Comments on Lara Buchak's "Risk Sensitivity"
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- **First reaction: the idea of risk sensitivity has some plausibility. Agents who exhibit it seem not crazy. Are they rational?**

- **We are familiar with agents' risk-aversion with respect to gains in, for example, $ received.**

  – **Much of this behavior may be captured by the shape of the agent's utility curve.**

  – **Agents may also be risk-seekers with respect to $ lost.**

A utility curve (Tversky & Kahneman, 1979)

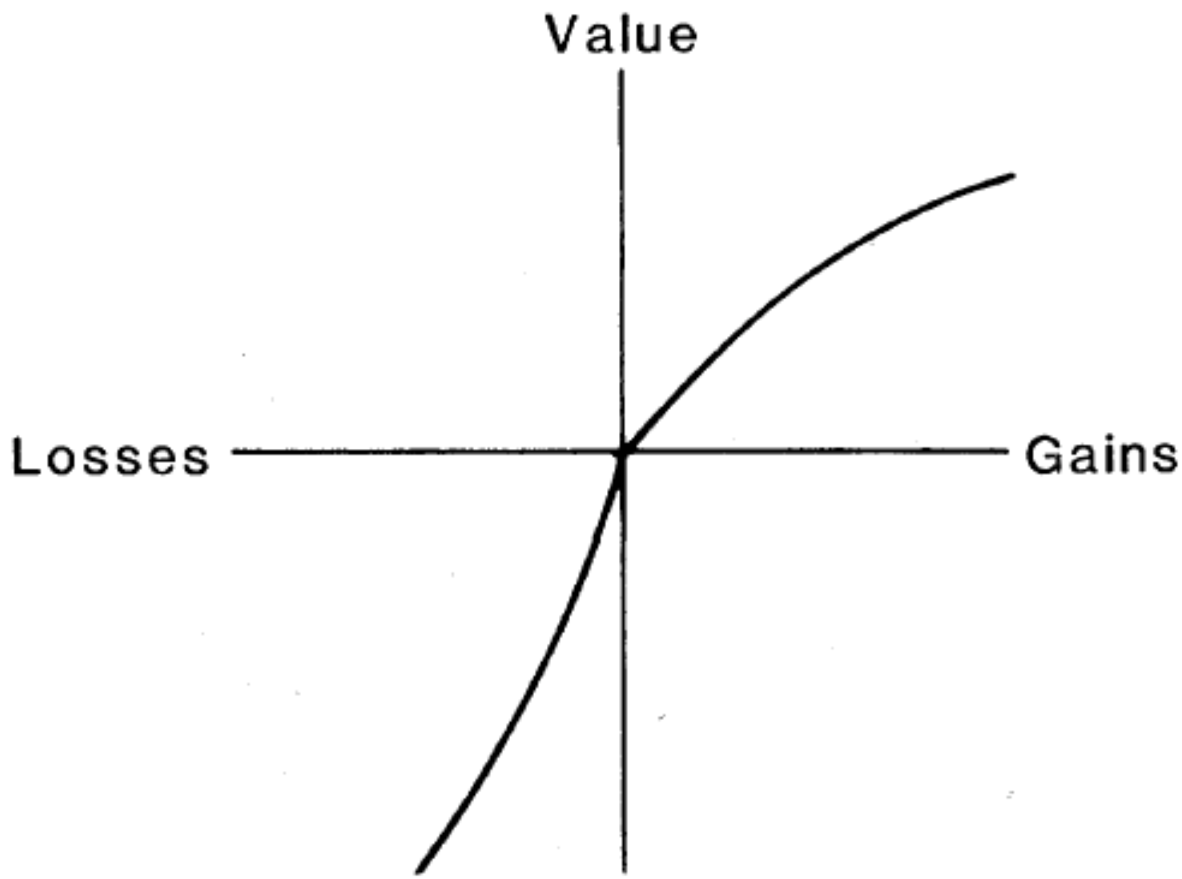


Fig. 1. A hypothetical value function.

