A Defence of the Fine-Tuning Argument for the Multiverse

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In this paper I will defend the fine-tuning argument for multiple universes against Hacking's claim that it commits the 'inverse gambler's fallacy'. Hacking has identified a fallacy, but I hope to show by an examination of his own dice example that it does not occur in the fine-tuning argument. In particular, a condition must be satisfied for an inverse gambler's fallacy to be made that it is not satisfied in the cosmology case.

Preliminaries

Confirmation**:** I will take confirmation to be measured by likelihoods, P(E|H). In particular, I will say that E favours H over -H iff P(E|H) > P(E|-H).

Objective vs Subjective Probability: All uses of 'probability' will refer to credences, or subjective probability.

Possible worlds vs Possible universes: Possible worlds will refer to ways the world could have been, in line with the normal use of the term. Some of these possible worlds contain many universes, some only one. Those that contain only one are Universes (with a capital U). Those that contain many universes are Multiverses.

The Fine-Tuning Argument

We appear to live in a universe that is fine-tuned for life. That is, if the values of the constants of the laws of nature were much different from what they are, life could not exist here[1]. This fact appears to require explanation. The fine-tuning argument says it would be explained if there were many actual universes, with different values of constants i.e. a Multiverse. The argument can be put formally as follows:

UV = There is only one universe

MV = There are many universes

E = Some universe has the right constants for life

= ∏x such that x is a universe and x has the right constants for life

n, m = Number of universes

N, M = The proposition that there are n, m universes

p = Probability that any given universe will NOT have the right constants for life.

Assuming the properties of the universes are independent, the probability that the right constants will exist somewhere given n universes is 1-(p^n). It follows that for any n>m, life is more likely to exist given n.

(1) $P(E \mid N) > P(E \mid M)$ iff $N > M$[2]

So for our specific hypotheses,

[1] The initial conditions are also crucial. For ease of exposition, I will take 'the right constants' to include the right initial conditions.

[2] This is false if $P(E) = 1$. I will ignore the problem of old evidence in this paper.

(2) P(E| MV) > P(E|UV)

The more universes there are, the more chances there are for a universe to have the right constants for life. This argument can be modeled with the following analogy.

Dice Analogy

You step into a room where a dice game is in progress. (Only one die is thrown at a time). You have to decide between two hypotheses:

One Throw: There have been few throws so far tonight (we will assume 1 throw).

Many Throws: There have been many throws so far tonight (2 throws)

Your initial degrees of belief in these hypotheses will depend on the time, the appearance of the players, and other such considerations. We are more concerned here, however, with how your degrees of belief *change* as you acquire certain items of information. Suppose you ask whether any sixes have been thrown so far tonight. You are told that there have been. This supports the hypothesis that there have been Many Throws. This shift is analogous to the shift to the Multiverse above, and is based on a similar likelihood inequality.

E_d = A six has been thrown tonight

= ∏x such that x is a throw and x is a 6

(2_d) P(E_d| Many Throws) > P(E_d|One Throw)

(To make the analogy explicit, each throw of the die corresponds to a universe. The throw coming up 6 corresponds to the universe having the right constants for life. We will individuate the throws as being either a 6 or Not.)

Inverse Gambler's Fallacy

Hacking then modifies the dice example to create a case where it would be a mistake to make the kind of inference shown above. I agree that Hacking has identified a fallacy. But I will argue that the case is not analogous to the situation we find ourselves in with regard to fine-tuning.

Hacking asks us to imagine the following reasoning. A gambler walking into the room is offered a bet on the number of past rolls that night.

'But slyly, he says, 'Can I wait and see how this roll comes out, before I lay my bet with you on the number of past plays made tonight?' The Kibitzer, no fool, agrees, although charging a slight fee for this 'information'. The roll is a double six. The gambler foolishly says 'Ha, that makes a difference - I think there have been quite a few rolls.'' p.333

This gambler is committing the inverse gambler's fallacy. The familiar gambler's fallacy is to think that a long run of trials make rare occurrences (that so far haven't happened) more likely. The new inverse gambler's fallacy is to think that a rare event is more likely to occur after a large number of trials than after a small number of trials. These are both fallacies, and for the same reason. Dice, and other gambling devices, have no memory. What happens on one throw is independent of any other throws there may be.

