A Different Conjunction Fallacy

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Abstract: Because the conjunction p-and-q implies p, the value of a bet on p-and-q cannot exceed the value of a bet on p at the same stakes. We tested recognition of this principle in a betting paradigm that (a) discouraged misreading p as p-and-not-q, and (b) encouraged genuinely conjunctive reading of p-and-q. Frequent violations were nonetheless observed. The findings appear to discredit the idea that most people spontaneously integrate the logic of conjunction into their assessments of chance.

1. Introduction

Investing more confidence in a conjunction of statements compared to one of its conjuncts is such a flagrant fallacy that it is natural to wonder whether perpetrators have understood the question. Doubts have thus arisen about the proper interpretation of 'Linda'-type problems (Tversky and Kahneman, 1983) which test appreciation of:

THE CONJUNCTION INEQUALITY: $\text{Prob}(p\text{-and-}q) \leq \text{Prob}(q)$.

One concern is whether reasoners intend to refer to the modern concept of chance through use of the word 'probability'. Hertwig and Gigerenzer (1999) describe various meanings that might plausibly be attached to this word, and observe that typical responses to the Linda problem are not fallacious for some of them. It has also been noted that no fallacy occurs if the bare conjunct q of a conjunction p-and-q is interpreted as not-p-and-q (see Morier and Borgida, 1984; Macdonald and Gilhooly, 1986; Politzer and Noveck, 1991; Dulany and Hilton, 1991). Similarly, there is no fallacy if the conjunction p-and-q is interpreted disjunctively (that is, as p-or-q; see Mellers, Hertwig and Kahneman, 2001). Such departures from literal meaning are often thought to be consistent with pragmatic principles of discourse that encourage nonliteral interpretation of odd questions like those involved in the conjunction fallacy (Hilton, 1995; Hacking, 2001, p. 66).

Most of these objections were foreseen in Tversky and Kahneman (1983), but the procedures they used in response were not entirely convincing. Sides,

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Osherson, Bonini and Viale (2002) discussed these issues and presented new experiments that rely on betting paradigms to avoid locutions involving the word 'probability'. In addition, they formulated conjunctions both with and without 'and' to ensure that errors did not hinge on non-standard interpretation of this word. To block the interpretation of q as not-p-and-q, participants were instructed to choose between bets on q versus p-and-q. Choices were communicated by obliterating the non-chosen option. It was explained that payoffs would be determined by an independent judge with access to only the legible option. It was thus clear that such a judge could not be influenced by the presence of p-and-q to read q as not-p-and-q. Despite these precautions, conjunction fallacies were committed by large majorities of participants across several experiments. For example, college students in Houston considered whether the following events would take place by March 1, 2000.

(a) Parents in Houston will be required to attend a 'good sportsmanship' workshop before their children are allowed to participate in organized sports.
(b) Several incidents will take place involving parental conflict at little league games, *and* parents in Houston will be required to attend a 'good sportsmanship' workshop before their children are allowed to participate in organized sports. [Emphasis added.]
(c) Several incidents will take place involving parental conflict at little league games, *after which* parents in Houston will be required to attend a 'good sportsmanship' workshop before their children are allowed to participate in organized sports. [Emphasis added.]

A majority of a group of 43 students preferred to bet on the truth of (b) compared to (a). Similarly, a majority of another 40 students preferred to bet on (c) compared to (a).

In personal communication to us, sceptics of the conjunction fallacy have expressed reservations about the foregoing results, and renewed their conjecture that the alleged fallacy rests on no more than misunderstanding between subjects and experimenters. It is claimed that q might have been construed as not-p-and-q, our precautions notwithstanding. The possibility is also raised that the crucial word 'and' might not have been understood conjunctively, that is, p-and-q might not have been understood as implying q (and similarly for 'after which'). Such scepticism invites further studies examining, for example, whether college students believe that p-and-q guarantees the truth of q. If they do, then the calculus of (subjective) probability commits them to the conjunction inequality. For, a central principle of probability is:

IMPLICATION PRINCIPLE: For any statements A, B, $\text{Prob}(A) \leq \text{Prob}(B)$ if A implies B.

Hence, the belief that p-and-q implies q requires the belief that $\text{Prob}(p\text{-and-}q) \leq \text{Prob}(q)$, i.e., the conjunction inequality. Since many students' preferences among bets seem to belie the latter inequality, it must be concluded (if they accept that p-and-q implies q) that their beliefs violate some principle of probability, e.g., the implication principle.

