After all, (3.1) is true for ***all*** $X$ — irrelevant or otherwise.
  (b) (3.1) is *not true for all* $\mathfrak{c}$'s (*e.g.*, it fails for $r(H,E) \overset{\text{def}}{=} \frac{\Pr(H \mid E)}{\Pr(H)}$).
  (c) (3.1) only applies to cases of *deductive* evidence.
    - Arguably, this is not such an important case, since most interesting applications of confirms$_i$ involve *statistical* $H$'s.
    - Moreover, as we'll see below, a *more general problem* of *irrelevant* conjunction plagues confirmation$_i$-theory — in *both* the deductive *and* the non-deductive cases.
- Earman's is not the only confirms$_i$-approach one finds in the literature. Rosenkrantz offers a different approach...





- Rosenkrantz [8] offers a confirms$_i$-approach — based on the following [where $d(H,E) \overset{\text{def}}{=} \Pr(H \mid E) - \Pr(H)$]:

  (3.2) If $H$ entails $E$, then $d(H \,\&\, X, E) = \Pr(X \mid H) \cdot d(H,E)$.

- Rosenkrantz does try to address *some* of the problems with Earman's account. In particular, he seems sensitive to (a):

  > ...I hope you will agree that the two extreme positions on this issue are equally unpalatable, (i) that a consequence $E$ of $H$ confirms $H \,\&\, X$ not at all, and (ii) that $E$ confirms $H \,\&\, X$ just as strongly as it confirms $H$ alone. ...In general, intuition expects intermediate degrees of confirmation that depend on the degree of compatibility of $H$ with $X$.

- Adopting $\Pr(X \mid H)$ as his measure of the "degree of compatibility of $H$ with $X$", and $d$ as his measure of confirmation$_i$ yields the kind of result that Rosenkrantz wants: (3.2). Is this an *improvement* on Earman?
- This depends on whether Rosenkrantz really has adequately addressed worries (a)–(c), above. I don't think he has...





- In a way, Rosenkrantz is *trying* to address (a) here. He seems to be thinking of $\Pr(X \mid H)$ as a kind of measure of "the degree of relevance" of $X$ — *qua conjunct* in $H \,\&\, X$.
- But, this is a *very peculiar* way for a *Bayesian* to explicate "relevance"! Moreover, $\Pr(X \mid H)$ can tell us nothing about "degrees of relevance" involving $X$, $H$ — *and* $E$.
- Moreover, when it comes to (b), Rosenkrantz is in even worse shape than Earman. Rosenkrantz's approach works *only* for confirmation$_i$-measures that are *very similar* to $d$.
- Finally, Rosenkrantz is still only addressing the *deductive* case. So, his account lacks *generality* in the same ways that Earman's approach does. Thus, he has not addressed (c).
- I think $\mathfrak{c}_i$-theorists need to *re-think* the problem of irrelevant conjunction, and its possible resolution(s).
- To that end, let's see how $\mathfrak{c}_i$-theory handles *irrelevant* conjunctions, in the general, *inductive* case...

- First, we need to say what it *means* for $X$ to be an *irrelevant conjunct* to a hypothesis $H$, with respect to evidence $E$.
- Here's a natural explication, for a confirmation$_i$-theorist [6]:
  - $X$ is an *irrelevant conjunct to* $H$*, with respect to evidence* $E$, just in case $\Pr(E \mid H \,\&\, X) = \Pr(E \mid H)$ [*i.e., if* $X \perp\!\!\!\perp E \mid H$].
- This is a more natural explication of "irrelevant conjunct" than Rosenkrantz's (implicit) explication, since:
  - It makes use of *probabilistic independence*, which is a standard way for $\mathfrak{c}_i$-theorists to explicate *irrelevance*.
  - It's a relation involving $X$, $H$, *and* $E$ (as it intuitively should be).
  - It's a natural (likelihood-based) *generalization* of the special, *deductive* case that has been traditionally discussed.
- With this explication of "irrelevant conjunct" in hand, we can now *state* a *more general problem of irrelevant conjunction — for confirmation*$_i$*-theory* — as follows:

(4) If $E$ confirms$_i$ $H$, and $X$ is an irrelevant conjunct to $H$, with respect to evidence $E$, then $E$ *also* confirms$_i$ $H \,\&\, X$.





- So, we *have* a (general) "problem of irrelevant conjunction" for for confirmation$_i$-theory. What can be said about it?

(4′) If $E$ confirms$_i$ $H$, and $X$ is an irrelevant conjunct to $H$, with respect to evidence $E$, then $\mathfrak{c}(H \,\&\, X, E) < \mathfrak{c}(H, E)$.

- What (4′) tells us is that — while irrelevant conjunctions will be confirmed$_i$ to *some* degree by ($H$-confirming evidence) $E$ — adding irrelevant conjuncts will lead to a *decrease* in $\mathfrak{c}_i$.
  - The *precise amount* by which $\mathfrak{c}_i$ is decreased by the addition of irrelevant conjuncts will depend on which relevance measure $\mathfrak{c}$ is used. But, "Rosenkrantz-like" equations [*i.e.*, (3.2)-like equations] can be deduced for each measure.
- (4′) is a generalization of Earman's (3.1). And, like Earman's (3.1), (4′) holds for *most* $\mathfrak{c}$'s (again, a notable exception being $r$).
- On the next slide, I'll return to Glymour's (1a) and the (SCC).
- Then, I'll address some objections to our approach that have appeared in the recent *Philosophy of Science* literature.





