Problems with Objective Probability for Conditionals

What Bennett Means by Objective Probability

- Objective probability is any sort of probability which demands inter-subjective agreement. Bennett distinguishes two types of objective probability.
    - Absolute objective probability: This is the sort of probability which assigns all true facts 1 and all false facts 0. Thus the probability booth shot lincoln is either 0 or 1. Only genuinely physically underdetermined future events like “This radioactive molecule will decay in the next 10 minutes” receive non-extreme truth values.
    - Relative objective probability: Gives the amount of probability a particular body of evidence confers on a statement. Interestingly on page 47 Bennett introduces this as a three place relation, R(P,Q,n), between a proposition P, body of evidence Q and a measure of probability n.
        - One can think of this as a Keynesian inductive probability where R(P,Q,n) gives the degree of support evidence Q confers on proposition P according to measure n.
            - What boolean algebra is this probability defined on? Perhaps Bennett thinks that this is built into parameter n (i.e. is just dom(n)) but this is unclear. To be fair perhaps Bennett is holding off identifying the space here because he thinks the best possibility is in terms of possible worlds.
        - In order to be objective in Bennett’s sense n must not depend on the person evaluating the claim. This too is consistent with a Keynesian, or Carnapian inductive probability where n is an inter-subjective truth.
        - Compatibility with Bennett’s other views (Ramsey test and hence conditionalization) requires something like R(P,Q,n)=n(P&Q)/n(Q). In other words the two place function R is equivalent to some ratio of one place functions.
- Bennett assumes we need to define the objective probability of particular conditionals in terms of relative objective probability, hence his insistence

we need to include pieces of background evidence. Though he never explicitly considers absolute objective probability it would seem any such notion still fails bennett's intuitions in cases of false antecedents.

- In particular since absolute objective probability assigns probability 0 to every false proposition all conditionals with false antecedents fall victim to 0 intolerance.

- What does an objective probability for a conditional mean? We know Bennett is assuming we start with a relative objective probability function but what we define using this might be something like an absolute objective probability function.
  - If we are defining something like an absolute objective probability function we are essentially just giving a test for the truth of a conditional, although truth in this case can take on non 0-1 values to account for genuinely random future contingents.
    - As above only future events genuinely undetermined by the current state of affairs get non-extreme probability. This includes most indicatives like "If Booth didn't shoot Lincoln someone else did."
  - Alternatively our objective probabilities give some measure of support by the evidence or assertability. Since this is objective it must be the same for everyone which will be problematic blow.
    - This must be what Bennett is after as he clearly is assuming this is part of the same project and he doesn't believe all conditionals about the past have extreme probability.
    - Note it is not clear that this option is necessarily exclusive with the absolute objective probability option. See OR_3

- First Attempts to define an objective probability
  - OR_1: $P(A\text{->}C)=\Pi(C/A\&P)$ where P is some true statement.
    - Remember $\Pi(C/A)=n(A\&C)/n(A)$ from before.
    - Problem: Drastically underdetermined. Probability varies with P so P(A->C) isn't even a function of A->C. Remember in order to be objective P cannot vary between individuals.
  - OR_2: $P(A\text{->}C)=\max \Pi(C/A\&P)$ for true P

- You might think this reasonable since by adding more and more facts to P we approximate truth better and better so if Π(C/A&P) is monotonic with respect to adding evidence to P this maximum is something like the limit as we approach the whole truth.

- Probability, unlike deductive logic, is not monotonic this way. Essentially this is saying only consider the body of evidence most favorable to A->C when evaluating P(A->C) which seems unsatisfactory.

- Example: Consider "If Chester can run the mile in two minutes he will receive an olympic medal. If we take P just to be 'The olympic record in the mile is greater than two minutes then Π(C/A&P) is reasonably large. If we take P* to be P combined with the fact that Chester has been taking large amounts of steroids and the olympic drug test is very sensitive. P* contains P yet Π(C/A&P*) is much lower than Π(C/A&P).

- OR3: P(A->C)=Π(C/A&P) where P is the whole truth.
  - Problem: Many reasonable conditionals fall victim to zero intolerance under the equation. In fact every conditional with a false antecedent requires conditionalizing on a contradiction.
    - Ex: If Booth did not shot lincoln someone else did.' becomes Π(someone else shot Lincoln/ Booth shot Lincoln & ~ Booth shot Lincoln). This happens anytime the antecedent is false as its negation is part of the whole truth.
  - Bennett thinks it might be okay to define Π(C/Contradiction) but this would have to be the same in every case.
    - Some theories of probability accept that Π(C/Contradiction)=1. Popper has such a theory.
      - This is required if you believe conditional probability gives something like inductive strength of an argument (as we are assuming in this section) and any classically deductively valid argument should be as inductively strong as possible.
    - If we accept conditionalizing on a contradiction gives probability 1 then we get something like absolute objective

probability and the equation naturally degrades into the horseshoe in most cases.