L.B.: Beyond this, though, agents display <u>risk-sensitivity</u>.

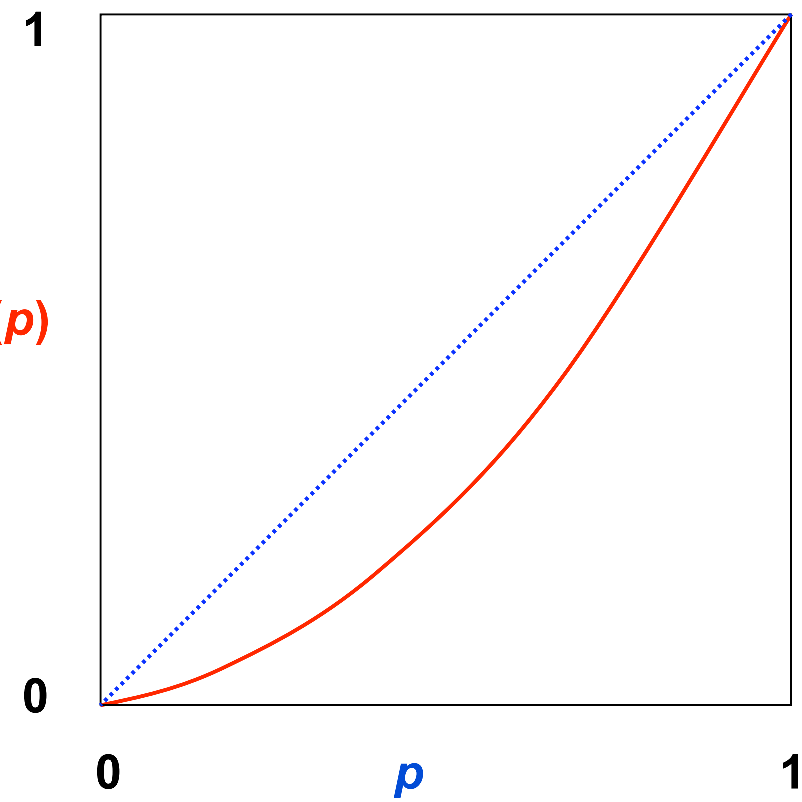
- **Consider a two-outcome risky choice G; let outcome A be preferred to B. Risk sensitivity is a reaction to the probability with which the agent can achieve G's better outcome.**
- **The contribution that A makes to G's value is discounted. It is diminished due to inherent undesirability of the riskiness attached to getting A.**
- **So the discounting factor is presented as a function *f* of the probability *p* that G yields A.**

$f \leq p$, non-decreasing, and = p at the values of 0, 1.

$G = pA \oplus (1-p)B$

$$
\begin{aligned}
des(G) &= des(\, pA \oplus (1-p)B \,) \\
&= p\,des(A) + (1-p)\,des(B) \\
&= des(B) + p[des(A) - des(B)]
\end{aligned}
$$

$G = pA \oplus (1-p)B$



$$des(G) = des(\, pA \oplus (1-p)B \,)$$
$$= p\,des(A) + (1-p)des(B)$$
$$= des(B) + p[des(A) - des(B)]$$

