Conditionals Seminar: Day 4

- Administrative:
  - I've added some papers to the website (also, OUP links to *all chapters*!)
  - Especially interesting are Stalnaker, Weatherson, Davis, and von Fintel
  - Today's my last day. Next: Peter on Ch. 6. Then, Alan H. on Ch. 5.
  - We'll skip Chs. 8 & 9, so we can do more $>$. Ch. 7 (Week 7) volunteer?
  - We have Wks. 8 (Aaron) and 11 (Ellen) covered. See web & volunteer!
  - I plan to meet with presenters Monday afternoons, prior to meeting.
- Today: Chapter 4 Stuff (and my quick take on the chapter 5 stuff)
  - Probability and Acceptability: Bennett's Strong Assumption About Them
  - Lottery Paradoxes: A Cautionary Tale about Probability and Acceptance
  - The Equation (and my take on the Lewisian attacks on it)
    - Bennett's Defence of the Equation + The Ratio "Formula"
    - My Favorite Lewis-Style "Triviality" Proof (Milne)
    - A Conflict Involving Import-Export, The Equation, and non-$\supset$-Analyses
    - Postscript: Confirmation, Indicative Conditionals, and Import-Export

Chapter 4: The Equation — §19 (Other Approaches)

- Strawson's (*non*-$\supset$) idea was that the *meaning* of $\rightarrow$ involves a *metaphysical* "connection" between antecedent and consequent. Counterexamples?

  'If she apologized to him, then he lied to me.'

  '(Even) if he apologizes, I shall (still) be angry.'
- Davis and Stalnaker give (*non*-$\supset$) "possible world" semantics for $\rightarrow$ (later on).
- Lycan has yet another (*non*-$\supset$) approach, which we'll also discuss later on.
- Bennett thinks the Ramsey Test is key to a correct (*and non*-$\supset$!) analysis of $\rightarrow$. "The *acceptability* of $A \rightarrow C$ for a person at a time is governed by the probability the person then assigns to C on the supposition of A [$\Pr(C \mid A)$]."
- Bennett makes some strong assumptions here about "acceptability" and Pr:

  > ...acceptability depends upon probability and nothing else: you ought not to accept what you do not find probable, and there is no obstacle to your accepting what you do find probable. So we can move to the thesis that the probability for you of $A \rightarrow C$ is proportional to your probability for C on the supposition of A.

Digression: Why Acceptability May Depend On Something More Than Probability

- Let $R(x)$ mean "there is no obstacle to your accepting x". Let $\Pr(x) \geqslant r$ say that x is "probable," according to your (rational) Pr. Consider the following claim about "high" Pr and there being "no obstacle to acceptance" (NOA).

(1) $R(x) \iff \Pr(x) \geqslant r$. [high (rational) Pr is necessary and sufficient for NOA]

- (1) faces challenges in both directions. First, the ($\Rightarrow$) direction:
- Let B_1 and B_2 be two propositions such that (*i*) $R(B_1)$ and $R(B_2)$, and (*ii*) B_1 and B_2 are *independent*, according to your (rational) credence function Pr.
- Now, assume (*iii*) $\Pr(B_1) = \Pr(B_2) = r < 1$. Then, B_1 & B_2 will *not* have high Pr: $\Pr(B_1 \,\&\, B_2) = r^2 < r$. Can't this be consistent with $R(B_1 \,\&\, B_2)$?
- Example: B_1: Bonds will hit another home run this season; B_2: it will rain in Berkeley sometime next January. *Mightn't* it be OK to accept B_1 & B_2?
- Generally, if you accept two independent propositions non-irrationally, and you (rationally) assign each of them Pr = r (the "threshold value"), then, if ($\Rightarrow$) is correct, you *cannot* rationally accept their conjunction. Too strong?

- Intuitively, high Pr may also not be *sufficient* for "there being no obstacle acceptance". Let L_i state that ticket i will not be the winning ticket in a lottery that you *know* to be very large (n) and to have exactly one winning ticket.
- Then, rational degrees of belief may be such that $\Pr(L_i)$ is (very) high, for each i. But, $\Pr(L_1 \,\&\, \cdots \,\&\, L_n) = 0$. Assuming ($\Leftarrow$), you *shouldn't* accept the conjunction of the L_i, but "there is no obstacle to your accepting" *each* L_i.
- So, ($\Leftarrow$) implies that $\mathcal{B}$ = "the set of propositions x such that $R(x)$" isn't closed under *known* logical consequence. *The Lottery Paradox*. If you think that $\mathcal{B}$ *is* closed under known-$\vDash$, you'll *reject* ($\Leftarrow$), on pain of contradiction.
- Perhaps a weaker claim can be salvaged here? What if we retreat to:

(2) $R(x) \Rightarrow \Pr(x)$ is *not low*. [*i.e.*, *low* $\Pr(x)$ is sufficient for $\neg R(x)$]

- This principle may seem plausible. But, again, if you think $\mathcal{B} = \{x \mid R(x)\}$ is closed under known-$\vDash$, then you'll reject *even* (2). Extend the first case above to n independent propositions, each NOA, and each with a Pr = r (high). Their conjunction will have *low* Pr, provided n is *large enough*.