        - If ~A then P(A->C)=1 as in above example with Lincoln. This is nicely what the horseshoe would tell us about the truth.

    - This might be a good notion for absolute objective probability, or even for the acceptability of propositions for an omniscient being but nearly all conditionals get extreme values so Bennett rejects this view.

- Davis-OR4 P(A->C)=Π(C/A&P) where P is as close to the whole truth as possible consistent with A.

    - Example: "If the british did not attack the suez the Israelis rigged the situation," would be true. We modify the truth the least by not changing all the various reports about the attack which must be explained but we are assuming it wasn't actually a british attack we conclude the Israelis rigged the situation.

    - Jackson suggests Davis's theory must conclude, "If I have misremembered the date of the battle of hastings it was not fought in 1066." is false. Since the 'simplest' way to make this consistent with the truth is to deny that I remember the date as 1066.

        - Bennett rejects this objection because it trades on reading the statement in one particular, and unlikely manner. We could more reasonably interpret this as "If I have misremembered the date of the battle of hastings as 1066 then it was not fought in 1066."

            - There seems to be a level of conventional implicature here. If we didn't think that the conditional above didn't imply my remembering the battle of hastings to be in 1066 it's truth would no longer be clear. It seems Jackson is merely forcing Davis's theory to ignore this non-explicit information.

        - Still this does expose the difficulty in deciding what is closest to the whole truth.

    - Bennett objects that davis only works when everyone has knows information which clearly decides the issue in question, i.e., the theory works in the lincoln case because we all have lots of evidence that booth killed lincoln. This is because these are the cases where no standoffs can occur.

- Bennett suggests as a counterexample cases where one person has every reason to believe A->C and another has every reason to believe A->~C. Notably these are all examples with A false, it is not clear whether this is required by Bennett's theory.
    - Example: A dam has one lever to open the east gate; one lever to open west gate. Water flows through whatever gate is open when the top gate is opened. A safety mechanism prevents the top gate from being opened when both west and east gate are open. Gates are open when their respective levers are down.
        - Wesla only observes the west gate is open and hence (rightly) concludes Π(water goes through west gate/top gate is opened) is high. Hence for her P(Top gate open -> Water through west gate) is high.
        - Esther only observes the east gate is open and hence (rightly) concludes Π(water goes through east gate/top gate is opened) is high. Hence for her P(Top gate open -> Water through east gate) is high.
    - This is only problematic as Bennett accepts the principle of conditional non-contradiction (CNC): ~[(A->C)∧(A->~C)].
        - This is required for non-contradictory A's by the combination of objective probability, the equation and the assumption A->C implies P(A->C)>.5
        - P(~C/A)=1-P(C/A) since conditionalizing on A gives us a probability function and objectivity requires that it be the same P on both sides.
    - Horseshoe fails CNC and it seems that intuitively we fail this. If given convincing evidence that "If top gate opens then water will flow west" and then convincing evidence for "If top gate opens than water will flow east" I am moved to accept both and then exclaim, "So, the top gate won't open." How else do we explain proof by absurdity in normal language?
        - Consider another example that avoids any concern about physical necessity. I learn that John only has 5 minutes

between his calculus and philosophy class and that the line at starbucks is quite long at this time. Therefore, I come to accept the conditional If John gets coffee after calculus john will be late to philosophy.

- I am further informed that John only drinks coffee when he is trying to be particularly studious. Therefore I come to accept that If john gets coffee after calculus he will not be late for philosophy. I accept this because I believe if john is getting coffee he is in a very studious mood and wouldn't risk being late for class.

- It seems perfectly reasonable that someone would really accept both these conditionals at the same time. In particular if I was to learn the above facts in order I would accept both conditionals. I would just happily conclude, "ohh so john won't get coffee after calculus." If this isn't what bennett's theory suggests it seems bad for bennett.

    - Importantly I feel that learning the second piece of evidence would not cause me to reject the first conditional. This is not simply a case where I originally assert one conditional and then change my mind about it. Instead I genuinely accept both simultaneously.

    - Notice this doesn't involve any casual or logical necessity and none of these conditionals can be wiggled out of as being independent.

- Bennett argues that we can't claim both conditionals in a standoff are false since this would make many other innocent conditionals with false antecedents false. I agree, this is why we take both conditionals in a standoff as true.

- Even if the CNC is correct as a statement about truth we still don't have a problem. The CNC must be a statement about assertability to have the requisite impact.