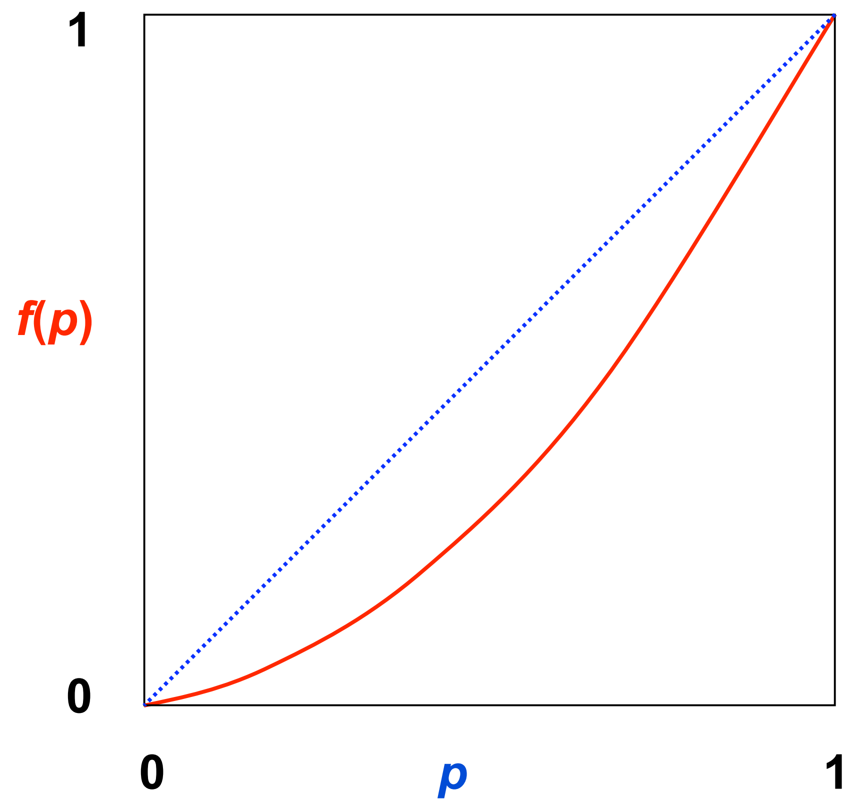
For a risk-sensitive agent,

$des_R(G) = des_R(B) + f(p)[des_R(A) - des_R(B)]$.

Since $f(p) < p$ in risk-sensitive cases, $des_R(G) < des(G)$.

des_R(G) =
des_R(B) + $f(p)[des_R$(A) – des_R(B)].

What function is *f* ?

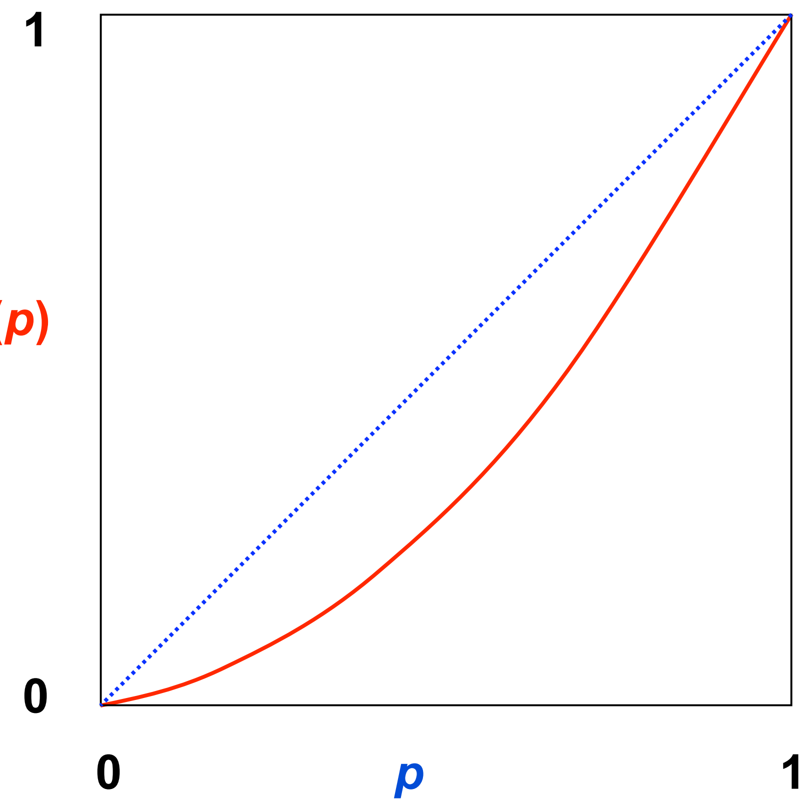


des_R(G) =
des_R(B) + $f(p)[des_R$(A) – des_R(B)].