§20: Kinds of Probability

- We have seen the formal theory of probability (axiomatically and algebraically presented). There are various *interpretations* (models) of this formal theory:
  - Physical Probabilities (*a.k.a.*, objective chances of events): If the universe is indeterministic, then even knowing all the laws and all the history of the universe will yield at most knowledge of *probabilities of* future events.
    - Propensities of some token events to bring about others (in set-ups).
    - Hypothetical limiting frequencies of event-types in chance set-ups.
  - Statistical Frequencies of Event-Types. Even in deterministic universes, systems will exhibit statistical regularities in the frequencies of event-types (relative to fixed reference classes of events). *E.g.*, Classical models of games of chance, and classical systems in statistical physics.
  - **Personalistic Probabilities (Degrees of belief of "rational" agents).**
    - ***Pragmatically* rational degrees of belief (expected utility theory, etc.)**
    - ***Epistemically* rational degrees of belief (probabilistic epistemology)**
- Hájek's SEP entry (online) is *the* thing to read on "kinds of probability".

§21: Elements of "Probability Logic"

- Here, Bennett uses "probability logic" more generally than usual (typically, this refers to the study of the transmission of probabilities by inferences). He means "probability calculus" here (later, Bennett does discuss Pr-*logic*).
- We've already seen the probability calculus (NOTE: our presentation is more rigorous and illuminating, which will be important for giving a more accurate and comprehensible story about the issues raised in chapters 5–8).
- A few more theorems Bennett discusses here (obvious, *algebraically*):
  - If q entails r, then $\Pr(q) \leqslant \Pr(r)$.
  - Therefore, if q is logically equivalent to r, then $\Pr(q) = \Pr(r)$.
  - $\Pr(\neg p) = 1 - \Pr(p)$
- **Definition**. p and q are *independent* under Pr iff $\Pr(p \,\&\, q) = \Pr(p) \cdot \Pr(q)$.
- **Definition**. p and q are *correlated* under Pr iff $\Pr(p \,\&\, q) > \Pr(p) \cdot \Pr(q)$.
- **Definition**. p and q are *anti-correlated* under Pr iff $\Pr(p \,\&\, q) < \Pr(p) \cdot \Pr(q)$

§22: The Ratio "Formula"

- The conditional probability of p, *given* q is written $\Pr(p \mid q)$, and it is supposed to represent (for our purposes) the degree of belief an agent [whose credence function is $\Pr(\cdot)$] assigns to *p on the supposition that q is true*.
- We've seen that $\Pr(p \mid q)$ is, traditionally, *defined* in terms of the unconditional probability function $\Pr(\cdot)$, as the *ratio* $\frac{\Pr(p \& q)}{\Pr(q)}$ [*i.e.*, when $\Pr(q) > 0$].
- A ratio *definition* implies that $\Pr(p \mid q)$ is *undefined* when $\Pr(q) = 0$. Bennett does *not* want to take the ratio as a *definition* or an *analysis* of $\Pr(\cdot \mid \cdot)$. He just wants to view the "ratio *formula*" as a *truth* about $\Pr(p \mid q)$, when $\Pr(q) > 0$.
- He thinks the *Ramsey Test* is "definitive" of conditional probability, and (therefore) that the ratio formula is (somehow) *implied by* the Ramsey Test.
- He concedes that there are cases in which the ratio can't be determined (or has no value), despite the fact that $\Pr(\cdot \mid \cdot)$ can be (or does). He avoids those cases (mainly, Hájek's $\vDash$ examples) by eschewing "independent conditionals".

