A DEFINITION OF "DEGREE OF CONFIRMATION"

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1. *The problem.* The concept of confirmation of an hypothesis by empirical evidence is of fundamental importance in the methodology of empirical science. For, first of all, a sentence cannot even be considered as expressing an empirical hypothesis at all unless it is theoretically capable of confirmation or disconfirmation, i.e. unless the kind of evidence can be characterized whose occurrence would confirm, or disconfirm, the sentence in question. And secondly, the acceptance or rejection of a sentence which does represent an empirical hypothesis is determined, in scientific procedure, by the degree to which it is confirmed by relevant evidence.

The preceding remarks, however, are meant only as accounts of methodological tendencies and are not intended to imply the existence of clear-cut criteria by means of which the scientist can decide whether—or, in quantitative terms, to what degree—a given hypothesis is confirmed by certain data. For indeed, no general and objective criteria of this kind are at present available; in other words, no general definition of the concept of confirmation has been developed so far. This is a remarkable fact in view of the importance of the concept concerned, and the question naturally suggests itself whether it is at all possible to set up adequate general criteria of confirmation, or whether it may not rather be necessary to leave the decision in matters of confirmation to the intuitive appraisal of the scientist.

This latter alternative would be highly unsatisfactory; for firstly, it would clearly jeopardize the objectivity—in the sense of intersubjectivity—of scientific procedure. Secondly, it would run counter to a view of confirmation which is now widely accepted; according to this view, statements about confirmation assert nothing regarding an observer's subjective appraisal of the soundness of a hypothesis; rather, they concern a certain objective relation between a hypothesis and the empirical evidence with which it is confronted; this relation depends exclusively on the content of the hypothesis and of the evidence, and it is of a purely logical character in the sense that once a hypothesis and a description of certain observational findings are given, no further empirical investigation is needed to determine whether, or to what degree, the evidence confirms the hypothesis; the decision is a matter exclusively of certain logical criteria which form the subject matter of a formal discipline which might be called inductive logic.

Of course, the widespread acceptance of this view does not prove that it is sound and that the program implicit in it can actually be carried out. The best—and perhaps the only—method of settling the issue seems to consist in actually constructing an explicit and general definition and theory of confirmation. To do this is the purpose of this article. It is intended to present in outline, and with emphasis on the general methodological issues, a theory of

confirmation which was developed by the present authors jointly with Dr. Olaf Helmer.[1]

As is illustrated by the terminology used in the preceding discussion, the concept of confirmation may be construed as a metrical (quantitative) as well as a purely classificatory (qualitative) concept. These two different forms are exemplified, respectively, in the phrases "The degree of confirmation of the hypothesis H relatively to the evidence E is such and such," and "The evidence E is confirming (disconfirming, irrelevant) for the hypothesis H." The theory here to be presented deals with the metrical concept of confirmation; its objective is to construct a definition of the concept of degree of confirmation and to derive, from this definition, a number of consequences, which may be called theorems of inductive logic.[2]

2. *The language L.* The planned definition or "rational reconstruction" of the concept of degree of confirmation in precise terms can be carried out only on the assumption that all the hypotheses to be considered and all the reports on observational data are formulated as sentences of a language, L, whose logical structure and means of expression are precisely determined. More specifically, we shall presuppose that L contains the following means of expression, which will be symbolized, but for minor and obvious changes, in accordance with the familiar notation of *Principia Mathematica*:

2.1 The statement connectives '$\sim$' ("it is not the case that"), '$\cdot$' ("and"), 'v' ("or"), '$\supset$' ("if . . . then")

2.2 Parentheses

2.3 Individual constants, i.e. names of individual objects (which may be physical bodies, events, space-time regions, or the like), 'a_1', 'a_2', 'a_3', $\cdots$
The number of individual constants in L may be finite or denumerably infinite.

[1] A detailed technical exposition of the theory will be given by Olaf Helmer and Paul Oppenheim in a forthcoming article, in vol. 10 of The Journal of Symbolic Logic.

The present issue of Philosophy of Science contains an article by Professor Rudolf Carnap which likewise sets forth a definition and theory of confirmation. The approach to the problem which is to be developed in the present paper is independent of Professor Carnap's and differs from it in various respects. Some of the points of difference will be exhibited subsequently as the occasion arises. We wish to express our thanks to Professor Carnap for valuable comments he made in the course of an exchange of ideas on the two different studies of confirmation.

We also wish to thank Dr. Kurt Gödel for his stimulating remarks.

[2] For a definition and theory of the classificatory concept of confirmation, see the following two articles by Carl G. Hempel: A purely syntactical definition of confirmation; The Journal of Symbolic Logic, vol. 8(1943), pp. 122–143; Studies in the logic of confirmation; Mind, n.s. vol. 54 (1945).

The technical term "confirmation" should not be construed in the sense of "verification" —an interpretation which would preclude, for example, its application to a hypothesis about an event which is temporally posterior to the data included in the evidence. Rather, as is suggested by the root "firm," the confirmation of a hypothesis should be understood as a strengthening of the confidence that can rationally be placed in the hypothesis.

2.4 Some finite number p of one-place predicates, i.e. names of properties which any one of the individuals referred to under 2.3 may or may not have:

$$\text{'}P_1\text{'}, \text{'}P_2\text{'}, \text{'}P_3\text{'}, \cdots, \text{'}P_p\text{'}$$

These predicates are undefined concepts in L; we shall therefore refer to them as the primitive predicates of L, and to their designata as the primitive properties referred to in L.

2.5 Individual variables: 'x', 'y', 'z', $\cdots$ in any number.

2.6 The symbols of universal and existential quantification, as illustrated in '$(x)P_1x$' and '$(Ey)P_1y$'.

We further assume that these symbols can be combined in the customary ways to form sentences in L[3], and that the usual rules of deductive inference govern the language L.

Briefly, then, we assume that L has the logical structure of the so-called lower functional calculus without identity sign, and restricted to property terms only. These assumptions involve a considerable oversimplification from the viewpoint of the practical applicability of the theory here to be presented, for the language of empirical science includes a great deal of additional logical apparatus, such as relation terms, expressions denoting quantitative magnitudes, etc. However, in appraising the significance of this restriction, the following points might well be borne in mind: 1. In the case of the concept of degree of confirmation, for which no explicit definition has been available at all, and for which even the theoretical possibility of a definition has been subject to serious doubt, it seems to be a significant achievement if such a definition can be provided, even if its applicability is restricted to languages of a comparatively simple structure. 2. While the means of expression of L are relatively limited, they still go beyond the logical machinery which forms the subject matter of traditional Aristotelian logic. 3. The formulation of a definition for languages of our restricted type may serve as a guide in the construction of an extension of the definition to more complex language forms.