What function is f ?

L.B. considers several possibilities; one example is $f(p) = p^2$; others are partially specified.

But f is clearly a function of p. Its effect on the value of G is fixed by p.

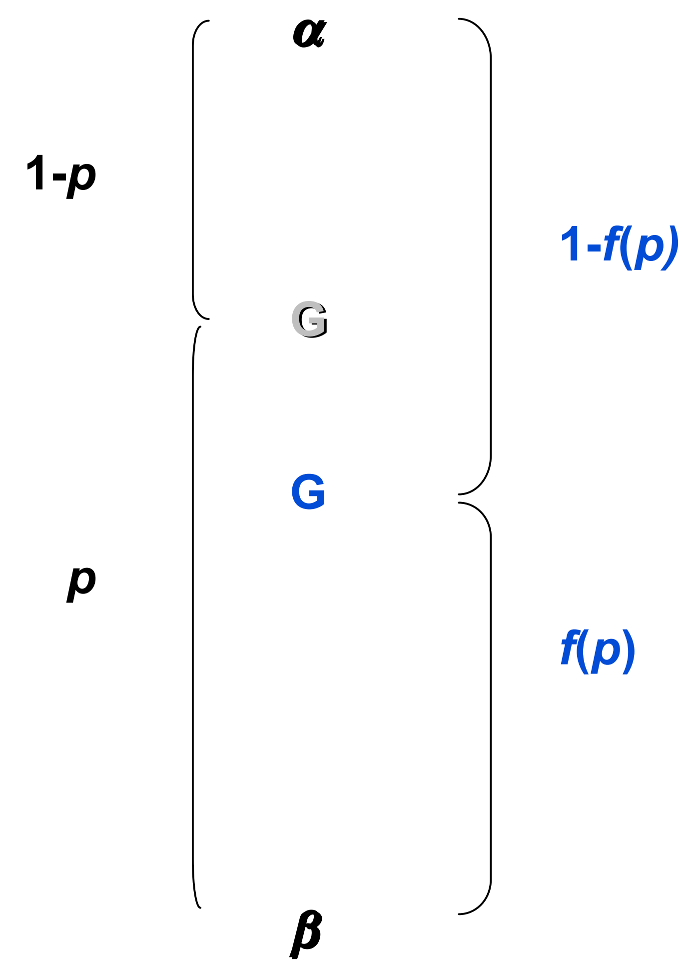


So it seems that for a given agent (and time) risk-sensitivity is uniform over risky choices.

Every choice of the form

$$G = p\alpha \oplus (1\text{-}p)\beta$$

is discounted by the same *f*(*p*).



As L.B. mentions, *f* is similar to decision weight functions proposed by a number of decision theorists, including to Kahneman & Tversky in their "prospect theory."

K&T sought to account for risk-aversion, risk-seeking, neglect of small probabilities, framing, rounding …

