

Stalnaker and Lance on Import-Export (If-And)

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Stalnaker and Lance (S&L) give what they claim to be a counterexample to the probabilistic import-export law (what Bennett calls “If-And”). This is the principle which says that

$$\Pr(A \rightarrow (B \rightarrow C)) = \Pr((A \& B) \rightarrow C)$$

Note: given The Equation (which is *assumed* in this context), this is equivalent to

$$\Pr(C | A \& B) = \Pr(A \rightarrow C | B)$$

Bennett discusses the example of Stalnaker and Lance in §40. I think his response is inadequate, and I also think the analyses of S&L and Bennett are incorrect. However, I think there is a way to bolster the argument of S&L (for a slightly different conclusion). Below, I will present their analyses, along with my commentary/alternative analysis.

The Example: There are 2 urns containing 100 balls each with the following constitutions

Urn 1: 90 red iron balls and 10 green copper balls

Urn 2: 90 green iron balls and 10 red copper balls

Agnes picks an urn at random, draws a ball (*a*) out, and returns the ball without noting its color or composition. Then she draws a second ball (*b*) from the same urn. Let

Ra: the first ball she picked was red.

Gb: the second ball she picked was green.

Cb: the second ball she picked was copper.

Bennett says that (assuming The Equation) our probability for $(Ra \& Gb) \rightarrow Cb$ should be 0.1 “because adding the supposition of *Ra* & *Gb* to our stock of beliefs doesn't make either urn likelier than the other to be the one Agnes drew both balls from. And 1/10 of all the balls are copper (in both urns combined).” I think this reasoning (also made by S&L) is incorrect. More rigorously, letting U_1 = the urn she drew was urn 1, and U_2 = the urn she drew was urn 2, we have the following, by The Equation (Bennettian Ramsey Test)

$$\Pr((Ra \& Gb) \rightarrow Cb) = \Pr(Cb | Ra \& Gb)$$

which (since U_1 and U_2 are m.e.e) by the Law of Total Probability expands to

$$\Pr(Cb | Ra \& Gb \& U_1) \cdot \Pr(U_1 | Ra \& Gb) + \Pr(Cb | Ra \& Gb \& U_2) \cdot \Pr(U_2 | Ra \& Gb)$$

Now, we know that $\Pr(Cb | Ra \& Gb \& U_1) = 1$ and $\Pr(Cb | Ra \& Gb \& U_2) = 0$, since U_1 entails $(\forall x)(Gx \supset Cx)$, and U_2 entails $(\forall x)(Gx \supset \sim Cx)$. Combining these results yields

$$\Pr((Ra \& Gb) \rightarrow Cb) = \Pr(U_1 | Ra \& Gb)$$

Moreover, given standard assumptions about random sampling (with replacement) and Bayes's Theorem, we can calculate $\Pr(U_1 | Ra \ \& \ Gb)$, as follows

$$\Pr(U_1 | Ra \ \& \ Gb) = \frac{\Pr(Ra \ \& \ Gb | U_1) \cdot \Pr(U_1)}{\Pr(Ra \ \& \ Gb | U_1) \cdot \Pr(U_1) + \Pr(Ra \ \& \ Gb | U_2) \cdot \Pr(U_2)} = \frac{(\frac{9}{10} \cdot \frac{1}{10}) \cdot \frac{1}{2}}{(\frac{9}{10} \cdot \frac{1}{10}) \cdot \frac{1}{2} + (\frac{1}{10} \cdot \frac{9}{10}) \cdot \frac{1}{2}}$$

Note: this equals $\frac{1}{2}$ (*not* $\frac{1}{10}$). Next, let's see what S&L and B say about $\Pr(Ra \rightarrow (Gb \rightarrow Cb))$.

S&L argue that the probability of $Ra \rightarrow (Gb \rightarrow Cb)$ should be 0.9. "Because when we add Ra to our stock of beliefs, our probability that she drew from Urn 1 becomes 0.9. Under the supposition she picked Urn 1, our probability for $Gb \rightarrow Cb$ becomes 1, since all green balls are copper in Urn 1. Under the supposition she picked Urn 2, our probability for $Gb \rightarrow Cb$ becomes 0, because no green copper balls there. Our probability for $Ra \rightarrow (Gb \rightarrow Cb)$ should then be $0.9 \cdot 1 + 0.1 \cdot 0$, which equals 0.9." Again, this is rather sloppy probability reasoning. But, there is some truth to their conclusions here. Let me explain.