E_d' = This throw is a 6

= ∏x such that x is a throw and x is a 6 and x is this throw

(2_d') P(E_d'| Many Throws) = P(E_d'|One Throw)

What's happening here is that the general evidence that some throw was a 6 supports Many throws. But the specific evidence about which throw was a 6 undercuts this support. Someone who incorrectly gets a shift to Many Throws from the specific information is committing the inverse gambler's fallacy. Hacking argues that the proponent of the fine-tuning argument is making a similar mistake; he is arguing from the fact that *this* universe is fine-tuned for life to the conclusion that there are Many Universes. But the features of *this* universe are independent of how many other universes there might be, just like the dice case.

E' = This universe has the right constants for life

= ∏x such that x is a universe and x has the right constants for life and x is this universe

(2') P(E'| Many Universes) = P(E'|One Universe)

I agree that Hacking has identified a fallacy. But I aim to show that it does not apply to the proponent of fine-tuning arguments. I will show that it is not just the presence of specific evidence that undercuts the shift to MV - the nature of the specific evidence matters as well. The shift to MV is only undercut if the specific evidence satisfies condition (C). I will argue that the evidence in the fine-tuning case does not satisfy this condition. As such, no inverse gambler's fallacy is committed, and the argument for the Multiverse goes through.

(C) The universe that the specific evidence refers to has the same probability of existing in both possible worlds

(or: The throw that the specific evidence refers to has the same probability of existing in both possible worlds)

The relevance of this condition is non-obvious so I will demonstrate how it works with a cards example.

Cards

Suppose your friend is dealt cards from a deck consisting only of Aces and Kings. A fair coin is flipped to determine whether he will be dealt one card or two[3] . Table 1 shows the possible results.

Coin Result	Possible Outcomes	Probability
Heads (Few)	A	2/8
	K	2/8
Tails (Many)	AA	1/8
	AK	1/8
	KA	1/8
	KK	1/8

Table 1

We are concerned with whether there are two cards (Many) or one (Few).

[3] Assume for mathematical ease that they are dealt with replacement.

P(Many) = P(Few) = 1/2

We ask your friend if he has an Ace. He replies that he has.

E_a = He has been dealt at least one Ace

This supports the hypothesis that he has two cards rather than one.

P(Many|He has been dealt at least one ace) = 3/5

This shift is analogous to the shift to MV on discovering that there is a universe with the right constants. But what happens if we then discover that the first card is an Ace?

E_{fa}= The first card dealt was an ace

P(Many|The first card dealt was an ace) = 1/2

The previous shift to Many is undercut when we find that the Ace is on the first card. Why? The reason is that the fist card would have been dealt anyway, so it cannot help us discriminate Many from Few. It is the fact that the Ace could have been on the second card that gives us the original shift to Many. When the Ace turns out to be on the first card, the shift is undercut. We will discuss this in more detail later, but for now we can see that the same thing happens in the dice example.

Dice

One Throw: There have been few throws so far tonight (we will assume 1 throw).

Many Throws: There have been many throws so far tonight (2 throws).

Assume we assign each of these possible worlds a subjective probability of 0.5. You are then told the general evidence that a 6 has been thrown at some point.

E_g = ∏x such that x is a throw and x is a 6

Does this support Many Throws? Yes. You have learnt that a six has been thrown somewhere, and this is more likely given more throws. This is based on the general evidence that there is a six somewhere.

$P(E_g|\text{Many Throws}) > P(E_g|\text{One Throw})$

But now suppose you get a specific piece of evidence about which throw was the six (which throw you are looking at). Suppose the throws are numbered 1 and 2, you learn that you are looking at throw 1.