Tversky & Kahneman's weighting function π

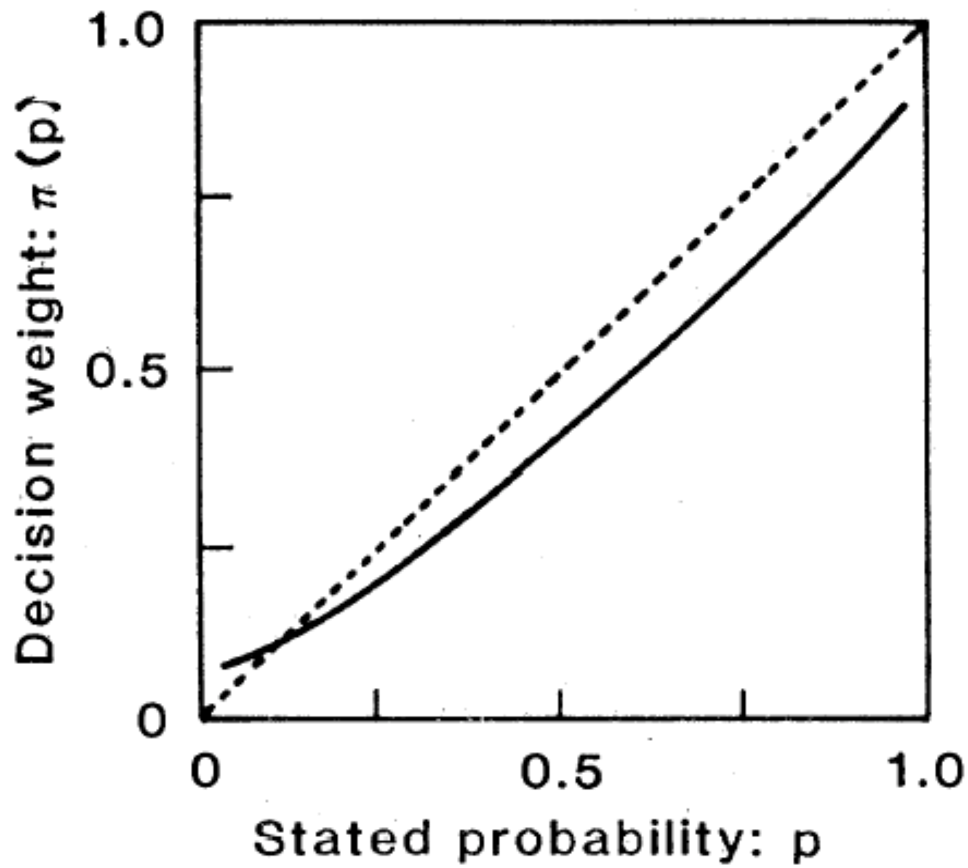


Fig. 2. A hypothetical weighting function.

- **K&T explicitly present prospect theory as a descriptive account of decision-making under risk.**
- **L.B. is interested in drawing conclusions about <u>rational</u> decision-making.**

Questions:

- **Does a different kind of risk-sensitivity, where risk carries positive value to an agent, deserve a similar treatment?**
- **Is L.B.'s risk-sensitivity more significant for normative theory than are other observed effects that EU theory has difficulty in capturing?**
- **If so, why?**

- **No dispute that L.B.'s risk-sensitive agent will not have preferences provably representable by EU theory.**

  As L.B. points out, her treatment of risk-sensitivity over compounded gambles can violate the Sure-thing Principle.

- **Nor do I dispute that there are difficulties in running the Dutch book argument against an agent who exhibits this sort of risk-sensitivity in the betting scenario.**

- **In fact, I think L.B.'s discussion of the Dutch book argument is quite interesting.**

Let "V(Bp)" stand for the value of a bet on p, (understood to be measured by the betting quotient of the least favorable bet the agent would accept on p).

One way of violating $V(Bp) + V(Bq) = V(Bp \oplus Bq)$

is to have $V(Bp) + V(Bq) > V(Bp \oplus Bq)$.

In the past I argued, and would still say, that this looks motivated by a reaction to compounded losses when buying the bets $Bp \oplus Bq$ (or of compounded gains when selling them).

But any such reaction would also be appropriate when one is offered a single bet on p for doubled stakes. And so the idea that we have stable degrees of belief in p, q, … over a range of stakes would be a non-starter.

On the other hand, violation of addition in the other direction $V(Bp) + V(Bq) < V(Bp \oplus Bq)$, seems different.

To fix the direction, suppose we are buying the bets.

Why does the bet on the disjunction (rhs) get extra value?
Not because of compounded gain—its payoff is the same as either individual bet. If there is a reason, it's the greater chance of winning, compared to just one of the individual bets. The invariance of belief to the stakes does not speak to this.

