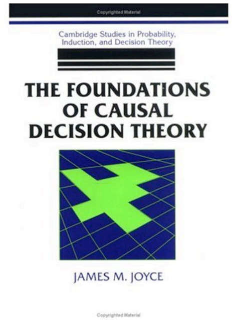
Probabilistic Measures of Causal Strength

Branden Fitelson
University of California, Berkeley

Christopher Hitchcock
California Institute of Technology

Happy Tin Anniversary!



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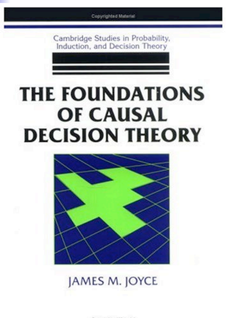
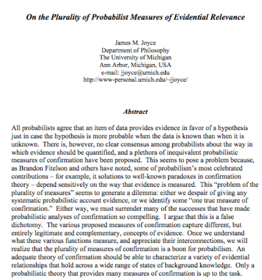


I can still remember the day when I turned the page!

Jim on Measures of Evidential Relevance



On the Plurality of Probabilist Measures of Evidential Relevance

James M. Joyce
Department of Philosophy
The University of Michigan
Ann Arbor, Michigan, USA
e-mail: jjoyce@umich.edu
http://www-personal.umich.edu/~jjoyce/

Abstract

All probabilists agree that an item of data provides evidence in favor of a hypothesis just in case the hypothesis is more probable when the data is known than when it is unknown. There is, however, no clear consensus among probabilists about the way in which evidence should be quantified, and a plethora of inequivalent probabilistic measures of confirmation have been proposed. This seems to pose a problem because, as Brandon Fitelson and others have noted, some of probabilism's most celebrated contributions – for example, it solutions to well-known paradoxes in confirmation theory – depend sensitively on the way that evidence is measured. This "problem of the plurality of measures" seems to generate a dilemma: either we despair of giving any systematic probabilistic account evidence, or we identify some "one true measure of confirmation." Either way, we must surrender many of the successes that have made probabilistic analyses of confirmation so compelling. I argue that this is a false dichotomy. The various proposed measures of confirmation capture different, but entirely legitimate and complementary, concepts of evidence. Once we understand what these various functions measure, and appreciate their interconnections, we will realize that the plurality of measures of confirmation is a boon for probabilism. An adequate theory of confirmation should be able to characterize a variety of evidential relationships that hold across a wide range of states of background knowledge. Only a probabilistic theory that provides many measures of confirmation is up to the task.

SAT Analogies

"Probabilistic Measures of Causal Strength"

is to

"On the Plurality of Probabilistic Measures of Evidential Relevance"

as

The Foundations of Causal Decision Theory

is to

The Logic of Decision

Theories of Causation

Probabilistic:

C causes E in background condition A_i just in case:

$P(E|CA_i) > P(E|{\sim}CA_i)$

Theories of Causation (cont'd)

Counterfactual:

C causes E in just in case:

The actual probability of E is higher than the probability of E would have been if C had not occurred

Theories of Causation (cont'd)

Interventionist:

C causes E in just in case:

Interventions that produce C yield a higher probability for E than interventions that produce ~C

Assumptions

Condition on a fixed background state so that there is no spurious correlation between C and E

If C_1 and C_2 are distinct causes of E, C_1 and C_2 are independent given this background condition

Notation

$CS(E, C) \stackrel{def}{=}$ causal strength of C for E

$CS(E, C_1; C_2) \stackrel{def}{=}$ causal strength of C_1 for E when C_2 is also present