The present study explores an alternative means of breaking the interpretive deadlock about the conjunction inequality. We examine a different kind of conjunctive principle, namely:

> CONJUNCTION DOMINANCE: It is irrational to wager any sum of money on the truth of p-and-q if the money can be wagered on the truth of p instead [except in the degenerate case where $\text{Prob}(p\text{-and-not-}q) = 0$].

Conjunction dominance is intuitively compelling, as well as required policy for a utility maximizer (assuming that money and utility are monotonically related). Note that the conjunction inequality does not imply conjunction dominance since it is not in general true that a rational better should concentrate all her stake on the most probable event.[1] The two principles both hinge on the logic of conjunction, however. Thus, if genuine violations of conjunction dominance can be demonstrated, there would be additional grounds for doubting whether conjunctions are successfully integrated into reasoning about chance. This would support the case that fallacy also arises for the conjunction inequality.

We therefore invited subjects to choose bets among all three of p-and-q, p, and p-and-not-q. The presence of p-and-not-q as an explicit alternative makes it pragmatically impossible to interpret p as p-and-not-q (since it would be uncooperative to needlessly repeat one of the options in altered form). Moreover, our second experiment makes the conjunctive reading of p-and-q inescapable. As documented below, substantial violation of conjunction dominance persists, suggesting that it arises from misapprehension of chance rather than from misleading pragmatics or ambiguity about conjunction.

2. Experiment 1

2.1 Materials

Each participants faced thirteen blocks of three statements describing events that may or may not come true by November 15, 2002. Each block was an invitation to divide 7 euros among the three statements with the understanding that the amount assigned to a given statement could be won if the statement comes true. The statements were all 'singular' in character, that is, not easily assimilated to a set

[1] For example, on a certain roll of a fair die, let G be the event of obtaining 1 or 2, and let H be the event of obtaining 3, 4, 5, or 6. A rational better might prefer (a) receiving 1 euro in the event of G and 1 euro in the event of H to (b) receiving 2 euros in the event of the more probable H. Bet (b) has higher expected monetary value but (a) may be rationally preferred since unlike (b) it guarantees a profit.

of similar events in the past. Instructions avoided mention of 'probability' and its cognates. The specific wording was as follows.[2]

> This booklet is composed of 13 blocks each of which holds three bets. The bets concern events that might occur between June 15 and November 15 of 2002. For each block you are hereby given 7 euros (at no charge to you). The money for a given block must be divided among the three statements that make up the block.
> You can apportion the money as you like. For example, you can bet all 7 euros on one statement and nothing on the two others. Or you can bet 4.4 euros on one bet, 2.6 euros on another, and 0 on the remaining one. Or you can bet 1 euro on one statement and 3 on the two others, etc. You must assign all 7 euros within a given block (money cannot be saved for yourself, or moved from one block for use in another).
> On November 16, 2002, one of your thirteen blocks will be selected. You will win whatever you bet on the statements in the block that have come true between June 15 and November 15.

The thirteen blocks included five with statements of logical forms *p*, *p*-and-*q*, and *p*-and-not-*q*. These will be called *conjunction blocks*. The remaining eight blocks had the logical forms shown in Table 1. They were 'fillers' used only to mask the intent of the experiment. The five conjunction blocks are presented in the Appendix. They were designed to elicit more reasons for believing *p*-and-*q* (or *p*-and-not-*q*) than *p* in the minds of our subjects. For example, in block #7 (see the Appendix) the additional measure mentioned in *p*-and-*q* compared to *p* is a plausible step for reducing traffic fatalities.

#	*statement 1*	*statement 2*	*statement 3*
1)	not-*p*	*q*	*r*-and-not-*q*
2)	*p*-and-*q*	*r*-and-*s*	*t*
4)	not-*p*-and-*q*	*r*	*s*
6)	*p*	*q*-and-*r*	*s*
8)	not-*p*	*q*-and-*r*	not-*s*
10)	*p*-and-*q*	*r*	not-*s*
12)	*p*	*q*-and-*r*	*s*
13)	not-*p*	*q*-and-*r*	*s*-and-*t*

Table 1 *Logical forms of filler-blocks*

Note: Each line represents a block in Experiments 1 and 2 that does not involve conjunction dominance.

[2] All materials are translated from Italian.

Booklets holding all thirteen blocks were assembled. The order of blocks was either that indicated by the numbers in Table 1 and the Appendix, or else the reverse.