    - We might accept the CNC for metaphysical truth values, like those given by an absolute objective probability function and yet have assertability still fail the CNC.

    - Consider the deductive case, surely we are prepared to admit there is an objective truth (which we could think of as a 0-1 valued probability) about provable mathematical theorems. But Bennett's argument would have us reject the objective interpretation of mathematical truth.

- Suppose Fermat's last theorem turned out to be false. Surely someone who was aware of the continued failure to find counterexamples would be rationally warranted to believe it is probably true. Yet someone who has seen the counterexample would surely be rationally warranted in believing it is false.

  - This is a direct counterexample to Gibbard's assertion (accepted by bennett) that 'one sincerely asserts something wrong only when one is mistaken about something germane.' (assuming we treat this charitably and not count the assertion itself as the thing one is mistaken about)

- Stepping back from the details standoffs do allow Bennett to make a very convincing argument that objective probability can give an analysis of the assertability of a conditional for individuals without complete omniscience.

  - Standoffs establish that different, equally reasonable, individuals give different acceptability values to the same conditionals.

  - Objective probability, by definition, requires everyone assign the same probability.

  - Therefore objective probability cannot be an explanation of acceptability the way Bennett desires.

Subjective Probability Instead

- Bennett now attacks the existence of subjective truth values. How this is not a reducto of his position given that we commonly talk about the truth of indicatives and this seems to be what we are seeking to explain I don't understand. We will have to wait and see what sort of things indicatives have if it isn't truth values.

- Subjective probability in this account is a probability function taking propositions *and* belief systems, usually the beliefs of the person asserting the conditional. This can be thought of as assigning to each set of beliefs a probability function. While different people will have different subjective probability for different propositions they will likely have intersubjective agreement on the two-place function from propositions and beliefs.

- Subjectivity through self-description. Conditionals are implicitly like indexicals and assert an objective relation between the belief system of the person making the statement and the conditional asserted. Under the self-description hypothesis the belief system is referred to by a description for the belief system (like "Amanda's beliefs")

  - For this section we are regarding a description as some label which makes the relation of the beliefs to the individual clear. So "Amanda's belief's" would count as subjectivity through self-description. "The beliefs with godel number 2,4 , 8...." would not constitute self-description.

  - Going back to our objective relative probability function we might try to define Amanda's subjective probability $P_{A,}$ for proposition Q where E_A is an description of Amanda's background beliefs as $P_A(Q)=R(Q,E_A,n)$. (whether n can vary from person to person is unclear but not important for Bennett's point).

  - Bennett objects this doesn't seem right as it now seems that people are talking about themselves when they assert a conditional. If I ask you is "If it rains the grass will get wet" true because of your beliefs the response is "No! it's true because of the world, or physics."

    - This is a compelling objection that this can't be what a conditional *means*. Once again though Bennett's lack of clarity about the nature of the project gives us reason to pause.

    - If the project is to explicate the *meaning* of the conditional this is a strong objection. However, if we were explicating the meaning we would likely want an absolute objective probability function (i.e. for most things we are asserting they are true not probable).

    - Given Bennett's past statements and his insistence that we many conditionals get non-extreme probability values tells us he is instead trying to explicate the assertability (or acceptability) of the conditionals. There is no reason why the acceptability conditions for a statement give the meaning of a statement, in fact they usually don''t.

      - If I say "It is raining outside" the reason this is okay for me to assert may be because Kenny told me it is raining. This surely doesn't mean that my statement was a statement about

    Kenny.

    - This argument doesn't even hold if we are analyzing the truth conditions of conditionals. Lewis has strongly argued that just because the truth conditions of having five fingers contingently mention a counterpart doesn't make the statement, "I have five fingers contingently" a statement about my counterpart.

  - Bennett seems to offer the following principle as a defense of his conflation of truth conditions and meaning. If some information P is necessary for the hearer to understand what the speaker says then that information is part of the meaning of what the speaker said.

    - This falls victim to the same objection. Intuitively, what your beliefs are is irrelevant to what you *mean* when you assert a conditional and they are also not needed to understand what you are asserting. However, this doesn't mean your beliefs aren't necessary to understand why this was an acceptable conditional for you.

- Bennett seems to imply that under this interpretation we could still use the fact that there is high assertability (for someone) for A->C and the same for A->~C we would also gain evidence for ~A. (p90) This simply isn't always true, and at the same time we give a standoff with a possible but unlikely antecedent.

    - Let the random variable X be choose uniformly on the integers 0-100 inclusive. Let A be the proposition $x \leq 49$, let C be $X \geq 45$.