- There are other cases in which $\Pr(p \,\&\, q)$ is not known, but $\Pr(p \mid q)$ is. For this problem, Bennett introduces another formula (he says is "arithmetically equivalent" to RF), which can be used (sometimes) to determine $\Pr(p \,\&\, q)$:

  $\Pr(p \,\&\, q) = \Pr(p \mid q) \cdot \Pr(q)$ [note: says nothing useful when $\Pr(q) = 0$]
- He ignores continuous magnitudes (points with "measure zero"). He says that "no such theoretical possibility could have any bearing on our lives". This seems pretty reasonable to me, since we're talking about *credences*.
- Bennett *needs* the ratio formula to be true, and he argues for its plausibility.
- He examines the (true) dilemma $\Pr(p \,\&\, q) < \Pr(q)$ or $\Pr(p \,\&\, q) = \Pr(q)$. He says the ratio gives intuitive answers for both horns. But, so do *other* functions $f(\Pr(p \,\&\, q), \Pr(q))$. Q: Why the *ratio*? A: f must be a *probability*!
- It's the cases where $\Pr(A) = 0$ that give the ratio the most trouble, intuitively. And, these are the only cases – in which all unconditional probabilities are known/crisp – where Hájek's arguments have force. *E.g.*, $\Pr(A \mid A) = 1$.
- This is why Bennett devotes a whole section to "zero intolerance" – §23.

§23: Indicative Conditionals are Zero-Intolerant

- Bennett argues that the RF is not missing anything by falling silent when $\Pr(A) = 0$. He thinks indicative conditionals are "zero intolerant":

  > ...combine the Ramsey test thesis with the Ratio Formula ... the acceptability to a person of $A \to C$ is given by his probability for $A \,\&\, C$ divided by his probability for A: 'Believe $A \to C$ to the extent that you think $A \,\&\, C$ is nearly as likely as A'. You can do nothing with this in the case where your $\Pr(A) = 0$.

- *Careful*! If $\Pr(A) = 0$, then $\Pr(A \,\&\, C) = 0$. So, they are *equally* likely! Only the *ratio* measure of "how close" these are is undefined in this case!
- He acknowledges three kinds of counterexamples, which he then ignores:
  1. **Conversational Stretches**. We assert $A \to C$ when *for us* $\Pr(A) = 0$, because the *hearer* is *not* such that *for them* $\Pr(A) = 0$. But, this seems to apply only to *assertibility*, and not *acceptability* of the conditional.
  2. **Inferred →s**. "The thesis that indicative conditionals are zero-intolerant should be confined to ones that stand on their own feet, and not applied to

inferred ones." $A \to C$ may be accepted, *despite* $\Pr(A) = 0$, since it's a *known consequence of* $A' \to C'$; $R(A' \to C')$, and $\Pr(A') \neq 0$. *Closure?*

  3. **Independence, Non-Interference, and something "in between"**.
     (a) With independent conditionals, we can simply move straight to certainty in the consequent without worrying about how to "accommodate" $\Pr(A) = 1$ – we can just "dump it in there and go straight to $\Pr(C) = 1$."
     (b) Non-interference conditionals are accepted on the ground that the person holds C to be true and thinks that A's being true would not interfere with that (no need for Ramsey here either, according to JB).
     (c) Other examples don't fit exactly into either of these classes. Hájek:
     (1) '(Even) if (A) there has never been a God, (B) the first sentence of the Book of Genesis (still) says that God created the heaven and the earth'
     (2) 'If (A) there's never been a God, then (C) the first sentence of Gen. is false'
     I can accept (1) $A \to B$ as a non-interference conditional, even as a devout theist. B asserts the existence of some item (here, a statement) b, and C assigns to b a relational property (here 'is false') which it must have if A is true. So I accept (2) $A \to C$, but it's *not* non-interference.

§24: The Equation

- **The Equation** asserts that the acceptability (for S) of the indicative conditional is the conditional probability of C, given A (that S assigns):
  $\Pr(A \to C) = \Pr(C \mid A) = \frac{\Pr(A \& C)}{\Pr(A)}$ if $\Pr(A) > 0$.
- Something like this thesis has been defended by Stalnaker, Adams, and Jeffrey (among others). I add the ratio at the end, since *Bennett* accepts it as true.
- Bennett suggests here that '$A \to C$' *does not express a proposition*. He says, for this reason, that there is no account of the *truth-conditions* of '$A \to C$'. But, he still thinks this consistent with the "Ramseyan spirit" of The Equation.
- This will take a lot of explaining on his part! If '$A \to C$' does not express a proposition, then how can it *have* a *probability*? More on this later [Bennett will take the position that its "probability" is not a probability of *truth*].
- In Chapter 5, various "triviality" arguments against The Equation are discussed. Bennett thinks these, too, point to his "anti-propositionality" thesis.