L.B.'s risk-sensitive agent responds to risk in very much this way. The compound bet is less risky than either of the component bets (obviously), and so has enhanced value for just that reason (or, is discounted less for just that reason).

- **This seemed to me a gap in the argument, but not a particularly well motivated possibility.**
- **LB envisions a systematic way in which the preferences of rational agents are altered by an inherent disutility of the risk that many (most) of their available choices carry.**
- **This is a substantial and interesting project if it can be fruitfully carried out.**
- **Maybe it can be, but I have a methodological worry about her treatment, and also an (insecure) intuition that seems to conflict with it.**

Methodological worry: assuming the probability function *p*.

- **What can we say about the beliefs and probabilities that guide decision-making?**
- **L.B. assumes *p* is given.**

So do K&T: "objective", "stated" probabilities.

- **Ramsey was more interested in probability than in decision theory; the Dutch book argument is supposed to show something about rational degrees of belief.**
- **Not L.B.'s project, perhaps, to give a simultaneous account of rational belief and rational choice; plenty of decision theories do presuppose the former as they address the latter.**
- **But how would we proceed if L.B. is right about risk sensitivity?**

Finding *f* and *p*.

- **Choices that L.B.'s rational agents make among risky options depend on *p* and *f*. The intermingled influences of these factors are indicated in L.B.'s discussion of Ramsey's theory.**

- **Given a rational agent and his preference ordering, what is his *f*?**

  If the relevant probabilities are all known/given, and if *f* depends entirely on them in a way that is consistent across all risky choices, then showing a simultaneous representation of the agent's preferences by utility and *f* can be imagined.

  It would be interesting to see that done with an appropriate set of preference axioms.

Finding *f* and *p*.

What's the source of information about the presupposed probabilities? Possible sketch:

- **Begin with specified lotteries, or with Ramsey's test for $p = ½$.**
  (Assumptions of identifiable independence, neutrality.)
- **Develop a range of risky gambles by iterating that test.**
  (Need precise method for treating compound gambles.)
- **Identify the function *f* according to how preference among those gambles are arranged;**
- **Invoke the assumption that *f* is unchanging over all probabilistically similar risks;**
- **Locate *p* values for all the propositions in the agent's system of beliefs.**

Finding *f* and *p*.

I can imagine the preceding, but haven't carried it out.

I can ask why it is a plausible assumption that rational discount functions exist and have exactly the features and influence L.B. describes.

Why are they purely functions of probability/belief, fixed over all choices with the same probability profiles?

Why does negative risk-sensitivity merit treatment as a rational influence on decision-making, whereas other effects do not?

(If other effects also merit inclusion, the task of providing an account becomes even more complex.)

- **L.B. presents an account of risk-sensitivity, and finds no effective arguments that the phenomenon is irrational.**
- **Is there an effective positive argument that it is consistent with rationality?**

  In the presence of the developed theories that exclude it, one effective argument would be a well developed and well motivated theory that incorporates it.

- **Point is not that the accounts and arguments that L.B. is dissatisfied with cannot be improved or elaborated. They are clearly models of rational choice and rational belief that have limitations.**

  Describing their limitations is one thing; producing a better model is another. It would be great to see that done.