3. *Some auxiliary concepts.* By an *atomic sentence* we shall understand any sentence of the kind illustrated by 'P_1a', which ascribes a primitive property to some individual. Any sentence such as '$(P_1a \supset \sim P_2a) \cdot P_1b$', which contains no quantifiers, will be called a *molecular sentence.*

Let 'Ma_i' be short for some molecular sentence such as '$P_1a_i \cdot (P_2a_i \vee \sim P_3a_i)$', which contains only one individual constant, 'a_i'; then we shall say that 'M' designates a *molecular property*—in the example, the property $P_1 \cdot (P_2 \vee \sim P_3)$. By a *statistic* we shall understand any sentence of the type

$$\pm Ma_i \cdot \pm Ma_j \cdot \cdots \cdot \pm Ma_t$$

where the constants 'a_i', 'a_j', $\cdots$, 'a_t' are all different from one another, and where the symbol '$\pm$' indicates that any one of the components may be either

[3] Illustrations: '$(P_1a_1 \vee P_2a_1) \supset (P_3a_1 \cdot \sim P_4a_1)$' stands for "If a_1 has at least one of the properties P_1, P_2 then it has the property P_3, but not the property P_4"; '$\sim(x)P_1x \supset (Ex)\sim P_1x$' stands for "If it is not the case that all objects have the property P_1, then there is at least one object which does not have the property P_1".

negated or unnegated. If we wish to indicate specifically the molecular property about whose incidence the statistic reports, we shall call a sentence of the above kind an *M-statistic*.

A sentence which contains at least one quantifier will be called a *general sentence*; in particular, all general laws, such as '$(x)(P_1x \supset P_2x)$', are general sentences.

By means of the p primitive predicates, we can form exactly $k = 2^p$ different conjunctions of the following kind: Each conjunction consists of exactly p terms; the first term is either 'P_1' or '$\sim P_1$', the second is either 'P_2' or '$\sim P_2$', and so on; finally, the pth term is either 'P_p' or '$\sim P_p$'. We call these expressions Q-expressions and, in lexicographic order, abbreviate them by 'Q_1', 'Q_2', $\cdots$, 'Q_k'. These Q-expressions designate certain molecular properties, which we shall call *Q-properties*. Alternatively, we may also say that each Q-expression designates a class, namely the class of all those individuals which have the Q-property in question. The classes designated by the Q-expressions clearly are mutually exclusive and exhaustive: every object belongs to one and only one of them. Moreover, they are the narrowest classes which can be characterized in L at all (except for the null class, which is designated, for example, by '$P_1 \cdot \sim P_1$'); for brevity, we shall refer to them as *(L-)cells*. In intuitive terms, we may say that if for a given individual we know to which L-cell it belongs, then we know everything about that individual that can be said in L at all; it is completely determined—relatively to the means of expression of L.

4. *A model language and a model world.* Our assumptions and definitions for L concern only the logical structure of that language and leave room for considerable variation in material content. For illustrative purposes, it will be useful to be able to refer to a specific model L_W of such a language and to a "model world" W of which it speaks.

Let us assume that the individuals a_1, a_2, a_3, $\cdots$ of which L_W speaks are physical objects, and that L_W contains just two primitive predicates, 'Blue' and 'Round'. Then L_W determines exactly four cells, Q_1 = Blue·Round, Q_2 = Blue·~Round, Q_3 = ~Blue·Round, Q_4 = ~Blue·~Round. All the hypotheses and evidence sentences expressible in L refer exclusively to the characteristics of blueness and roundness of the objects in W. Thus, e.g., the evidence sentence E might report, in the form of a statistic, on a sample of individuals in the following manner:

4.1 E = 'Blue a_1·Round a_1·Blue a_2·Round a_2·~(Blue a_3·Round a_3)
·Blue a_4·Round a_4'

and the hypothesis might be

4.2 H = 'Blue a_5·Round a_5'

In this case, E reports on four objects, three of which were found to be blue and round, while one was not; and H asserts that a fifth object, not yet examined (i.e. not referred to in E), will be blue and round. The question then arises: What degree of confirmation shall be assigned to H on the basis of E? We shall return to this case in the following section.

5. *Restatement of the problem.* Our basic problem can now be restated as follows: To define, in purely logical terms, the concept "degree of confirmation

of H relatively to E"—or briefly, 'dc(H, E)'—where H and E are sentences in a language L of the structure characterized in section 2, and where E is not contradictory.

The restriction of E to logically consistent sentences is justifiable on pragmatical grounds: No scientist would consider a contradictory "evidence sentence" as a possible basis for the appraisal of the soundness of an empirical hypothesis. —But the same restriction is demanded also, and more urgently, by considerations of generality and simplicity concerning the formal theory of confirmation which is to be based on our definition: We shall try to define dc in such a way that the following conditions, among others, are generally satisfied:[4]

5.1 $dc(H, E) + dc(\sim H, E) = 1$

5.2 If H is a logical consequence of E, then $dc(H, E) = 1$

But these requirements cannot be generally satisfied unless E is non-contradictory. For if E is a contradictory sentence, then any hypothesis H and its denial $\sim$H are consequences of E, and therefore, by virtue of 5.2, both have the dc 1 with respect to E; hence $dc(H, E) + dc(\sim H, E) = 2$, which contradicts 5.1.[5]

As the illustration 4.1 suggests, it might seem natural further to restrict E by the requirement that it has to be a molecular sentence, for in practice, E will usually consist in a report on a finite number of observational findings. However, it also happens in science that the evidence adduced in support of a hypothesis (such as Newton's law of gravitation) consists of general laws (such as Kepler's and Galileo's laws), and in the interest of the greatest possible adequacy and comprehensiveness of our definition, we shall therefore allow E to be any non-contradictory sentence in L. The sentence H, which represents the hypothesis under consideration, will be subject to no restrictions whatever; even analytic and contradictory hypotheses will be permitted; in these latter two cases, no matter what the evidence may be, the dc will yield the values 1 or 0, respectively, provided that dc is defined in such a way as to satisfy 5.1 and 5.2.