- I think this is a wrongheaded reaction to the "triviality" arguments.
- I'll let Alan summarize the most compelling arguments he sees for "triviality". And, I'll let him respond to my challenges to them.
- I'll argue that *Bennett* should think they rest on a false presupposition about *iterated* →s. This will be *despite* his attempts to argue *in support of* this presupposition. Interestingly, he doesn't seem to address my worries here.
- Bennett is committed to (simultaneously):
  – *Rejecting* the horseshoe analysis (outright): (i) $\Pr(A \to C) = \Pr(A \supset C)$.
  – *Accepting* two theses (in his *reductio* of the propositionality of '$A \to C$'):
  (ii) The Equation: $\Pr(A \to C) = \Pr(C \mid A)$.
  (iii) Pr-Importation-Exportation: $\Pr(B \to (A \to C)) = \Pr((A \,\&\, B) \to C)$.
- (ii) and (iii) jointly entail (i), if '$A \to C$' expresses a proposition. This is why Bennett rejects propositionality. Bennett defends (ii) and (iii). But, his defense of (iii) is rather weak. And, he doesn't discuss Gibbard's argument about IEL.

(IEL) $B \to (A \to C)$ is *logically equivalent* to $(A \,\&\, B) \to C$.

Chapter 5: Lewisian "Triviality Proofs" — A Brief, Opinionated Overview

- A.H. will present this stuff in more detail (and Lewis-style) on Oct. 5. Meanwhile, here's my take on it. I begin with my favorite "proof" (Milne):

  Premise 1. S's (*propositional*!) degree of belief structure $\langle \mathcal{B}, \Pr \rangle$ is a *probability model* (all we *need* is the assumption that S's degrees of belief Pr over *propositions* [$\mathcal{B}$] satisfy the two *expansion laws* **T1** and **T2**, above).

  Premise 2. S's algebra of propositions (propositional belief structure) $\mathcal{B}$ is *closed* under indicative implication. That is, for all $a, c \in \mathcal{B}$, $a \to c \in \mathcal{B}$.

  Premise 3. S's Pr satisfies The Equation: $\forall a, c \in \mathcal{B}$, $\Pr(a \to c) = \Pr(c \mid a)$.

  Premise 4. S's Pr satisfies the following constraint (Lewis): $\forall a, b, c \in \mathcal{B}$,
  $$\Pr(a \to c \mid b) = \Pr(c \mid a \,\&\, b)$$

  Conclusion. S's Pr is such that, for all *contingent* [$\Pr \in (0,1)$] $a, c \in \mathcal{B}$,
  $$\Pr(a \to c) = \Pr(a \supset c)$$

- This argument is valid (Milne). Of course, Bennett denies its conclusion! So, he must deny at least one of its four premises. In fact, he denies Premise 2.

- If I were an Equation Guy, my reaction to this "proof" would *not* be to deny that $\mathcal{B}$ is closed under $\to$. To be honest, I'm not even sure what that would mean (more later). Another potential problem with the argument is Premise 4.
- Lewis calls Premise 4 "closure of the class of probability functions under conditionalization", and Bennett reports a "proof" of Premise 4 due to Alan H.
- I think this "proof" is fallacious (more when Alan is here!). Moreover, Lewis' constraint has nothing in particular to do with conditionalization. In fact, it is equivalent to a constraint on *unconditional* probabilities of *iterated* $\to$s!
- From Premises 1–3 (*i.e.*, The Equation), we can derive the following pair:
  - For all $a, b, c \in B$, $\Pr(b \to (a \to c)) = \Pr(a \to c \mid b)$.
  - For all $a, b, c \in B$, $\Pr(c \mid a \,\&\, b) = \Pr((a \,\&\, b) \to c)$.
- Therefore, given Premises 1–3, Lewis' constraint (Premise 4) is *equivalent to*:

  Premise 4*. S's Pr satisfies the following constraint: $\forall a, b, c \in \mathcal{B}$,
  $$\Pr(b \to (a \to c)) = \Pr((a \,\&\, b) \to c)$$

- This is just the Pr-*importation/exportation* for $\to$, which we saw above.

- There are independent reasons (even for *Bennett*!) to reject Premise 4*. As you might expect, Bennett later *defends* Premise 4*, which he calls "And-If" (§25, §40). Unfortunately, he doesn't discuss Gibbard's argument, below.
- Unless $\to$ satisfies (IEL), why should one accept Premise 4* (Bennett concurs: §40)? Gibbard argues that if $\to$ satisfies (IEL), then $\to = \supset$.
- Assume that $\to$ satisfies (IEL). Then, (1) is logically equivalent to (2):

  (1) $(A \supset C) \to (A \to C)$
  (2) $((A \supset C) \,\&\, A) \to C$

  Substitutivity of classical logical equivalents into $\to$s implies that (2) $\dashv\vDash$ (3):