$PS(E, C) \stackrel{def}{=}$ preventative strength of C for E

Measures of Causal Strength

Eells:	$CS_e(E, C) = P(E\mid C) - P(E\mid {\sim}C)$
Galton:	$CS_g(E, C) = P(C)P({\sim}C)[P(E\mid C) - P(E\mid {\sim}C)]$
Cheng:	$CS_c(E, C) = (P(E\mid C) - P(E\mid {\sim}C))/P({\sim}E\mid {\sim}C)$
Lewis ratio:	$CS_{lr}(E, C) = P(E\mid C)/P(E\mid {\sim}C)$
	$CS_{lr1}(E, C) = [P(E\mid C) - P(E\mid {\sim}C)]/[P(E\mid C) + P(E\mid {\sim}C)]$
	$CS_{lr2}(E, C) = [P(E\mid C) - P(E\mid {\sim}C)]/P(E\mid C)$
Suppes:	$CS_s(E, C) = P(E\mid C) - P(E)$
Suppes ratio:	$CS_{sr}(E, C) = P(E\mid C)/P(E)$
	$CS_{sr1}(E, C) = [P(E\mid C) - P(E)]/[P(E\mid C) + P(E)]$
	$CS_{sr2}(E, C) = [P(E\mid C) - P(E)]/P(E\mid C)$
Good:	$CS_{ij}(E, C) = P({\sim}E\mid {\sim}C)/P({\sim}E\mid C)$
	$CS_{ij1}(E, C) = [P({\sim}E\mid {\sim}C) - P({\sim}E\mid C)]/[P({\sim}E\mid {\sim}C) + P({\sim}E\mid C)]$
	$CS_{ij2}(E, C) = [P({\sim}E\mid {\sim}C) - P({\sim}E\mid C)]/P({\sim}E\mid {\sim}C)$

Issues and Properties

- Scaling
- Continuity Properties
- Independence
- Comparisons Between Measures

Scaling

For purposes of comparing measures, we will convert all measures to a unit scale

If C causes E, $CS(E, C) \in (0, 1]$

If C prevents E, $PS(E, C) \in [-1, 0)$

Ordinal Equivalence

CS and CS' are ordinally equivalent just in case:

For all C, E, C', E'

$CS(E, C) \geq CS(E', C')$

Iff

$CS'(E, C) \geq CS'(E', C')$

Re-scaling

For $CS(E, C) \in (1, \infty]$

Fitelson re-scaling:

- $CS \rightarrow (CS - 1)/(CS + 1)$
- $A/B \rightarrow (A - B)/(A + B)$

Hitchcock re-scaling:

- $CS \rightarrow 1 - 1/CS$
- $A/B \rightarrow 1 - B/A = (A - B)/A$

Continuity Properties

- Causation-Prevention Continuity (CPC)
- Causation-Omission Continuity (COC)

Causation-Prevention Continuity

When C prevents E

PS(E, C) = –CS(~E, C)

CS satisfies CPC iff

PS(E, C) = CS(E, C)

Causation-Omission Continuity

When the failure or omission of C is a cause of E, we can measure its strength:

CS(E, ~C)

CS satisfies COC just in case:

CS(E, ~C) = –CS(E, C)

Causal Independence

C_1 and C_2 are independent in causing E just in case:

$CS(E, C_1; C_2) = CS(E, C_1; \sim C_2)$

We will write $I_{CS}(E, C_1, C_2)$

Causal Independence

Is $I_{CS}(E, C_1, C_2)$ symmetric in C_1 and C_2?

Does $I_{CS}(E, C_1, C_2)$ impose constraints on $CS(E, C_1)$ and $CS(E, C_2)$?

Comparing Measures

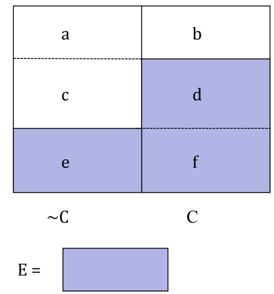
- Mathematical inter-definability
- Ordinal equivalence
- Agreement in independence judgments

The Eells Measure

$CS_e(E, C)$

$= P(E|C) – P(E|\sim C)$

$= c + d$



The Eells Measure (cont'd)

Related to:

Causal Effect

Pearl's PNS

The Galton Measure

$CS_g(E, C)$

$= P(C)P(\sim C)[P(E|C) – P(E|\sim C)]$

$= P(C)P(\sim C)CS_e(E, C)$

The Galton Measure (cont'd)

$CS_g(E, C) = Cov(X_c, X_e)$

$= E(X_c X_e) - E(X_c)E(X_e)$

where X_c, X_e are the indicator functions of C and E

The Galton Measure (cont'd)

Scaling:

$CS_g(E, C)$ takes values in $[-\frac{1}{4}, \frac{1}{4}]$

To convert to $[-1, 1]$

$Corr(X_c, X_e) = Cov(X_c, X_e)/\sigma_{Xc}\sigma_{Xe}$

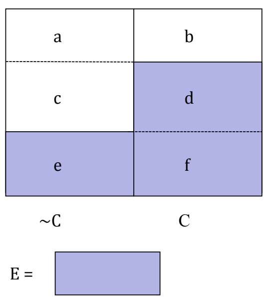
Or just multiply by 4

The Cheng Measure

$CS_c(E, C)$

$$= \frac{P(E \mid C) - P(E \mid \sim C)}{P(\sim E \mid \sim C)}$$

$= d/(b + d)$



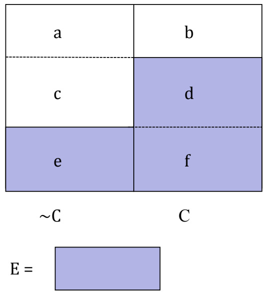
The Cheng Measure (cont'd)