E_s = Throw 1 is a 6

In this case, Hacking and the others get exactly what they want. The shift to Many Throws gets undercut by the specific evidence about where the double six is.

$$P(E_S|\text{Many Throws}) = P(E_S|\text{One Throw})$$

The reason is that the double six occurred on a throw that had the same probability of existing (100%) in both possible worlds. This structure may not be obvious, so let's enumerate all the possibilities:

Possible World	First Roll	Second Roll	Probability out of 72
a	6	-	6
b	Not	-	30
c	6	Not	5
d	Not	6	5
e	Not	Not	25
f	6	6	1

Table 2

When we learn that a 6 has been thrown, we can rule out (b) and (e). The probability that there have been two rolls rises from 1/2 to 11/17. Many Throws is confirmed Now suppose we learn that the first roll was a 6. Does this specific piece of information undercut the shift to Many Throws we just got? Yes. We can now rule out (d) as well as (b) and (e). The probability falls from 11/17 to 1/2. We are back where we were before. This is a case where we would be committing an inverse gambler's fallacy if we took the evidence to favour Many Throws. The reason the original shift is undercut is that the throw we have found out about exists with the same probability in both possible worlds. There would always have been a first throw.

Contrast this with discovering that the roll the six is on is the second throw. The second throw is much more likely to exist in the Many Throws world than the One Throw world (in fact with probabilities of 1 and 0). In such a case, we would get conclusive evidence that Many Throws is true. So we would get a further shift for Many Throws - a first shift from the general evidence that there is a six somewhere, then a further shift from discovering that the six is on the second throw.

Notice that we have a symmetry here. Finding that the six was on the first throw favours One Throw, while discovering that the six was on the second throw favours Many Throws. This kind of symmetry is required when the pieces of evidence are mutually exclusive. Any possible evidence in favour of one hypothesis must be balanced by possible evidence *against* that hypothesis. Do we have that symmetry in the cosmology case? I will argue that we do not.

Before getting to cosmology, I want to point out that our current position might appear odd. The worry is that it no longer matters that the second throw was a 6. Finding out that a second throw *exists* gives us the maximal shift to Many Throws . We have been told that the second throw was a six, but this doesn't matter - we might as well have been told that the second throw was something between 1 and 5, or even between 1 and 6. It is the mere existence of the second throw that supports Many Throws. Something similar can be said for the first throw. When we are told that the first throw was a six, this is equivalent to being told that some throw has been randomly selected, and it is throw 1. To see why this is equivalent, notice that a 6 is equally likely to come up on either throw 1 or 2 (if they both exist). If both are sixes, one is chosen at random. So you are essentially being given information about a randomly selected throw. The randomly selected throw is more likely to be the first one given One Throw (probability of 1) than Many Throws (probability of 1/2). So, the worry is that it is no longer some feature of the throws that gives us the shift, but discovering that the throw exists.

But this is exactly as it should be. Nothing has gone wrong. This in fact is exactly *why* the general information that some throw was a 6 supported Many Throws in the fist place - it is because if there were many throws, there would have been throws that would not have *existed* if there were one throw. The 6 may have been thrown on one of these throws. And *this* is why we get support for Many Throws - the relevant evidence is that a 6 has been thrown, and it might have been thrown on one of the throws that would not have existed if there had been only one throw. This is why we lose the support if we learn that the 6 came on the first throw. The first throw exists in both possible worlds, so it can't help us decide between the possible worlds.

And this is why the Inverse Gambler's Fallacy is a fallacy in this case. The original shift to Many Throws occurred because the six might have been rolled on one of the throws that would not have existed if there were few throws. If we then find out that the 6 came up on one of the throws that would have existed either way, we no longer have support for Many Throws.

This leads us back to C:

> (C) The throw / universe that has the six / right constants must have the same probability of existing in both possible worlds.

If the throw or universe we find out about satisfies condition (C), then we do not get support for Many Throws or Universes. To think that we do would be to commit the Inverse Gambler's Fallacy. We now have to decide whether this holds in the cosmology case. If it does, Hacking is right, and we are left with no support for the Multiverse. I will argue that it does not.

