

is, at the same time, a suggestion—arising from earlier usage, and because Bacon and Mill never quite freed themselves from it—of argument by mere multiplication of instances. I have thought it best, therefore, to use the term *pure induction* to describe arguments which are based upon the *number* of instances, and to use *induction* itself for all those types of arguments which combine, in one form or another, pure induction with analogy.

NOTES ON PART III

(i.) ON THE USE OF THE TERM *INDUCTION*

1. *INDUCTION* is in origin a translation of the Aristotelian *ἐπαγωγή*. This term was used by Aristotle in two quite distinct senses—first, and principally, for the process by which the observation of particular instances, in which an abstract notion is exemplified, enables us to realise and comprehend the abstraction itself; secondly, for the type of argument in which we generalise after the complete enumeration and assertion of *all* the particulars which the generalisation embraces. From this second sense it was sometimes extended to cases in which we generalise after an *incomplete* enumeration. In post-Aristotelian writers the induction *per enumerationem simplicem* approximates to induction in Aristotle's second sense, as the number of instances is increased. To Bacon, therefore, "the induction of which the logicians speak" meant a method of argument by multiplication of instances. He himself deliberately extended the use of the term so as to cover all the systematic processes of empirical generalisation. But he also used it, in a manner closely corresponding to Aristotle's *first* use, for the process of forming scientific conceptions and correct notions of "simple natures."¹

2. The modern use of the term is derived from Bacon's. Mill defines it as "the operation of discovering and proving general propositions." His philosophical system required that he should define it as widely as this; but the term has really been used, both by him and by other logicians, in a narrower sense, so as to cover those methods of proving general propositions, which we call empirical, and so as to exclude generalisations, such as those of mathematics, which have been proved formally. Jevons was led, partly by the linguistic resemblance, partly because in the one case we proceed from the particular to the general and in the other from the general to the particular, to define Induction as the inverse process of Deduction. In contemporary logic Mill's use prevails; but there

¹ See Ellis's edition of Bacon's *Works*, vol. i. p. 37. On the first occasion on which Induction is mentioned in the *Novum Organum*, it is used in this secondary sense.

(ii.) ON THE USE OF THE TERM *CAUSE*

1. Throughout the preceding argument, as well as in Part II., I have been able to avoid the metaphysical difficulties which surround the true meaning of *cause*. It was not necessary that I should inquire whether I meant by *causal* connection an invariable connection in fact merely, or whether some more intimate relation was involved. It has also been convenient to speak of causal relations between objects which do not strictly stand in the position of cause and effect, and even to speak of a *probable cause*, where there is no implication of necessity and where the antecedents will sometimes lead to particular consequents and sometimes will not. In making this use of the term, I have followed a practice not uncommon amongst writers on probability, who constantly use the term *cause*, where *hypothesis* might seem more appropriate.¹

One is led, almost inevitably, to use 'cause' more widely than 'sufficient cause' or than 'necessary cause,' because, the necessary causation of particulars by particulars being rarely apparent to us, the strict sense of the term has little utility. Those antecedent circumstances, which we are usually content to accept as causes, are only so in strictness under a favourable conjunction of innumerable other influences.

2. As our knowledge is partial, there is constantly, in our use of the term *cause*, some reference implied or expressed to a limited body of knowledge. It is clear that, whether or not, as Cournot² maintains, there are such things as independent series in the order of causation, there is often a sense in which we may hold that there is a closer intimacy between some series than between others. This intimacy is relative, I think, to particular information, which is actually known to us, or which is within our reach. It will be useful, therefore, to give precise definitions of these wider senses in which it is often convenient to use the expression *cause*.

¹ Cf. Czuber, *Wahrscheinlichkeitsrechnung*, p. 139. In dealing with Inverse Probability Czuber explains that he means by *possible cause* the various *Bedingungskomplexe* from which the cause can result.

² See Chapter XXIV. § 3.