One of the guiding ideas in our attempt to construct a definition of confirmation will be to evaluate the soundness of a prediction in terms of the relative frequency of similar occurrences in the past. This principle appears to be definitely in accordance with scientific procedure, and it provides certain clues for a general definition of dc. Thus, e.g., in the case stated in 4.1 and 4.2, we shall want dc(H, E) to be equal to 3/4. And more generally, we shall want our definition to satisfy the following condition:

5.3 If E is an M-statistic and H a sentence ascribing the property **M** to an object not mentioned in E, then dc(H, E) is to be the relative frequency of the occurrence of 'M' in E.[6]

This rule is closely related to Reichenbach's rule of induction. This is no coincidence, for Reichenbach's theory, too, aims at giving a strictly empiricist

[4] Here and at some later places we use statement connective symbols autonomously, i.e., roughly speaking, as designations of the same symbols in the "object language" L.

[5] This argument was suggested by Professor Carnap.

[6] On this point, see also sections 10 and 16 in Professor Carnap's article.

account of the inductive procedure of science.[7] The applicability of the rule 5.3 is obviously restricted to the case where E is a statistic and H has the special form just described. And since we cannot presuppose that in science H and E are generally of this very special type, it becomes an important problem to find a rule whose scope will include also more complex forms of H and of E. In fact, this rule will have to be applicable to any H and any consistent E in L, and in cases of the special type just considered, it will have to yield that value of dc which is stipulated in 5.3.—We shall now develop, in a number of steps, the ideas which lead to a definition of the desired kind.

6. *Frequency distributions.* We have seen that for a given language L, the p primitive predicates determine $k = 2^p$ cells $Q_1, Q_2, \cdots, Q_k$. Each one of these cells may be occupied or empty, i.e. there may or there may not be individuals having the property which characterizes the elements of that cell. Whether a given cell is empty, and if not, how many objects it contains, is of course an empirical question and not a matter to be settled by logic. At any rate, if the number of all objects is finite, say N, then each cell Q_s has a certain occupancy number (i.e., number of elements) N_s, and a certain relative frequency $q_s = N_s/N$. Obviously, $N_1 + N_2 + \cdots + N_k = N$, and

6.1 $$q_1 + q_2 + \cdots + q_k = 1$$

If the class of all individuals is infinite—and here we restrict ourselves to the case of a denumerably infinite set of objects—then we shall assume that they are arranged in a fixed sequence, and by q_s we shall now generally understand the limit of the relative frequency with which elements belonging to cell Q_s occur in that sequence.

Now, while we do not actually know the values q_s, we may nevertheless consider certain hypothetically assumed values for them and develop the consequences of such an assumption. By a frequency distribution Δ in L, we shall understand any assignment of non-negative numbers $q_1, q_2, \cdots q_k$ to the cells $Q_1, Q_2, \cdots Q_k$ in such a fashion that 6.1 is satisfied. We shall briefly characterize such a distribution by the following kind of notation:

$$\Delta = \{q_1, q_2, \cdots q_k\}$$

It follows immediately that in every Δ,

6.2 $$0 \leqq q_s \leqq 1 \quad (s = 1, 2, \cdots, k)$$

Example: In the case of L_W, one of the infinitely many possible frequency distributions is $\Delta = \{\frac{1}{2}, \frac{1}{3}, 0, \frac{1}{6}\}$, which represents the case where one half of all objects are blue and round, one third of them blue and not round, none of them round and not blue, and one sixth of them neither blue nor round.[8]

[7] Cf. Hans Reichenbach, Wahrscheinlichkeitslehre, Leiden 1935, especially §§75–80, and Experience and Prediction, Chicago 1935, Chapter V.

[8] Note that distributions cannot be characterized in L and that, therefore, they cannot form the content of any hypothesis that may be formulated in L; we speak about them in a suitable meta-language for L. In our case, this meta-language is English, supplemented by a number of symbols, such as 'H', 'E', 'q_1', 'q_2', $\cdots$, 'Δ', etc. It might be well to emphasize at this point that the definition and the entire theory of dc for L is formulated in that

7. *Probability of a hypothesis.* If a fixed frequency distribution Δ is given or hypothetically assumed for the cells determined by L, then it is possible to define a concept 'pr(H, E, Δ)'—in words: "the probability of H relatively to E according to the distribution Δ,"—which we shall then use to define dc(H, E).

The meaning of this probability concept will first be explained by reference to our model. Let us consider the process of establishing evidence sentences, and of testing hypotheses by means of them, in analogy to that of drawing samples from an urn and using the evidence thus obtained for the test of certain hypotheses. The latter may concern either the distribution of certain characteristics over the whole population of the urn, or the occurrence or non-occurrence of certain characteristics in objects subsequently to be drawn from the urn. For the sake of simplicity we shall assume from now on that the totality of all objects to which L refers is denumerably infinite. (This does not necessarily mean that L contains infinitely many individual constants, but it does mean that the universal and existential quantifiers occurring in the general sentences of L refer to an infinite domain.)—Now let us imagine that for our model world W we are given the frequencies associated with the four cells determined by the two predicates of L_W; let this distribution be $\Delta_1 = \{\frac{1}{2}, \frac{1}{3}, 0, \frac{1}{6}\}$. Suppose further that the hypothesis H_1 = 'Blue a_1·Round a_1' is under consideration. We wish to show that a definite probability pr(H_1, E, Δ_1) can be assigned to H_1 with respect to any given E (with a restriction to be mentioned subsequently), according to the frequency distribution Δ_1.

I. We first consider the case where no information besides Δ_1 is available; in this case, E may be taken to be some analytic sentence, say 'Blue a_1 v ~Blue a_1', which we shall designate by 'T'. Thus, we are concerned with an explanation of pr(H_1, T, Δ_1), i.e. the probability of H_1 according to the frequency distribution Δ_1. We shall construe the problem of defining this magnitude in strict analogy to the following question: Given the distribution Δ_1 for the population of an urn W, what is the probability that the first object drawn will be both blue and round? And since there is no discrimination among the objects except in terms of their properties referred to in L_W, this latter probability will be the same as the probability that some object, chosen at random from the urn, will be both blue and round. The latter probability, however, is uniquely determined by the given distribution: it is the relative frequency assigned to Q_1 in Δ_1. In our case, therefore, pr(H_1, T, Δ_1) = $\frac{1}{2}$. Now let H_2 = 'Blue a_1', which is logically equivalent with '(Blue a_1·Round a_1)v(Blue a_1·~Round a_1)'. This sentence asserts that a_1 belongs to one of two cells whose occupancy frequencies, according to Δ_1, are $\frac{1}{2}$ and $\frac{1}{3}$, respectively. And since the cells are mutually exclusive, we set pr(H_2, T, Δ_1) = $\frac{1}{2} + \frac{1}{3}$. Similarly, for H_3 = '~Blue a_2', pr(H_3, T, Δ_1) = $\frac{1}{6}$. Finally, let H_4 = 'Blue a_1·Blue a_2·~Blue a_3'; then we set pr(H_4, T, Δ_1) = $\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6}$.[9]

meta-language, not in L itself: In the meta-language, we speak about the sentences of L and about the degrees to which certain sentences confirm others.