(C) fails:

This universe does not have the same probability of existing in all possible worlds

I think that once the criterion is identified, it is clear that it is not satisfied in the cosmology case. Any possible universe is more likely to exist if there are many universes than if there is only one universe. And this is simply because it has more chances to exist if there are lots of universes. The model I am using here is that there is a plethora of possible universes. For every actual universe, one of these possible universes gets selected at random. The more actual universes there are, the greater the probability that a given possible universe will exist.

In order to avoid a shift to the Multiverse, this universe would have to have the same probability of existing in any possible world. Furthermore, as we are dealing with credence, we would have to know that that this universe had the same probability of existing in any possible world. One way we could know this is if our universe necessarily existed. Then it would have a probability of 1 of existing in all possible worlds. But we have no reason to think that our universe necessarily exists. That seems like chauvinism of the highest order.

Chauvinistic though it may be, this position cannot be dismissed automatically. We are dealing with subjective probability, so it is open for someone to assign a probability of 1 to this universe existing in any possible world. What can we say to an agent with such a subjective probability distribution?

One answer is to admit that the argument doesn't apply to him - it only applies to agents with certain distributions, of which he is not one. I think this dialectical move is perfectly acceptable, but it is unsatisfying, and should be treated as a last resort. A better response would be to argue that there are rational constraints on probability distributions. Notice that the agent we are discussing, who thinks this universe necessarily exists, assigns P(This universe exists | UV) = 1. While this may not violate any norms of coherence, it does seem like it is rationally flawed in a weaker sense. Suppose we challenge our agent to justify this assignment. He is free to reply, ‘Nothing. I have to have some assignment, and that is as good as any.’

My opinion is that is it not as good as any, because it makes the agent certain about something which he has no right to be certain of. Surely when we are highly uncertain, as we are concerning the probability of this universe existing in a one universe possible world, we shouldn’t assign a high probability to anything. A reasonable principle seems to be to respond to uncertainty with low probabilities. Our agent does the opposite. He responds to uncertainty with high probabilities, and this seems rationally unwarranted[4] .

(Unfashionable though it may be, I think that we could appeal to some Principle of Indifference in cases where the world divides itself up neatly into the different possibilities. This seems like such a case, but I won’t pursue this idea here.)

The usual strategy of proponents of the Inverse Gambler’s Fallacy is to name the universe. Call our universe Alpha. Does it help if we discover that universe Alpha is the one with the right constants? No, because it might be the case that Alpha only exists in the Multiverse. Which means we end up with the same shift to the Multiverse that we got from discovering that some universe has the right constants.

To drive this point home, recall the symmetry considerations discussed above. We start with a shift to the MV from the general evidence that some universe has the right constants for life. If then finding that one *particular* universe (i.e. the first) has the right constants gives a shift back to UV, then finding that some *other* universe (i.e. the second) has the right constants must support MV again. This shift to MV could only happen if we found we were in a universe that would only exist given MV (or at least was overwhelmingly likely to). But we have no reason to think this is the case. The same goes for the purported shift back to UV from the specific evidence that Hacking defends. We only get that shift if we found that we were in a universe that

[4] Our agent need not think that this universe is certain to exist in all possible worlds, only that it has the same probability of doing so. Nevertheless, the agent would have to assign P(E|UV) greater than seems warranted.

had the same probability of existing in both possible worlds. But we have no way of knowing this, even if it were true.

What we would need would be something like discovering our universe is the first in chronological order (where we would not get a shift to MV). Or that it was last (where we would get a maximal shift to MV). But we have no such information, and no way of getting it. In the absence of such information, we keep our shift to the Mulitverse.

Conclusion

The Inverse Gambler's Fallacy is only committed if the specific evidence refers to a trial that has the same probability of existing in any relevant possible world. Our universe is more likely to exist given that there is a Multiverse rather than a Universe. So this objection to the Fine-Tuning argument for the Multiverse does not work.

References

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