We must first distinguish between assertions of law and assertions of fact, or, in the terminology of Von Kries,¹ between nomologic and ontologic knowledge. It may be convenient in dealing with some questions to frame this distinction with reference to the special circumstances. But the distinction generally applicable is between propositions which contain no reference to *particular* moments of time, and existential propositions which cannot be stated without reference to specific points in the time series. The Principle of the Uniformity of Nature amounts to the assertion that natural laws are all, in this sense, timeless. We may, therefore, divide our *data* into two portions *k* and *l*, such that *k* denotes our formal and nomologic evidence, consisting of propositions whose predication does not involve a particular time reference, and *l* denotes the existential or ontologic propositions.

3. Let us now suppose that we are investigating two existential propositions *a* and *b*, which refer two events A and B to particular moments of time, and that A is referred to moments which are all prior to those at which B occurred. What various meanings can we give to the assertion that A and B are *causally connected*?

(i.) If $b/ak = 1$, A is a sufficient cause of B. In this case A is a cause of B in the strictest sense. *b* can be *inferred* from *a*, and no additional knowledge consistent with *k* can invalidate this.

(ii.) If $b/ak = 0$, A is a necessary cause of B.

(iii.) If *k* includes all the laws of the existent universe, then A is *not* a sufficient cause of B unless $b/ak = 1$. The Law of Causation, therefore, which states that every existent has to some other previous existent the relation of effect to sufficient cause, is equivalent to the proposition that, if *k* is the body of natural law, then, if *b* is true, there is always another true proposition *a*, which asserts existences prior to B, such that $b/ak = 1$. No use has been made so far of our existential knowledge *l*, which is irrelevant to the definitions preceding.

(iv.) If $b/akl = 1$ and $b/kl \neq 1$, A is a sufficient cause of B under conditions *l*.

(v.) If $b/akl = 0$ and $b/kl \neq 0$, A is a necessary cause of B under conditions *l*.

(vi.) If there is any existential proposition *h* such that $b/ahk = 1$ and $b/hk \neq 1$, A is, relative to *k*, a possible sufficient cause of B.

(vii.) If there is an existential proposition *h* such that $b/ahk = 0$ and $b/hk \neq 0$, A is, relative to *k*, a possible necessary cause of B.

(viii.) If $b/ahkl = 1$, $b/hk \neq 1$, and $h/akl \neq 0$, A is, relative to *k*, a possible sufficient cause of B under conditions *l*.

(ix.) If $b/ahkl = 0$, $b/hkl \neq 0$, $h/akl \neq 0$, and $h/akl \neq 0$, A is, relative to *k*, a possible necessary cause of B under conditions *l*.

Thus an event is a possible necessary cause of another, relative to given nomologic data, if circumstances can arise, not inconsistent with our existential data, in which the first event will be indispensable if the second is to occur.

(x.) Two events are *causally independent* if no part of either is, relative to our nomologic data, a possible cause of any part of the other under the conditions of our existential knowledge. The greater the scope of our existential knowledge, the greater is the likelihood of our being able to pronounce events causally dependent or independent.

4. These definitions preserve the distinction between 'causally independent' and 'independent for probability,'—the distinction between *causa essendi* and *causa cognoscendi*. If $b/ahkl \neq b/ahk$, where *a* and *b* may be any propositions whatever and are not limited as they were in the causal definitions, we have 'dependence for probability,' and *a* is a *causa cognoscendi* for *b*, relative to data *kl*. If *a* and *b* are causally dependent, according to definition (x.), *b* is a possible *causa essendi*, relative to data *kl*.

But, after all, the essential relation is that of 'independence for probability.' We wish to know whether knowledge of one fact throws light of *any kind* upon the likelihood of another. The theory of causality is only important because it is thought that by means of its assumptions light *can* be thrown by the experience of one phenomenon upon the expectation of another.

¹ *Die Principien der Wahrscheinlichkeitsrechnung*, p. 86.