  (3) $(A \,\&\, C) \to C$

  Since (3) is a logical truth (on any theory of $\to$?), (IEL) allows us to infer that (2) and (1) are also logical truths. But, $(p \to q) \vDash (p \supset q)$. So (1) entails:

  (4) $(A \supset C) \supset (A \to C)$

  So (4) is a logical truth. Thus, $(A \supset C)$ entails $A \to C$, hence $\to = \supset$. *QED*.
- $\therefore$ IEL entails the $\supset$-analysis. Bennett rejects the $\supset$-analysis, but he (I think) would accept the premises of Gibbard's argument, *and* he defends IEL. *Huh?*

My Last Rant?: Confirmation and Belief/Assertion of $A \to C$

- I recommend augmenting the probabilistic (Ramsey) story about $A \to C$.
- Many people (*e.g.*, Jackson) think that there is more to assertibility and/or believeability of $A \to C$ than just high conditional probability of C, given A.
- I'm inclined to agree. But, what else? How about *no negative relevance*?
- **Definition**. X *disconfirms* Y (for S) just in case X and Y are *anti-correlated* under S's Pr. *I.e.*, if the likelihood-ratio $l(Y, X)$ is < 1: $l(Y, X) = \frac{\Pr(X \mid Y)}{\Pr(X \mid \neg Y)} < 1$.
  - ∗ $A \to C$ is assertible/believable (by S) *only if* A does not disconfirm C.
- The requirement of non-negative Pr-*relevance* makes sense. What if A *lowers* the probability of C (even if $\Pr(C \mid A)$ is high)? Do we want to say '$A \to C$'? [Example: 'If you smoke two packs a day, then you won't get lung cancer.']
- NOTE: This is consistent with 'even if' conditionals being assertible, since those are such that $l(C, A) = 1$. It's also consistent with $\Pr(C) = 1$ or $\Pr(A) = 0$ cases being assertible [since $l(C, A)$ is *undefined* in those cases].

Supposition, Conjunction, Confirmation, and "Import-Export" for $\to$

- If A's not disconfirming C is a necessary condition for the assertibility of '$A \to C$', then "Importation-Exportation" is implausible for assertibility.
- The nature of supposition *vs* conjunction in Pr-confirmation theory makes it possible to have *both* (NOTE: *not* possible with *deductive* confirmation!):
  - A confirms C, *given* B. $[\Pr(C \mid A \,\&\, B) > \Pr(C \mid B)]$
  - $A \,\&\, B$ disconfirms C, *unconditionally*. $[\Pr(C \mid A \,\&\, B) < \Pr(C)]$
- Here's an example (fictional!) involving admissions, departments, and gender.

	Male	Female	All People
Geography	$\frac{11}{45}$ (0.24)	$\frac{28}{55}$ (0.51)	$\frac{39}{100}$ (0.39)
History	$\frac{54}{55}$ (0.982)	$\frac{44}{45}$ (0.978)	$\frac{98}{100}$ (0.98)
All Departments	$\frac{65}{100}$ (0.65)	$\frac{72}{100}$ (0.72)	$\frac{137}{200}$ (0.685)

- Given these statistics (assume they are accurate/representative, and that it would be rational to emulate them), and letting A = Hilary is a female, C = Hilary is admitted into graduate school, and B = Hilary applies to Geography:
  - A confirms C, *given* B. $[\Pr(C \mid A \,\&\, B) = \frac{28}{55} > \Pr(C \mid B) = \frac{39}{100}]$
  - $A \,\&\, B$ disconfirms C, *given* $\top$. $[\Pr(C \mid A \,\&\, B) = \frac{28}{55} < \Pr(C \mid \top) = \frac{137}{200}]$
- So, in this example, we have:
  - The conditional '$B \to (A \to C)$' passes the no disconfirmation test.
  - The conditional '$(A \,\&\, B) \to C$' fails the no disconfirmation test.
- Thus, '$(A \,\&\, B) \to C$' is not assertible in this case. But, *ceteris paribus*, '$B \to (A \to C)$' is. I think this is a counterexample to "Import-Export" *i.e.*, the claim that '$B \to (A \to C)$' is assertible *iff* '$(A \,\&\, B) \to C$' is.
- NOTE: *Even if* $\Pr(A \to C) = \Pr(A \supset C)$, we can *still* have *both* $\Pr(A \to C \mid B) > \Pr(A \to C)$ *and* $\Pr(C \mid A \,\&\, B) < \Pr(C)$ *simultaneously*.
- So the issue of disconfirmation and "Import-Export" *cross-cuts* the $\supset$ debate. If there's more to assertibility than $\Pr(C \mid A)$, then who's afraid of "triviality"?