[9] In the case of a finite total population, the application of the simple product rule presupposes that the objects constituting a sample are taken from the urn one at a time, and that each of them is replaced into the urn before the next one is drawn. In order to avoid complications of this sort, we assume the population to be infinite.

After these illustrations, we shall now outline a general method of determining pr(H, T, Δ) for any given H and Δ. For this purpose, we introduce an auxiliary concept. By a *perfect description*, we shall understand a conjunction each of whose terms assigns some particular individual to some definite L-cell, and in which no individual is mentioned more than once. Thus, e.g., '$Q_1a_1 \cdot Q_1a_2 \cdot Q_2a_4 \cdot Q_4a_6$' is a perfect description.

IA. Now consider first the case that H is a molecular sentence. Then H can always be transformed into a conjunction of perfect descriptions. We omit the elementary but somewhat lengthy proof of this theorem here and rather illustrate it by an example: Let L contain exactly two primitive predicates, 'P_1' and 'P_2', and let H_5 = '$P_1a_1 \cdot P_2a_2$'; then H_5 can readily be expanded into the following expression:

$$'((P_1a_1 \cdot P_2a_1) \vee (P_1a_1 \cdot \sim P_2a_1)) \cdot ((P_1a_2 \cdot P_2a_2) \vee (\sim P_1a_2 \cdot P_2a_2))',$$

which in turn is equivalent to '$(Q_1a_1 \vee Q_2a_1) \cdot (Q_1a_2 \vee Q_3a_2)$'; and this can be transformed into the following disjunction of perfect descriptions:

$$'(Q_1a_1 \cdot Q_1a_2) \vee (Q_1a_1 \cdot Q_3a_2) \vee (Q_2a_1 \cdot Q_1a_2) \vee (Q_2a_1 \cdot Q_3a_2)'.$$

Once H has thus been transformed, the determination of pr(H, T, Δ) follows simply the following two rules, which were illustrated above: (a) The probability of H with respect to T and Δ is the sum of the probabilities of the perfect descriptions whose disjunction is equivalent to H; (b) The probability of a perfect description with respect to T and Δ is the product of the relative frequencies assigned by Δ to the Q-expressions occurring in the perfect description. Thus, if $\Delta = \{q_1, q_2, q_3, q_4\}$, then $pr(H_5, T, \Delta) = q^2 + q_1q_3 + q_2q_1 + q_2q_3$.

IB. If H is a general sentence, then two cases have to be distinguished$_1$:

a) If the number N of all objects to which L refers is finite, then H can obviously be transformed into a molecular sentence. Thus, e.g., the hypothesis '$(x)(P_1x \supset P_2x) \cdot (Ey)P_3y$' is equivalent to the following molecular sentence, which will also be called the *molecular development of H for the class of individuals* $\{a_1, a_2, \cdots, a_N\}$ or, briefly, $D_N(H)$: '$(P_1a_1 \supset P_2a_1) \cdot (P_1a_2 \supset P_2a_2) \cdots (P_1a_N \supset P_2a_N) \cdot (P_3a_1 \vee P_3a_2 \vee \cdots \vee P_3a_N)$'. Now we simply define pr(H, T, Δ) as $pr(D_N(H), T, \Delta)$; and the latter magnitude can be determined according to the rules laid down in IA.

b) If the class of all individuals is denumerably infinite and ordered in a sequence $a_1, a_2, a_3, \cdots$—and this is the case with which we are principally concerned—then we define pr(H, T, Δ) as the limit, for indefinitely increasing N, of $pr(D_N(H), T, \Delta)$. It can be shown that this limit exists in all cases. (In particular, we note that when H is a general sentence containing no individual constants the limit in question is either 0 or 1).

II. We now turn to the concept 'pr(H, E, Δ)', which refers to those cases where, besides the distribution Δ, some additional information E is given. To illustrate this case by means of the urn analogue and by reference to L_W: Let again $\Delta_1 = \{\frac{1}{2}, \frac{1}{3}, 0, \frac{1}{6}\}$, and let H_6 = '$\sim$Round a_5'. Then $pr(H_6, T, \Delta_1) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$. Now suppose that we are given the additional information E_1 = 'Blue a_5'. In the light of the thus enlarged total information, H_6 will acquire a

different probability: Since, according to E_1, a_5 is blue, and since, according to Δ_1, the frequency of the non-round objects among the blue ones is $\frac{1}{3} \div (\frac{1}{2} + \frac{1}{3}) = \frac{2}{5}$, we shall set $pr(H_6, E_1, \Delta_1) = \frac{2}{5}$.

A completely generaly definition of pr(H, E, Δ) can be given in terms of the narrower concept 'pr(H, T, Δ)':

7.1 $$pr(H, E, \Delta) = \frac{pr(H \cdot E, T \cdot \Delta)}{pr(E, T, \Delta)}$$

This definition presupposes that $pr(E, T, \Delta) \neq 0$; when this condition is not satisfied, pr(H, E, Δ) will not be defined.

This definition is suggested by the following consideration: We wish pr(H, E, Δ), for any fixed Δ, to satisfy the standard principles of probability theory.[10], including the general multiplication principle. Now the latter demands that

$$pr(E \cdot H, T, \Delta) = pr(E, T, \Delta) \cdot pr(H, E \cdot T, \Delta)$$

In view of the fact that $E \cdot H$ is logically equivalent to $H \cdot E$ and $E \cdot T$ logically equivalent to E, this leads to 7.1.

It can be proved that the concept thus defined satisfies all the customary postulates of probability theory.[11]

8. *Optimum distributions relatively to given evidence.* Our problem of defining dc(H, E) could now readily be solved if it were generally possible to infer from the given evidence E the frequency distribution Δ characteristic of the L-cells in the language under consideration; for we could then simply identify dc(H, E) with pr(H, E, Δ). Unfortunately, however, no evidence sentence that is expressible in L can be strong enough to permit such an inference. Nonetheless, a closely related but somewhat weaker procedure is indeed available for the definition of dc(H, E). This procedure is based on the fact that while a given E does not uniquely determine a fixed Δ, it may confer different degrees of likelihood—in a sense presently to be explained—upon the different possible distributions. Under favorable circumstances it may even be possible to characterize one particular distribution; Δ_E, as the one which is most likely on the basis of E; and in this case, dc(H, E) might be defined as $pr(H, E, \Delta_E)$. We shall eventually extend this idea to the case where E does not uniquely determine just one most likely distribution; but before going into the details of this method, which will be done in the subsequent section, we have first to clarify the idea of likelihood referred to in the preceding discussion.