- To illustrate our approach, consider the following example:
  - Suppose we'll be sampling a card at random from a standard deck. Let $E$ be the proposition that the card is black. Let $X$ be the hypothesis that the card is an ace, and let $H$ be the hypothesis that the card is a spade.
- The preconditions for our (4) and (4′) are met here, since:
  - $E$ confirms$_i$ $H$.
  - $\Pr(E \mid H \,\&\, X) = \Pr(E \mid H)$.
- Therefore, (4) and (4′) entail the following:
  - $E$ confirms$_i$ $H \,\&\, X$.
  - $\mathfrak{c}(H \,\&\, X, E) < \mathfrak{c}(H, E)$, for "most" relevance measures $\mathfrak{c}$.
- Finally, we *also* have the following:
  - $E$ does *not* confirm$_i$ $X$.

☞ We think all these predictions of our confirms$_i$-explications line-up well with the support$_1$-relations. ∴ We think ours is *no mere* rejection of (SCC). It's *principled* (and explanatory).





- Patrick Maher [7] complains that our approach doesn't *address* the problem of irrelevant conjunction (PIC), because he thinks the PIC is *grounded on the following intuition*:

($\star$) If $X$ is an irrelevant conjunct to $H$, with respect to evidence $E$, then $E$ *does not support*$_1$ $H \,\&\, X$.

- ∴ Maher thinks that *the* way to resolve the PIC is *merely* to point out that ($\star$) is *false* (as we do in our example above).
- We agree that ($\star$) is false. We also agree that *some* people *may* be worried about PIC *because* they accept ($\star$).
- But, we *disagree* with Maher on the following two points:
  - We *don't* think acceptance of ($\star$) is *essential* to the problem and/or its motivation [did ($\star$) ground PIC for *Glmyour*?].
  - We think our approach and analysis *further illuminates* what is going on — from a confirmation$_i$ point of view.
- So, we are not moved by Maher's worries about our approach. Next, we'll discuss a more recent objection…

- Crupi *et al.* have recently argued [2] that our approach yields incorrect predictions — in cases of *dis*confirmation.
- To understand their worry, it helps to state the results we had in mind in a slightly more general (and revealing) way.
- Let's assume that confirmation$_i$-measures ($\mathfrak{c}$) take *negative* values in cases of *dis*confirmation$_i$ and *positive* values in cases of *con*firmation$_i$. And, assume we're talking about the measures $\mathfrak{c}$ we had in mind when we put forward our (4′).
- Given these assumptions, we can *actually* show that:
  - (†) If $X$ is an irrelevant conjunct to $H$, with respect to $E$, then $|\mathfrak{c}(H \,\&\, X, E)| < |\mathfrak{c}(H, E)|$.
  - ($4^\dagger$) ∴ If $E$ *dis*confirms $H$, and $X$ is an irrelevant conjunct to $H$, with respect to $E$, then $\mathfrak{c}(H \,\&\, X, E) > \mathfrak{c}(H, E)$.

☞ For the measures $\mathfrak{c}$ we had in mind when we put forward (4′), we get the result that *adding irrelevant conjuncts to E-**dis**confirmed hypotheses **increases** degree of confirmation.*





- Crupi *et. al.* think our (†) and ($4^\dagger$) are counter-intuitive.
- In fact, they defend the following *contrary* claim:
  - $\sim(4^\dagger)$ If $E$ *dis*confirms $H$, and $X$ is an irrelevant conjunct to $H$, with respect to $E$, then $\mathfrak{c}(H \,\&\, X, E) < \mathfrak{c}(H, E)$.
- And, they use $\sim(4^\dagger)$ to bolster their (pre-existing) case for a "piece-wise" confirmation measure $z$, which treats *confirmation* and *disconfirmation* as *different functions*:

$$z(H,E) = \begin{cases} \frac{\Pr(H|E)-\Pr(H)}{\Pr(\sim H)} & \text{if } \Pr(H \mid E) \geq \Pr(H) \\ \frac{\Pr(H|E)-\Pr(H)}{\Pr(H)} & \text{if } \Pr(H \mid E) < \Pr(H) \end{cases}$$

- I won't be able to discuss the very clever (independent) argument in favor of $z$ that Crupi *et. al.* had previously published [1]. But, our response *here* is to *bite the bullet*.

  ☞ It seems to us that an irrelevant conjunct (one that doesn't alter the likelihood $H$ attributes to the evidence) adds nothing but "extra mass" to the hypothesis. This "extra mass" just makes the incremental confirmation *and disconfirmation* of $H \,\&\, X$ "more sluggish" than for $H$ alone.






[1] V. Crupi and K. Tentori and M. Gonzalez, *On Bayesian Measures of Evidential Support: Theoretical and Empirical Issues*, *Philosophy of Science*, 2007.

[2] V. Crupi and K. Tentori, *Irrelevant Conjunction: Statement and Solution of a New Paradox*, *Philosophy of Science*, 2010.

[3] J. Earman, *Bayes or Bust?*, MIT Press, 1992.

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[5] C. Glymour, *Theory and Evidence*, Princeton U. Press, 1980.

[6] J. Hawthorne and B. Fitelson, *Re-solving irrelevant conjunction with probabilistic independence*, *Philosophy of Science*, 2005. URL: `http://fitelson.org/ic_2.pdf`.

[7] P. Maher, *Bayesianism and Irrelevant Conjunction*, *Philosophy of Science*, 2004.

[8] R. Rosenkrantz, *Bayesian confirmation: Paradise regained*, British Journal of Phil. of Sci., 1994.