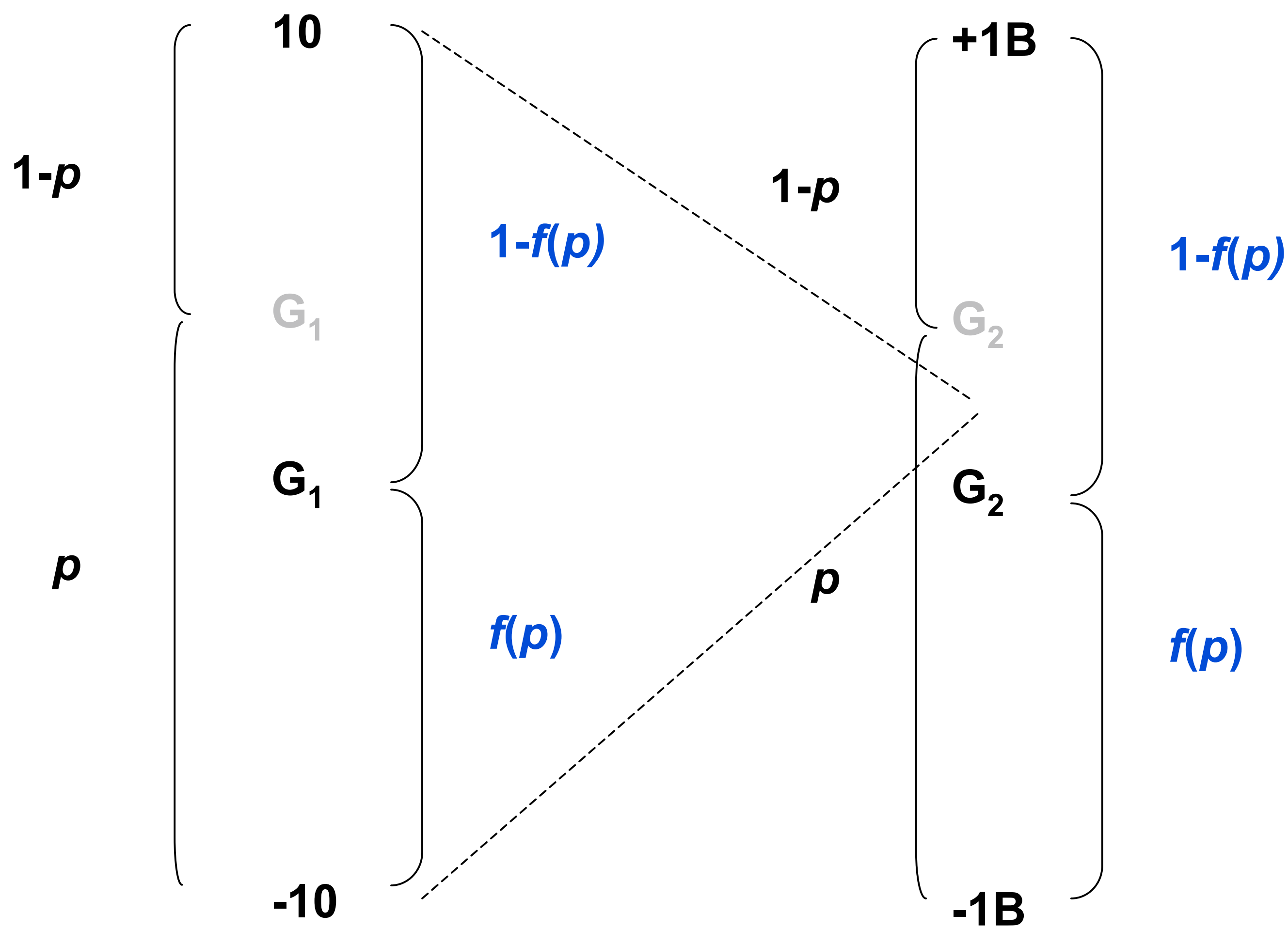
Is risk sensitivity uniform over risky choices?
(Ought it to be?)

G_1: p(10 utiles) $\oplus$ (1-p)(-10 utiles)