Let us illustrate the essential points by reference to L_W and the urn analogue. Suppose that E_1 is a report asserting that among 12 objects selected at random, 6 were blue and round, 4 blue and not round, and 2 neither blue nor round. If no additional information is available, we would say that in the light of the given evidence, $\Delta_1 = \{\frac{1}{2}, \frac{1}{3}, 0, \frac{1}{6}\}$ is more likely than, say, $\Delta_2 = \{\frac{4}{9}, \frac{2}{9}, 0, \frac{3}{9}\}$, and that

[10] These are stated in section 10 of the present article.

[11] This probability concept was developed by Olaf Helmer; a detailed exposition of the theory of this concept is included in the article by Helmer and Oppenheim mentioned in footnote 1.

the latter is more likely than, say, $\Delta_3 = \{\frac{1}{10}, \frac{5}{10}, \frac{2}{10}, \frac{2}{10}\}$. Precisely how is the meaning of "more likely" to be construed here? It was shown in the preceding section that on the basis of any given frequency distribution Δ, and in the absence of any further information, it is possible to assign to every sentence S of L a definite probability pr(S, T, Δ); here, S may be a hypothesis under test or any other sentence in L. In particular, we may consider the case where S is our given evidence sentence E; i.e., we may ask: What is the probability pr(E, T, Δ) which E would possess on the basis of a certain hypothetical distribution Δ, and in the absence of any other information? If E is made more probable, in this sense, by a certain distribution Δ_1 than by another distribution Δ_2, then we shall say that Δ_1 has a *greater likelihood* relatively to E than does Δ_2.

Illustration: In our last example, we have

$$\begin{aligned} \mathrm{pr}(E_1, T, \Delta_1) &= (\tfrac{1}{2})^6 \cdot (\tfrac{1}{3})^4 \cdot (\tfrac{1}{6})^2 \\ \mathrm{pr}(E_1, T, \Delta_2) &= (\tfrac{4}{9})^6 \cdot (\tfrac{2}{9})^4 \cdot (\tfrac{3}{9})^2 \\ \mathrm{pr}(E_1, T, \Delta_3) &= (\tfrac{1}{10})^6 \cdot (\tfrac{5}{10})^4 \cdot (\tfrac{2}{10})^2 \end{aligned}$$

and indeed, as can readily be verified, we have here

$$\mathrm{pr}(E_1, T, \Delta_1) > \mathrm{pr}(E_1, T, \Delta_2) > \mathrm{pr}(E_1, T, \Delta_3),$$

in accordance with our earlier judgment as to the order of likelihoods involved.

Relatively to some given evidence E, therefore, the infinitely many theoretically possible frequency distributions fall into a definite order of likelihood. By an *optimum distribution relatively to* E, we shall understand a distribution Δ such that the probability pr(E, T, Δ) which Δ confers upon E is not exceeded by the probability that any other distribution would assign to E.[12]—Now it cannot be expected that every possible E determines exactly one optimum distribution: there may be several distributions each of which would give to E the same, maximum, probability. Thus, e.g.—to mention just one simple case—the probability of the evidence sentence 'Blue a_1' in L_W will clearly be maximized by any distribution which makes the frequency of the blue objects equal to 1, i.e. by any distribution of the form $\{q_1, 1 - q_1, 0, 0\}$, where q_1 may have any arbitrary value between 0 and 1 inclusive. It can be shown, however, that every E determines at least one optimum distribution; if there are several of them, then, of course, they all will confer the same probability upon E. We shall use the symbol 'Δ_E' to refer to the optimum distribution or distributions relatively to E; Δ_E is, therefore, a generally plurivalued function of the evidence E.

[12] An alternative to this approach would be to determine, by means of Bayes' theorem, that distribution upon which E confers the greatest probability (in contradistinction to our question for that distribution which confers upon E the maximum probability); but this approach presupposes—to state it first by reference to the urn analogue—an infinity of urns, each with a different frequency distribution; and to each urn U, there would have to be assigned a definite a priori probability for the sample to be taken from U. Applied to our problem, this method would involve reference to an infinity of possible states of the world, to each of which there would have to be attached a certain a priori probability of being realized; and for such a "lottery of states of the world," as it were, it seems very difficult to find an empiricist interpretation.

The determination of Δ_E for given E is a mathematical problem whose treatment will be discussed here only in outline. Consider again the model language L_W and the four cells Q_1, Q_2, Q_3, Q_4 determined by it. Let a specific evidence sentence E be given. To find Δ_E, consider the general case of a hypothetical distribution $\Delta = \{q_1, q_2, q_3, q_4\}$, where the four components of Δ are parameters satisfying the conditions

8.1 $$0 \leqq q_s \leqq 1 \qquad (s = 1, 2, 3, 4)$$

8.2 $$q_1 + q_2 + q_3 + q_4 = 1$$

The probability pr(E, T, Δ) which Δ confers upon E will be a function $f(q_1, q_2, q_3, q_4)$ of the parameters, as is illustrated at the end of IA in section 6. Δ_E can now be found by determining those values of the parameters which satisfy 8.1 and 8.2, and for which $f(q_1, q_2, q_3, q_4)$ assumes an absolute maximum. These values are found by partial differentiation of the function f. By equating the partial derivatives to 0, a system of simultaneous equations is obtained whose solution (or solutions) yield the value (or values) of Δ_E for the given evidence E. Explicit formulae for the solution of such systems of equations will be available only in special cases; but in many other cases, methods of computation can be indicated which will at least approximate the solutions. We mention here only one result of particular importance:

8.3 If E is a perfect description—as, for example, '$Q_1a_1 \cdot Q_1a_2 \cdot Q_1a_3 \cdot Q_2a_4 \cdot Q_2a_5 \cdot Q_3a_6$' in L_W—then Δ_E is unique, and its components are simply the relative frequencies with which the cells are represented in E—in our example, $\Delta_E = \{\frac{1}{2}, \frac{1}{3}, \frac{1}{6}, 0\}$.