$CS_c(E, C) = Cs_e(E, C)/P(\sim E \mid \sim C)$

Related to Pearl's PS

The Lewis Ratio Measure

$$CS_{lr}(E, C) = \frac{P(E \mid C)}{P(E \mid \sim C)} = (d+f)/d$$



The Lewis Ratio Measure (cont'd)

- Related to 'relative risk' or 'risk ratio' in epidemiology and tort law

The Lewis Ratio Measure (cont'd)

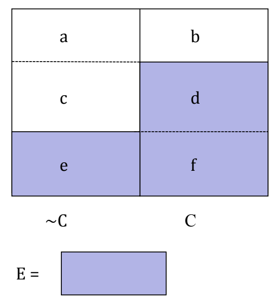
Scaling:

When C causes E,
$CS_{lr}(E, C)$ takes values in $(1, \infty]$

When C prevents E
$CS_{lr}(E, C)$ takes values in $[0, 1)$

The Lewis Ratio Measure (Fitelson re-scaling)

$$CS_{lr1}(E, C) = \frac{P(E \mid C) - P(E \mid \sim C)}{P(E \mid C) + P(E \mid \sim C)} = d/(d + e + f)$$



The Lewis Ratio Measure (Fitelson re-scaling cont'd)

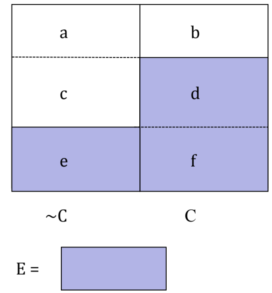
- $CS_{lr1}(E, C) = \dfrac{CS_e(E, C)}{P(E \mid C) + P(E \mid \sim C)}$

The Lewis Ratio Measure (Hitchcock re-scaling)

$CS_{lr2}(E, C)$

$= \dfrac{P(E \mid C) - P(E \mid \sim C)}{P(E \mid C)}$

$= d/(d + f)$



The Lewis Ratio Measure (Hitchcock re-scaling)

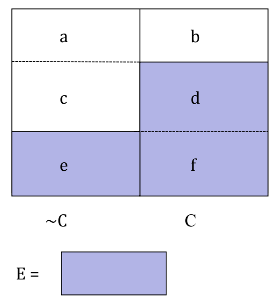
- $CS_{lr2}(E, C) = \dfrac{CS_e(E, C)}{P(E \mid C)}$
- $CS_{lr2}(E, C)$ is called the 'probability of causation' in epidemiology and tort law
- $CS_{lr2}(E, C)$ is related to Pearl's PN

The Suppes Measure

$CS_s(E, C)$

$= P(E|C) – P(E)$

$= c$



The Suppes Measure (cont'd)

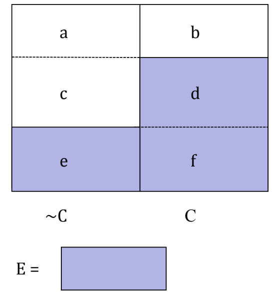
- $CS_s(E, C) = CS_e(E, C)P(\sim C)$
- $CS_s(E, C) = CS_g(E, C)/P(C)$
- $CS_s(E, C) = CS_c(E, C)P(\sim E \wedge \sim C)$

The Suppes Ratio Measure

$CS_{sr}(E, C)$

$= P(E|C)/P(E)$

$$= \frac{c + d + e + f}{d + e + f}$$



Re-scaling the Suppes Ratio Measure

Fitelson re-scaling:

$$CS_{sr1}(E, C) = \frac{P(E \mid C) - P(E)}{P(E \mid C) + P(E)}$$

$$= \frac{CS_s(E, C)}{P(E \mid C) + P(E)}$$

Re-scaling the Suppes Ratio Measure (cont'd)