2.2 Method

Sixty student volunteers were recruited from college campuses in Trento. Average age was 23.7 years (S.D. 3.79). There were 18 men and 42 women. They completed their booklets in small groups without time constraints. One of the two orders of blocks was assigned randomly to each participant. Explanation and amplification of the instructions were given as needed. Bets were randomly drawn and duly paid on November 16, 2002.

2.3 Results

Conformity to conjunction dominance requires assigning 7 euros to the conjunct p in all five conjunction blocks. In contrast, the mean average assignment to p across the five conjunction blocks was only 1.99 (S.D. 1.10), compared to 3.15 (S.D. .90) to p-and-q ($p < .01$ by correlated t-test).[3] The average bet on p was inferior to the average for p-and-q in four blocks, and inferior to the average bet on p-and-not-q in two blocks.

In three of the five blocks (considered individually) the average bet on p was reliably smaller than the average bet on p-and-q by correlated t-test. In one of these three blocks, the average for p is also reliably smaller than the average for p-and-not-q. See Table 2.

All sixty participants allocated money to p-and-q at least once across the five conjunction blocks. Fifty-seven participants allocated money to p-and-not-q at least once. The percent of participants who made such allocations is shown by block in Table 3. Of course, allocation of any money to p-and-q or to p-and-not-q violates conjunction dominance.

3. Experiment 2

In discussing the results of Experiment 1 with colleagues, we encountered the hypothesis that Italian 'e'—used to express statements of the forms p-and-q and p-and-not-q—was ambiguous between conjunction and disjunction. We find this idea no more credible than the claim that 'and' is ambiguous in the translations provided in the Appendix. Nonetheless, to remove any possible confusion about the conjunctive character of compound events in our stimuli, we added the following parenthetical remark after each one.

3 All tests are one-tailed in light of prior studies, including our own, that document conjunction fallacies.

Block	Form	Average (S.D.)
	p	1.73 (2.03)
#3	*p*-and-*q*	2.71 (2.12)*
	p-and-not-*q*	2.55 (1.90)*
	p	1.34 (1.74)
#5	*p*-and-*q*	4.91 (2.12)*
	p-and-not-*q*	.74 (1.09)
	p	2.35 (1.76)
#7	*p*-and-*q*	3.78 (2.05)*
	p-and-not-*q*	.87 (1.20)
	p	2.46 (1.94)
#9	*p*-and-*q*	2.15 (1.56)
	p-and-not-*q*	2.39 (1.71)
	p	2.08 (1.96)
#11	*p*-and-*q*	2.17 (1.99)
	p-and-not-*q*	2.75 (2.22)
Average for all 5 blocks	*p*	1.99 (1.10)
	p-and-*q*	3.15 (.90)*
	p-and-not-*q*	1.86 (.82)

Table 2 *Average bets in each conjunction block, Experiment 1* (N = *60*)

Note: *Average bets on starred conjunctions are reliably greater than bets on the corresponding conjunct* p *by correlated* t-*test* ($p < .05$, *one tailed*).

	Type of fallacy	
Block	*p*-and-*q*	*p*-and-not-*q*
#3	80	81.7
#5	95	45
#7	93.3	48.3
#9	83.3	83.3
#11	75	76.7

Table 3 *Percent of respondents violating conjunction dominance, Experiment 1* (N = *60*)

Note: *The column headed by* p-*and*-q *denotes the percentage of respondents who allocated any money to the conjunction* p-*and*-q *in a given block. Likewise, the column headed by* p-*and-not*-q *denotes the percentage of respondents who allocated any money to the conjunction* p-*and-not*-q *in a given block.*

Both events must happen for you to win the money placed on this bet.

3.1 Materials and Method

Except for the added remark, the instructions and stimuli for Experiment 2 were identical to those for Experiment 1.[4] For example, conjunction block #7 (see the Appendix) was modified to:

> In order to reduce traffic fatalities, the government will. . .
>
> launch a publicity campaign.
>
> launch a publicity campaign and penalize more harshly dangerous traffic violations (both events must happen for you to win the money placed on this bet).
>
> launch a publicity campaign and not penalize more harshly dangerous traffic violations (both events must happen for you to win the money placed on this bet).

Sixty new student volunteers were recruited from college campuses in Milan. Average age was 22.8 years (S.D. 2.39). There were 29 men and 31 women. The procedure was the same as in Experiment 1.