The method which has been used here to characterize optimum distributions goes back to a procedure introduced by R. A. Fisher as the *maximum likelihood method.*[13] We shall consider later the general character of our procedure, but first we turn to the definition of dc in terms of the concept of optimum distribution.

9. *Definition of dc(H, E).* In accordance with the program outlined in the beginning of the preceding section, we now define

9.1 $$\mathrm{dc}(H, E) = \mathrm{pr}(H, E, \Delta_E)$$

This definition embodies an empiricist reconstruction of the concept of degree of confirmation: On the basis of the given evidence E, we infer the optimum distribution (or distributions) Δ_E and then assign to H, as its degree of confirmation, the probability which H possesses relatively to E according to Δ_E.

As can be seen from 7.1, the definition 9.1 determines dc(H, E) in all cases where pr(E, T, Δ_E) $\neq 0$. Now it can be shown that this condition is satisfied if and only if E is logically consistent; so that, by 9.1, dc(H, E) is defined for every non-contradictory E.

[13] Cf. R. A. Fisher: The mathematical foundations of theoretical statistics, Phil. Trans. Roy. Soc. London 222 (1922), pp. 309–368. Also see M. G. Kendall: On the method of maximum likelihood, Journal Roy. Stat. Soc. 103 (1940), pp. 388–399, and the same author's work, Advanced Theory of Statistics (London 1943).

It should be noted, however, that, since Δ_E is not necessarily single-valued, dc(H, E) may have more than one value.[14] Thus, e.g., when H = 'P_1a' and E = 'P_2b', dc(H, E) turns out to have as its values all the real numbers between 0 and 1 inclusive. This is quite sensible in view of the fact that the given E is entirely irrelevant for the assertion made by H; E, therefore, can impose no restrictions at all upon the range of the logically possible values of the degree to which H may be confirmed.

However, dc(H, E) can be shown to be single-valued in large classes of cases; these include, in particular, the cases where E is a perfect description, as can readily be seen from theorem 8.3. Also, it can be shown that in all cases of the kind characterized in 5.3, our definition leads to a unique value of dc(H, E), and that this value is the relative frequency stipulated in 5.3.

We shall now analyze in some detail a special example which incidentally shows that dc can be single-valued in cases other than those just mentioned. Let L contain just one primitive predicate, 'P,' and let

$$E = \text{'}(Pa_1 \cdot Pa_2 \cdot \sim Pa_3) \vee (Pa_1 \cdot \sim Pa_2 \cdot \sim Pa_3) \vee (\sim Pa_1 \cdot \sim Pa_2 \cdot Pa_3)\text{'}$$

and

$$H_1 = \text{'}Pa_4\text{'}, \qquad H_2 = \text{'}Pa_1\text{'}.$$

In order to determine Δ_E, we have to find that $\Delta = \{q, 1 - q\}$ which maximizes the magnitude

9.2 $$pr(E, T, \Delta) = q^2(1 - q) + q(1 - q)^2 + (1 - q)^2 q = q^3 - 3q^2 + 2q$$

By equating the derivative of this function to 0 and solving for q, we obtain

9.3 $$q = \frac{3 - \sqrt{3}}{3}$$

and hence

9.4 $$\Delta_E = \left\{\frac{3 - \sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right\}$$

Substituting from 9.3 in 9.2 yields

9.5 $$pr(E, T, \Delta_E) = \frac{2\sqrt{3}}{9}$$

We similarly compute

9.6 $$pr(H_1 \cdot E, T, \Delta_E) = \frac{2\sqrt{3}}{9} \cdot \frac{3 - \sqrt{3}}{3}$$

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[14] The symbol 'dc(H, E)' is therefore used here in a similar manner as, say, '$\sqrt{x}$' in mathematics; both represent functions which are not generally single-valued. An alternative would be to stipulate that dc(H, E) is to equal pr(H, E, Δ_E) in those cases where the latter function is single-valued, and that in all other cases, dc(H, E) is to remain undefined. A third possibility would be to define dc(H, E) as the smallest value of pr(H, E, Δ_E); for of two hypotheses tested by means of the same evidence, that one will be considered more reliable for which that smallest value is greater. This definition, however, has a certain disadvantage, which is explained in footnote 17.

Hence

9.7 $$\mathrm{dc}(H_1, E) = \frac{\mathrm{pr}(H_1 \cdot E, T, \Delta_E)}{\mathrm{pr}(E, T, \Delta_E)} = \frac{3 - \sqrt{3}}{3}$$

As for H_2, we note that

$$H_2 \cdot E = \text{`}(Pa_1 \cdot Pa_2 \cdot \sim Pa_3) \vee (Pa_1 \cdot \sim Pa_2 \cdot \sim Pa_3)\text{'}$$

Hence

9.8 $$\mathrm{pr}(H_2 \cdot E, T, \Delta_E) = q^2(1 - q) + q(1 - q)^2 = q(1 - q) = \frac{\sqrt{3} - 1}{3}$$

and finally

9.9 $$\mathrm{dc}(H_2, E) = \frac{\mathrm{pr}(H_2 \cdot E, T, \Delta_E)}{\mathrm{pr}(E, T, \Delta_E)} = \frac{3 - \sqrt{3}}{2}$$

After having considered some examples involving non-general hypotheses, we now turn to the case of hypotheses in the form of general sentences. Let us assume, for example, that L contains again only one primitive predicate, 'P', and let H = '(x)Px', E_1 = 'Pa_1', E_2 = '$Pa_1 \cdot Pa_2 \cdot \cdots Pa_t$', E_3 = '$\sim Pa_1 \cdot Pa_2 \cdot Pa_3 \cdot \cdots Pa_t$'. To compute the values of dc for these cases, we note first that, as can readily be shown, conditions 5.1 and 5.2 are satisfied by dc as defined in 9.1, and that, as a consequence, $\mathrm{dc}(H, E) = 0$ whenever H contradicts E. Now, if again we assume the class of all objects to be infinite, we obtain

$$\mathrm{dc}(H, E_1) = \mathrm{dc}(H, E_2) = \lim_{N \to \infty} 1^N = 1; \qquad \mathrm{dc}(H, E_3) = 0,$$