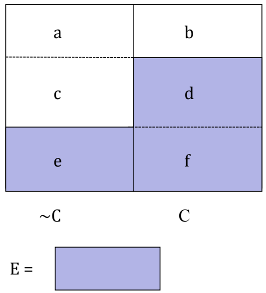
Hitchcock re-scaling:

$$CS_{sr2}(E, C) = \frac{P(E \mid C) - P(E)}{P(E \mid C)}$$

$= CS_s(E, C)/P(E| C)$

The Good Measure

$CS_{ij}(E, C)$
= P(~E|~C)/P(~E|C)
= (b + d)/b



The Good Measure (cont'd)

The measure Good actually proposed was the log of this quantity

If C causes E, $Cs_{ij}(E, C)$ takes values in $(1, \infty]$

Re-scaling the Good Measure

Fitelson re-scaling:

- $CS_{ij1}(E, C) = \dfrac{P(\sim E \mid \sim C) - P(\sim E \mid C)}{P(\sim E \mid \sim C) + P(\sim E \mid C)}$

Hitchcock re-scaling:

- $CS_{ij2}(E, C) = \dfrac{P(\sim E \mid \sim C) - P(\sim E \mid C)}{P(\sim E \mid \sim C)}$

Re-scaling the Good Measure (cont'd)

Hitchcock re-scaling:

$CS_{ij2}(E, C) = CS_c(E, C)$

Ordinal Equivalence

CS and CS' are ordinally equivalent just in case:

For all C, E, C', E'

CS(E, C) ≥ CS(E', C')

Iff

CS'(E, C) ≥ CS'(E', C')

Ordinal Equivalence (cont'd)

Different re-scalings of a measure are ordinally equivalent with the original measure and with each other (by definition)

Apart from re-scalings, only the Good and Cheng Measures are ordinally equivalent in general

Ordinal equivalence (cont'd)

For a fixed cause C, the Eells and Suppes Measures are ordinally equivalent

$CS_e(E_1, C) \geq CS_e(E_2, C)$

iff

$CS_s(E_1, C) \geq CS_s(E_2, C)$

Ordinal equivalence (cont'd)

For a fixed cause C, the Lewis Ratio and Suppes Ratio Measures are ordinally equivalent

For a fixed effect the Suppes and Cheng Measures are ordinally equivalent

Continuity Properties

The Eells, Galton, and Suppes measures all satisfy Causation-Prevention continuity: CS(E, C) = –CS(~E, C)

e.g. Eells:

$Cs_e(E, C) = P(E|C) – P(E|{\sim}C)$

Continuity Properties (cont'd)

Contrast with e.g. Cheng:

$$Cs_c(E, C) = \frac{P(E \mid C) - P(E \mid {\sim} C)}{P({\sim} E \mid {\sim} C)}$$

If you switch E and ~E you change not only the sign but also the denominator

Continuity Properties (cont'd)

The Eells and Galton Measures satisfy Causation-Omission continuity:

CS(E, C) = –CS(E, ~C)

e.g. Eells

$Cs_e(E, C) = P(E|C) – P(E|{\sim}C)$

The Suppes Measure does not:

$CS_s(E, C) = P({\sim}C)[P(E|C) – P(E|{\sim}C)]$

Continuity Properties (cont'd)

The Fitelson re-scalings of the Lewis Ratio and Good Measures satisfy COC.

The Hitchcock re-scalings of the same measures do not!

Causal Independence

C_1 and C_2 are independent in causing E just in case:

$CS(E, C_1; C_2) = CS(E, C_1; \sim C_2)$

We will write $I_{CS}(E, C_1, C_2)$

Causal Independence (cont'd)

Symmetry:

For all our measures CS:

$I_{CS}(E, C_1, C_2)$ iff $I_{CS}(E, C_2, C_1)$

Causal Independence (cont'd)

The Eells, Galton, and Suppes Measures all have constraints on independence:

$I_{CS}(E, C_1, C_2)$
$\rightarrow CS(E, C_1) + CS(E, C_2) \leq 1$

The other measures are unconstrained

Causal Independence (cont'd)

If two measures are ordinally equivelant, they yield the same independence relations

Re-scalings yield the same independence relations

The Cheng and Good Measures yield the same independence relations

Causal Independence (cont'd)

The Eells, Galton, and Suppes Measures all yield the same independence relations

The Lewis Ratio and Suppes Ratio Measures yield the same independence relations

Causal Independence (cont'd)

Apart from these cases, two measures will agree on whether $I(E, C_1, C_2)$ only in the trivial case where one of C_1 or C_2 has a CS of 0