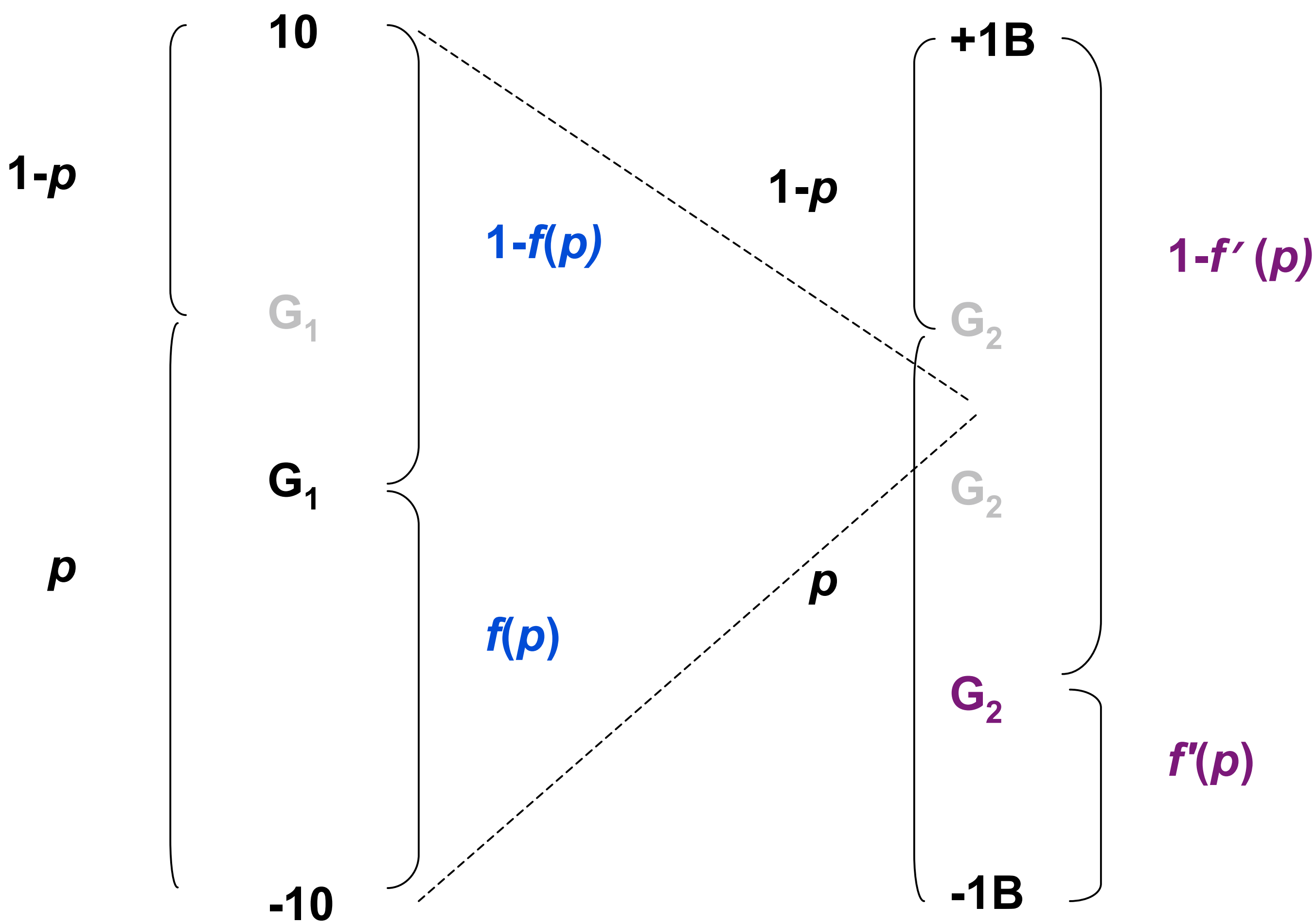
G_2: p(1 billion utiles) $\oplus$ (1-p)(-1billion utiles)

Cues: 10 utiles is the value of receiving an unexpected $100
- 1 billion utiles is something like slow death by torture, brutal incarceration for life

For a risk-sensitive agent, will the G_1 be discounted at the same rate as G_2?



Or might G_2 be discounted at a higher rate, by a different *f′* ?



- **Is this a plausible suggestion? I'm not sure.**
- **But if there is a difference in risk-sensitivity, the shape of the utility curve over goods/bads wouldn't explain it.**
- **If the discount function *f* does shift, the interesting question is what causes it?**