3.2 Results

The mean average assignment to *p* across the 5 conjunction blocks was 2.10 (S.D. 1.15) instead of 7 as required by conjunction dominance. This value is not significantly different from the 1.99 (S.D. 1.10) observed in Experiment 1 ($t(118) = .51$). The mean average for *p*-and-*q* in Experiment 2 was 2.98 (S.D. .88), reliably higher than for *p* ($p < .01$, correlated *t*-test). The average bet on *p* was inferior to the average for *p*-and-*q* in three blocks and inferior to the average bet on *p*-and-not-*q* in two blocks. In two blocks, the average bet on *p* is reliably smaller than the average bet on *p*-and-*q*. In one additional block, the average for *p* is reliably smaller than the average for *p*-and-not-*q*. See Table 4.

All sixty participants allocated money to *p*-and-*q* at least once across the five conjunction blocks, and likewise for *p*-and-not-*q*. The percent of participants who made such allocations is shown by block in Table 5.

4. Discussion

Do our subjects' choices embody a reasoning fallacy? One attempt to frame a negative answer starts from the observation that both the conjunction inequality and conjunction dominance are properly stated in terms of the truth functional

[4] The temporal interval in which the event was supposed to happen, however, was shifted one month to July 15–December 15, 2002.

connective $\wedge$ rather than English 'and'. It is well known that *p*-and-*q* is often used by English speakers to express more than $p \wedge q$, notably, temporal and causal relationships. For this reason, the truth value of $p \wedge q$ is preserved under commu-

Block	Form	Average (S.D.)
#3	*p*	1.90 (2.08)
	p-and-*q*	1.99 (1.85)
	p-and-not-*q*	3.11 (2.25)*
#5	*p*	1.18 (1.54)
	p-and-*q*	5.36 (2.01)*
	p-and-not-*q*	.47 (.965)
#7	*p*	2.46 (2.02)
	p-and-*q*	3.29 (2.23)*
	p-and-not-*q*	1.25 (1.98)
#9	*p*	2.21 (1.79)
	p-and-*q*	1.88 (2.09)
	p-and-not-*q*	2.92 (2.21)
#11	*p*	2.74 (1.87)
	p-and-*q*	2.39 (2.07)
	p-and-not-*q*	1.87 (1.65)
Average for all 5 blocks	*p*	2.10 (1.15)
	p-and-*q*	2.98 (.88)*
	p-and-not-*q*	1.92 (.84)

Table 4 *Average bets in each conjunction block, Experiment 2* (N $= 60$)

Note: Average bets on starred conjunctions are reliably greater than bets on the corresponding conjunct p *by correlated* t-*test* ($p < .05$, *one tailed*).

	Type of fallacy	
Block	*p*-and-*q*	*p*-and-not-*q*
#3	75	83.3
#5	96.7	23.3
#7	80	48.3
#9	63.3	83.3
#11	78.3	73.3

Table 5 *Percent of respondents violating conjunction dominance, Experiment 2* (N $= 60$)

Note: The column headed by p-*and*-q *denotes the percentage of respondents who allocated any money to the conjunction* p-*and*-q *in a given block. Likewise, the column headed by* p-*and-not*-q *denotes the percentage of respondents who allocated any money to the conjunction* p-*and-not*-q *in a given block.*

tation of p and q whereas this is not always true for p-and-q.[5] Doubt may therefore be expressed about the status of the two principles when 'and' is substituted for $\wedge$, as we have done in our discussion above.

It seems to us, however, that such doubts are misplaced. For all the sentences p-and-q in the experiments reported here (as well as in the experiments of Sides *et al.*, 2002), it is likely that subjects find it impossible for p-and-q to be true while either p or q is false. In other words, they take p-and-q to imply p and to imply q. Since $\{p, q\}$ implies $p \wedge q$, it follows from the transitivity of implication that p-and-q implies $p \wedge q$. The upshot is that violating the conjunction inequality when stated in terms of 'and' implies violation when stated in terms of $\wedge$. For, if someone judges that Prob(p-and-q) $>$ Prob(q) then they are committed to $\text{Prob}(p \wedge q) > \text{Prob}(q)$ by the transitivity of $>$ [because $\text{Prob}(p \wedge q) \geq$ Prob(p-and-q) by the Implication principle]. Likewise, someone preferring to wager on p-and-q instead of p commits a 'worse' violation than preferring $p \wedge q$ to p (since p-and-q is logically stronger than $p \wedge q$ if p-and-q encodes temporal or causal information).[6]