First, by The Equation (alone), what we know is

$$\Pr(Ra \rightarrow (Gb \rightarrow Cb)) = \Pr(Gb \rightarrow Cb | Ra)$$

which, by the Law of Total Probability, expands to

$$\Pr(Gb \rightarrow Cb | Ra \ \& \ U_1) \cdot \Pr(U_1 | Ra) + \Pr(Gb \rightarrow Cb | Ra \ \& \ U_2) \cdot \Pr(U_2 | Ra)$$

At this stage, S&L claim that $\Pr(Gb \rightarrow Cb | Ra \ \& \ U_1) = 1$ and $\Pr(Gb \rightarrow Cb | Ra \ \& \ U_2) = 0$. But, this does not follow from the probability calculus + The Equation. Here, S&L assume that $\Pr(Gb \rightarrow Cb | Ra \ \& \ (\forall x)(Gx \supset Cx)) = 1$, and $\Pr(Gb \rightarrow Cb | Ra \ \& \ (\forall x)(Gx \supset \sim Cx)) = 0$. But, surely, *they* don't think this is true! What's true (as above) is that $\Pr(Cb | Gb \ \& \ Ra \ \& \ (\forall x)(Gx \supset Cx)) = 1$, and $\Pr(Cb | Gb \ \& \ Ra \ \& \ (\forall x)(Gx \supset \sim Cx)) = 0$. So, what do they have in mind?

[Note: Bennett's "response" to S&L seems very weak to me (I don't really even follow it).]

Note: S&L's claim that $\Pr(U_1 | Ra) = 0.9$ [$\Pr(U_2 | Ra) = 0.1$] is correct. Bayes's Theorem:

$$\Pr(U_1 | Ra) = \frac{\Pr(Ra | U_1) \cdot \Pr(U_1)}{\Pr(Ra | U_1) \cdot \Pr(U_1) + \Pr(Ra | U_2) \cdot \Pr(U_2)} = \frac{\frac{9}{10} \cdot \frac{1}{2}}{\frac{9}{10} \cdot \frac{1}{2} + \frac{1}{10} \cdot \frac{1}{2}} = \frac{9}{10}$$

But, their argument for $\Pr(Ra \rightarrow (Gb \rightarrow Cb)) = 0.9 \neq \Pr((Ra \ \& \ Gb) \rightarrow Cb) = 0.1$ is incorrect. They *should* argue for $\Pr(Ra \rightarrow (Gb \rightarrow Cb)) = 0.9 \neq \Pr((Ra \ \& \ Gb) \rightarrow Cb) = 0.5$. But, *even that* just doesn't follow from The Equation (plus the probability calculus) *alone*. They never establish $\Pr(Gb \rightarrow Cb | Ra \ \& \ U_1) = 1$ and $\Pr(Gb \rightarrow Cb | Ra \ \& \ U_2) = 0$, which they seem to need for their argument to go through (as they and Bennett construe the argument).

Alternative S&L-Style Argument: Assume, for *reductio*, that *both* The Equation and "If-And" are true. Then, we *can* (rigorously!) infer *both* that $\Pr((Ra \ \& \ Gb) \rightarrow Cb) = \frac{1}{2}$ (first derivation above), *and* that $\Pr((Ra \ \& \ Gb) \rightarrow Cb) = \frac{9}{10}$.¹ This is absurd. So, *at least one of The Equation and "If-And" must be false*. I think this is a knock-down argument against The Equation + "If-And". And, it seems to me, this gets S&L exactly what they wanted. \square

¹"If-And" and The Equation jointly entail $\Pr(Gb \rightarrow Cb | Ra \ \& \ U_1) = \Pr(Cb | Gb \ \& \ Ra \ \& \ U_1) = 1$, and, hence, that $\Pr((Ra \ \& \ Gb) \rightarrow Cb) = \Pr(Ra \rightarrow (Gb \rightarrow Cb)) = \frac{9}{10}$. This fills the gap in the second derivation, above.