no matter now large t may be. The last value appears perfectly reasonable: Since E_3 contains one conjunctive term which contradicts H, E_3 itself contradicts H and thus disconfirms it to the highest degree that is theoretically possible. The value 1 in the first two cases, however, might seem counter-intuitive for two reasons: First, it seems strange that it should make no difference for the value of dc(H, E) how many confirming instances for H are included in E—as long as E contains no disconfirming evidence; and second, it is surprising that even one single confirming case for H should confirm the hypothesis H—which virtually covers an infinity of such cases—to the maximum extent. The significance of these results might become clearer if we distinguish between the retrospective and the prospective aspects of what has sometimes been called the probability, and what we call the degree of confirmation, of a universal hypothesis. Taken retrospectively, the magnitude in question is to characterize the extent to which H is confirmed "by past experience," i.e. by the given evidence E; taken prospectively, it is to constitute, as it were, a measure of the warranted assertability of the hypothesis, or of the rational belief to be placed in its validity in instances which have as yet not been examined. Now clearly, in our illustration, H is confirmed to the fullest possible extent by E_1 as well as by E_2: in both cases it is satisfied in 100 per cent of the instances mentioned by E. As to the prospective aspect, it is simply an inductivist attitude which directs us to assign the dc 1^N to the hypothesis that the next N instances will conform to the hypothesis, and finally, the limit of 1^N, for indefinitely increasing N, to the hypothesis itself, i.e. to the assumption that *all* objects conform to it.

10. *Probability and degree of confirmation.* Might dc(H, E) as well be called the probability of the hypothesis H relatively to the evidence E? Partly, of course, that is a matter of arbitrary terminological decision. However, the concept of probability has come to be used with reference to magnitudes which satisfy certain conditions which, for brevity, will be called here the postulates

of general probability theory.[15] We shall summarize them here in a form adapted from Janina Hosiasson-Lindenbaum's article "On confirmation."[16]

The probability of H relatively to E, or, briefly, p(H, E) is a single-valued function of two sentences, the second of which is non-contradictory. This function satisfies the following conditions:

10.1 If H is a consequence of E, then $p(H, E) = 1$

10.2 If E implies that H_1 and H_2 cannot both be true, then

$$p(H_1 \vee H_2, E) = p(H_1, E) + p(H_2, E)$$

(Special addition principle of probability theory)

10.3 $$p(H_1 \cdot H_2, E) = p(H_1, E) \cdot p(H_2, H_1 \cdot E)$$

(General multiplication principle of probability theory)

10.4 If E_1 and E_2 are logically equivalent, then

$$p(H, E_1) = p(H, E_2)$$

The concept 'pr(H, E, Δ)' can be shown to satisfy, for any fixed Δ, all of these conditions. But the concept 'dc(H, E)', which is defined by reference to it, does not. For, firstly, as we saw, dc(H, E) is not always a single-valued function of H and E. As to the four postulates listed above, the following can be shown: The first, second, and fourth postulates are generally satisfied by dc provided that when dc is plurivalued, "corresponding values"—i.e. values obtained from the same Δ_E—are substituted in the formulae. The third postulate, however, is not generally satisfied; the reason for this becomes clear when in 10.3, 'dc' is replaced by its definiens. Then the left hand side turns into '$pr(H_1 \cdot H_2, E, \Delta_E)$', and as 'pr' satisfies the general multiplication principle, we may transform the last expression into '$pr(H_1, E, \Delta_E) \cdot pr(H_2, H_1 \cdot E, \Delta_E)$'; but the right hand side of 10.3 transforms into '$pr(H_1, E, \Delta_E) \cdot pr(H_2, H_1 \cdot E, \Delta_{H_1,E})$'; and clearly, the second factors in these two expressions cannot generally be expected to be equal. However, the following restricted version of 10.3 is generally satisfied:

10.3′ "Corresponding values" of dc(H, E) satisfy 10.3 in particular if the two following conditions are satisfied:

(a) H_1 and H_2 have no individual constants in common,

(b) At least one of the hypotheses H_1, H_2 has no individual constants in common with E.[17]

[15] On this point, cf. also section 3 of Professor Carnap's article.

[16] The Journal of Symbolic Logic, vol. 5 (1940), pp. 133–148.

[17] In footnote 14, two alternatives to our definition of dc were mentioned. It can be shown that the concept determined by the first of these satisfies without exception the requirements 10.1, 10.2, 10.3′, and 10.4, whereas the concept introduced by the second alternative does not. Thus, e.g., if H = 'P_1a_1', E = 'P_2a_2', then the values of $pr(H, E, \Delta_E)$ are all the real numbers from 0 to 1 inclusive, so that the smallest value is 0. The same is true of $pr(\sim H, E, \Delta_E)$; hence these two smallest values violate the principle 5.1 and thus indirectly the postulates 10.1 and 10.2, of which 5.1 can be shown to be a consequence.

In view of the fact that dc as defined above does not satisfy all of the postulates of probability theory, we prefer not to call dc a probability.[17a]

Finally, it may be of interest to compare our way of defining dc with another method, which makes use of the concept of measure of a sentence. Briefly, this method consists in assigning, by means of some general rule, a measure m(S) to every sentence S in L in such a manner that the following conditions are satisfied:

10.51 For every S, $0 \leqq m(S) \leqq 1$

10.52 If S_1, S_2 are logically equivalent, then $m(S_1) = m(S_2)$

10.53 If S_1, S_2 are logically incompatible, then

$$m(S_1 \vee S_2) = m(S_1) + m(S_2)$$

10.54 For any analytic sentence T, $m(T) = 1$

The degree of confirmation of a hypothesis H with respect to a non-contradictory evidence sentence E is then defined as $\frac{m(H.E).}{m(E)}$ The stipulations 10.5 leave room for an infinite variety of possible measure functions; the choice of a particular function will be determined by the adequacy of the concept of degree of confirmation which is definable in terms of it.[18] Our concept 'dc(H, E)' can be introduced in a formally similar manner as follows: Instead of assigning to each sentence of L once and for all an apriori measure, as it is done in the method just described, we give to the sentences of L measures which depend on the given empirical evidence E. The E-measure of a sentence S in L can be defined thus:

10.6 $$m_E(S) = pr(S, T, \Delta_E)$$

In terms of this magnitude, we can express dc(H, E) as follows:

10.7 $$dc(H, E) = \frac{m_E(H \cdot E)}{m_E(E)},$$

for by virtue of 9.1, 7.1, and 10.6,

$$dc(H, E) = pr(H, E, \Delta_E) = \frac{pr(H \cdot E, T, \Delta_E)}{pr(E, T, \Delta_E)} = \frac{m_E(H \cdot E)}{m_E(E)}$$

[17a] The alternative term "likelihood" which suggests itself is inexpedient also, as it has already been introduced into theoretical statistics with a different meaning (cf. section 8 above). If a term customarily associated with "probability" should be desired, then "expectancy" might be taken into consideration.