The most straightforward interpretation of the results described above is therefore that people can be led to violate transparent applications of conjunction dominance. Such violations seem attributable neither to pragmatic reasoning (e.g., construing p as p-and-not-q) nor to misunderstanding of the word 'probable' (which never occurred in our procedure). It is therefore difficult to attribute the violations to 'intelligent inference' (as suggested in Hertwig and Gigerenzer, 1999). They seem rather to arise from failure to coordinate the logical form of statements with their estimated chance, as required by the axioms of subjective probability. The proper interpretation of subjective probability remains a controversial topic (see Kahneman and Tversky, 1996; Gigerenzer, 1996), but there is little doubt about the irrationality of the behavior documented here. Of course, the difficulties observed in connection with conjunction dominance reinforce the possibility that the conjunction inequality is similarly—and genuinely—violated in naive judgments of chance.

Why do people often fail to take the logical form of statements into account when comparing their probabilities? Part of the reason may lie in the way chances are evaluated. Tversky and Koehler (1994) showed that assessed probability tends to vary with perceived 'support', where the latter variable can often be assimilated to mentally available reasons to believe a given statement. The outcome can be violation of the additivity axiom when different respondents assess the probability of distinct elements of a partition.[7] Our conjunction blocks were designed to elicit

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5 A standard (if somewhat elliptical) example is the noncommutativity of 'Jones ate fish with ice cream and died'. Such contrasts between 'and' and $\wedge$ have been noted in logic texts for decades. See, for example, Strawson (1952, p. 80), Suppes (1957, p. 5), Mates (1965, Ch. 5) and Kleene (1967, p. 64).

6 For further discussion of the normative issues surrounding the conjunction inequality and similar principles, see Stanovich and West (2000), and accompanying commentary.

7 See also Rottenstreich and Tversky (1997). Nonadditivity typically arises when each partition element has substantial support, leading to a sum of probabilities that exceeds unity. But nonadditivity also arises when partition elements have a dearth of support, leading to a sum inferior to unity. See Idson, Krantz, Osherson and Bonini (2001).

more support for *p*-and-*q* than for *p* so they pull in the direction of fallacy. But why isn't the pull resisted by consideration of logical form? However this question is ultimately answered, our results serve to underline the importance of the issue to the psychology of reasoning. Conjunction and dominance fallacies are elementary cases of the failure to reconcile probability with logic. Their origin seems to involve more than miscommunication.

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Appendix: Conjunction blocks

The three statements composing each conjunction block are shown below. The notations for logical form (*p*-and-*q*, etc.) are for the reader's convenience. They did not appear in the stimuli.[8]

#**3**. Because of the Italian Rail's new policies aimed at encouraging voyages longer than 100 km, the number of passengers...

will decline by 5% on commuter trains and increase by 10% on long distance trains (*p*-and-*q*),
will decline by 5% on commuter trains and will not increase by 10% on long distance trains (*p*-and-not-*q*),
will decline by 5% on commuter trains (*p*).

#**5**. The Italian government will increase cigarette taxes by .30 euros...

with the stated goal of diminishing cigarette smoking in adolescents and without the stated goal of increasing revenues. (*p*-and-not-*q*)
with the stated goal of diminishing cigarette smoking in adolescents. (*p*)
with the stated goal of diminishing cigarette smoking in adolescents and with the stated goal of increasing revenues. (*p*-and-*q*)

#**7**. In order to reduce traffic fatalities, the government will...

launch a publicity campaign. (*p*)
launch a publicity campaign and penalize more harshly dangerous traffic violations. (*p*-and-*q*)

[8] Assigning logical form *p*-and-*q* to one conjunction and *p*-and-not-*q* to the other is sometimes an arbitrary decision.

launch a publicity campaign and not penalize more harshly dangerous traffic violations. (*p*-and-not-*q*)

#**9**. Thanks to new labor laws throughout Europe...

employment will increase by 5%. (*p*)
employment will increase by 5% and economic growth will not be less than 2%. (*p*-and-not-*q*)
employment will increase by 5% and economic growth will be less than 2%. (*p*-and-*q*)

#**11**. In Italy...

more than 90% of private schools and less than 70% of public schools will be connected to the Internet. (*p*-and-*q*)
more than 90% of private schools will be connected to the Internet. (*p*)
more than 90% of private schools and at least 70% of public schools will be connected to the Internet. (*p*-and-not-*q*).

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