    - Sara has also been told information K1, namely $X \geq 43$. Under K1 she feels confident asserting A->C since P(A->C)=Π(C/A&K1)=Π(X$\geq$45/(X$\geq$43)&(49$\geq$X))>.5

    - Joe has also been told information K2, namely 50$\geq$X. Under K2 he feels confident asserting A->~C since P(A->~C)=Π(X<40/(X$\leq$50)&(X$\leq$49))=41/50>.5

    - However, under the combined knowledge of K1 and K2 we know X is between 43 and 50 inclusive. Conditionalizing on this information we find A (X being less than 49) is *far* more probable than we found it initially.

- Bennett seems to imply that under this interpretation we could still use the fact that there is high assertability (for someone) for A->C and the same for A->~C we would also gain evidence for ~A. (p90). This would provide a defense for the equation against the worry above about proof by contradiction.

    - This simply isn't always true, and at the same time we give a standoff with a possible but unlikely antecedent.

    - Let the random variable X be choose uniformly on the integers 0-100 inclusive. Let A be the proposition $x \leq 49$, let C be $X \geq 45$.

    - Sara has also been told information K1, namely $X \geq 44$. Under K1 she feels confident asserting A->C since $P(A\text{->}C)=\Pi(C/A\&K1)=\Pi(X\geq 45/(X\geq 44)\&(49\geq X))>..75>.5$

    - Joe has also been told information K2, namely $50\geq X$. Under K2 he feels confident asserting A->~C since $P(A\text{->}{\sim}C)=\Pi(X<40/(X\leq 50)\&(X\leq 49))=41/50>.75>.5$

    - However, under the combined knowledge of K1 and K2 we know X is between 44 and 50 inclusive. Conditionalizing on this information we find A (X being less than 49) is *far* more probable than we found it initially. Note that our probabilities for the two different conditionals are both greater than .75 so intuitively the 'assertability' of both of them together (if this was possible) would be >.5.

    - Note this is only a problem for someone who accepts CNC since they must explain a proof by contradiction in terms of *others* assertability since they are apparently forbidden from having high assertability for both claims themselves. If assertability went by the ramsey test one individual would *never* have good reason to assert both parts of a CNC and hence wouldn't phrase his reasoning as a proof by contradiction.

    - A horseshoe analysis explains this perfectly. Proof by contradiction works based entirely on *my* knowledge. Since P(A->C)=P(~AVC) nothing prevents me from simultaneously having high acceptability for both parts of the CNC.

    - In fact one can fairly simply show that (assuming horseshoe) if P(A-

        >C)*P(A->~C) >.5 (which intuitively is the assertability of the conjunction) then P(~A)>.5.

        - The problem is not that someone who believes the equation can't support the *conclusion* of a proof by contradiction. They could simply take all the knowledge together (that supporting A->C and that supporting A->~C) and find that pr(A|all knowledge) is low. The problem is they would have to not assert one of the conditionals as well so we should *never* expect people to phrase an argument as a proof by contradiction.

        - Some might object that we are being unfair to those who believe in the equation as we require them to explain proof by contradiction based on assertions by two differnt people, or at least two differnt sets of evidence. While on the other hand we allow the horseshoe to explain proof by contradiction by guaranteing P(A->C) and P(A->~C) are high implies P(~A) is high all on the same evidence. This is only because the equation *cannot* hope to explain proof by contradiction conditionalizing on the same evidence sine it implies the CNC. The hope was that they could rephrase proof by contradiction as reasoning from *others* assert ability conditions (or assertability based on different evidence) but since this argument is invalid this doesn't seem plausible either.

- Bennett then considers subjectivity without self-description. The idea is that instead of P(A->C) being a ternary relation between A,C and a description capturing the relation of your beliefs to you it is instead a ternary relation between A,C and a proper name of your belief system.

    - Under this idea Amanda's subjective probability $P_A(Q)$=R(Q,'Henry',n) where Henry is a proper name for her belief system.

    - This avoids the problem of Amanda talking about herself when asserting a conditional. It introduces a new problem as she is now talking about belief system 'Henry'

        - What is 'Henry'. In other cases we expect there to be something readily apparent which we could substitute into our expressions to explicitly identify Henry but this doesn't appear to be the case here since as we have seen above 'my belief system' is *not* a valid option.

    - How are we supposed to interpret talk about conditionals now? Different people still assert different conditionals but we can’t think of them as talking about themselves.

- If someone is genuinely unsure if A->C holds and they ask is A->C true what are they asking? There are many equally valid options, namely F(Q,name, n) where name is a name of someone’s belief system. If there is not privileged belief system how do we explain the psuedo-objectivity we seem to assume in talk about conditionals (i.e. the assumption we are all somehow talking about the same thing).