[18] The method characterized above is illustrated by a definition of probability which F. Waismann (Logische Analyse des Wahrscheinlichkeitsbegriffs, Erkenntnis, vol. 1, pp. 228–248) has outlined following a suggestion made in L. Wittgenstein's Tractatus Logico-Philosophicus (New York and London 1922). Also, the regular c-functions introduced in Professor Carnap's article on inductive logic exemplify this way of defining dc. In that article, some special choices for the measure function m are presented and examined as to their suitability for the establishment of an adequate definition of the concept of degree of confirmation.

(As was pointed out in connection with 9.1, $m_E(E) = pr(E, T, \Delta_E)$ equals 0 only when E is contradictory; in this case, dc(H, E) is not defined.)

11. *Concluding remarks.* The concept of dc as it has been defined here is a purely logical concept in the following sense: Given two sentences H, E in L, dc(H, E) is completely determined by the formal, or syntactical, structure of the two sentences alone, and apart from possible mathematical complications, its value can be found by an analysis of that structure and the application of certain purely deductive mathematical techniques. Nevertheless, the proposed concept is empiricist and not "aprioristic" in character; for the degree of confirmation assigned to H is determined, generally speaking, by reference to relative frequencies derived from the evidence sentence E. With reference to the alternative definition 10.6, the matter can be stated as follows: dc is defined in terms of the concept of measure of a sentence; but whereas in an aprioristic theory, the measure of a sentence is determined once and for all on the basis of a mere analysis of its logical structure, the measure used in 10.6 is empiricist in that its determination requires reference not only to the structure of the sentence, but also to the given empirical evidence E.

The method employed to determine dc(H, E) consists essentially of two steps: First, by means of the maximum likelihood principle, a hypothetical assumption is formed, on the basis of E, as to the frequency distribution for the L-cells; second, on the basis of the hypothetical distribution thus assumed, a probability is assigned to H relatively to E. The rationale of this procedure is perhaps best exhibited by reference to a simple model case. Suppose that we are given a die about whose homogeneity and symmetry nothing is known. We have an opportunity to roll the die 20 times and are then to lay a bet on the hypothesis H that both the 21st and the 22nd throw will yield a six. The maximum likelihood principle would direct us, in this particularly simple case, to record, in a report E, the occurrence or non-occurrence of a six as the result of each of the first twenty throws, and then to form a hypothesis as to the limit of the relative frequency with which throws with the given die will yield a six. This limit is to be chosen in such a way that relatively to it, the distribution of the results found in E has a maximum probability. In the simple case under consideration, this means that we have to set the limit equal to the relative frequency with which the result six is reported in E; let this be $\frac{1}{10}$; then the distribution $\Delta_E = \{\frac{1}{10}, \frac{9}{10}\}$ for the cells corresponding to the results six and non-six is the optimum distribution, and on the basis of it, dc(H, E) becomes $\frac{1}{100}$; this value would be the basis for determining the rates of a fair bet on H, in the light of E.

In this special case, which is covered by the rule 5.3, the procedure dictated by the maximum likelihood principle clearly coincides with that which a "rational gambler" would use, and which is also used in statistical investigations of various kinds. It reflects an assumption which might be called the statistical version of the principle of induction, and which, stated in very crude terms, implies that relative frequencies observed "in the past" (i.e. in the instances so far examined) will remain fairly stable "in the future" (i.e. in those instances

which have not as yet been examined, no matter whether they belong to the past or to the future). The maximum likelihood principle in the form in which it has been used here for the general definition of dc is but an extension of this same idea to cases more complex than those covered by rule 5.3; and we may say that it represents a generalization and rational reconstruction of the statistical version of the principle of induction.

The theory obtained by our procedure provides criteria which establish, so to speak, a fair rate of betting on a specified hypothesis on the basis of given data. (In many cases, as we saw, dc will be single-valued and the betting rate will therefore be uniquely determined; in other cases, where the evidence is insufficient in a certain sense, dc will have several values, and then, the smallest of these might be used to establish a betting rate.) The decisions, however, which a gambler has to make concern not only the betting rate but also the amount he is going to risk; and while the rate is determined, generally speaking, by the relative frequency in the past of the event on which he wishes to bet, the gambler's stake will be determined by different factors, such as, e.g., the size of the sample which represents the evidence. Analogously, the concept of degree of confirmation as it has been defined in the present article, refers only to one among several factors which enter into an objective appraisal of the soundness or reliability of an empirical hypothesis. The remaining factors include, among others, the number of tested instances which are mentioned in E, and the variety of those instances.[19] Our theory of confirmation is intended to account exclusively for the first of these various aspects of the evaluation of a hypothesis by means of relevant evidence,—that aspect which is analogous to the betting rate in the preceding example.

The theory of confirmation which has been outlined in this article cannot claim to be more than a first contribution to the exploration of a field in which systematic logical research is only beginning. Among various problems which are suggested by the present study, we should like to point out a few which seem to deserve special attention in future research:

(1) The next step in the development of the theory of confirmation would be the extension of the definition of dc to the entire lower functional calculus and possibly even to the higher functional calculus.

(2) In section 1, we distinguished the metrical concept of degree of confirmation from the classificatory concepts of confirming and disconfirming evidence for a given hypothesis. In this connection, the question arises whether the meaning of the expressions "E is confirming evidence for H" and "E is disconfirming evidence for H" is adequately definable in terms of dc(H, E).

(3) In the practice of scientific research, observation reports are not all considered equally reliable; rather, their reliability will depend on certain characteristics of the observer, on the instruments of observation used, and on the circumstances under which the observation took place. Also, when general

[19] Cf. Ernest Nagel, Principles of the Theory of Probability; Internat. Encycl. of Unified Science, vol. 1, No. 6, Chicago 1939; pp. 68–71. Also, see section 15 of Professor Carnap's paper, which contains a discussion of this point.

sentences are included in the evidence E, these might be said to have different degrees of reliability (which, for example, might be determined on the basis of their dc relatively to all the relevant evidence known at the time). We might try to reflect this aspect of scientific testing by assuming in our theory that each evidence sentence is assigned a numerical "weight," whose value is a real number between 0 and 1, inclusive. The problem then arises of defining dc(H, E) in a manner which takes into consideration those weights attached to the evidence. The generalized definition here called for should comprehend, as one special case, our definition 9.1 (or another adequate definition of this kind); for the latter rests, as it were, on the tacit assumption that the weight of the given evidence is always 1.

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