Comments on *Judgment Aggregation without Paradox*

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May 25-29 2005 - FEW 2005 - UTA

Main Aim

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    - Instability: there are different non-equivalent, but *prima facie* equally good, ways of applying PBP (Bovens and Rabinowicz (2004)). [Discussed in section 1]
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  - Connection: applying majority-voting on isolated premises blinds the procedure to the logical connections between them. [Discussed in section 2]
- One more common drawback of PBP & Fusion
  Manipulability: aggregated outcome can be manipulated by strategic voting. [Discussed in section 3]

An Example

The San Francisco Art School is looking for models (of any sex) for their painting classes. Jerry shows up for the auditions. A seven-member committee votes on the following:

- A: Jerry is attractive
- B: Jerry is poor in social skills
- C: Jerry is accepted

- Integrity Constraint: $((A \,\&\, B) \vee (\sim A \,\&\, \sim B)) \equiv C$

	A	B	C	$A \,\&\, B$	$\sim A \,\&\, \sim B$	$A \equiv B$
Voter 1	1	1	1	1	0	1
Voter 2	1	1	1	1	0	1
Voter 3	1	1	1	1	0	1
Voter 4	1	0	0	0	0	0
Voter 5	1	0	0	0	0	0
Voter 6	0	1	0	0	0	0
Voter 7	0	1	0	0	0	0
Majority	1	1	0	0	0	0

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- Example of *instability* of PBP: if we identify A and B as premises, PBP's outcome on C is 1.
- If we collect majority on the complex propositions $A \& B$ and $\sim A \& \sim B$ as premises, PBP's outcome on C is 0.

Applying the Fusion Procedure

- $Mod(\mathcal{K}_1) = Mod(\mathcal{K}_2) = Mod(\mathcal{K}_3) = \{(1, 1, 1)\}$
- $Mod(\mathcal{K}_4) = Mod(\mathcal{K}_5) = \{(1, 0, 0)\}$
- $Mod(\mathcal{K}_6) = Mod(\mathcal{K}_7) = \{(0, 1, 0)\}$
- $Mod(IC) = \{(1, 1, 1), (0, 0, 1), (1, 0, 0), (0, 1, 0)\}$

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A	B	C	$\mathcal{K}_1$	$\mathcal{K}_2$	$\mathcal{K}_3$	$\mathcal{K}_4$	$\mathcal{K}_5$	$\mathcal{K}_6$	$\mathcal{K}_7$	$d_\Sigma(I,E)$
1	1	1	0	0	0	2	2	2	2	**8**
1	0	0	2	2	2	0	0	2	2	10
0	1	0	2	2	2	2	2	0	0	10
0	0	1	2	2	2	2	2	2	2	14

A Language Variation

- Let us now move to a language that does not contain A, and B but where there are atomic sentences $D :\approx (A \,\&\, B)$ and $E :\approx ({\sim}A \,\&\, {\sim}B)$.

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- Same vote as before, but now the integrity constraints are expressed in terms of D, E, and C.
- IC: $(D \vee E) \equiv C$ and $D \rightarrow \sim\! E$.
  - Note the extra-IC: it takes care of the connection between $(A \,\&\, B)$ and $(\sim\! A \,\&\, \sim\! B)$

Reversal!

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D	E	C	$\mathcal{K}_1$	$\mathcal{K}_2$	$\mathcal{K}_3$	$\mathcal{K}_4$	$\mathcal{K}_5$	$\mathcal{K}_6$	$\mathcal{K}_7$	$d_\Sigma(I, E)$
1	0	1	0	0	0	2	2	2	2	8
0	1	1	2	2	2	2	2	2	2	14
0	0	0	2	2	2	0	0	0	0	**6**

Instability and Linguistic Variance Compared

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  - (Proper) instability of PBP: different ways of applying the same procedure yield different outcomes–holding fixed the identification of the atoms.
  - In the current case, we need to carry out the fusion procedure exactly in the same way, but look at two different languages. (*linguistic instability*).
- The common feature between linguistic instability and instability proper is that in both cases the aggregation procedure sees a difference where there should be none.

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- Let $\mathcal{L}_2$ have three sentences X, Y, Z, such that: $Z :\approx C$, $Y :\approx B \equiv C$, $X :\approx A \equiv B$.

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- Let $\mathcal{L}_2$ have three sentences X, Y, Z, such that: $Z :\approx C$, $Y :\approx B \equiv C$, $X :\approx A \equiv B$.
- E.g.: A vote on $(A \,\&\, B) \equiv C$ becomes a vote on $[(X \equiv (Z \equiv Y)) \,\&\, (Z \equiv Y)] \equiv Z$.

Conclusion on Instability

My conclusion in this section is that, to defend her approach, and at the same time be able to use *instability* as an objection to PBP, Gabriella owes both of the following:

1. An explanation of why *linguistic* instability is harmless for the Fusion procedure.

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1. An explanation of why *linguistic* instability is harmless for the Fusion procedure.
2. An account of why the explanation in (1) cannot be used by a supporter of PBP against the instability criticism.

Reasons and Connections

- Distinction: (i) collective judgments in which the outcome must be supported by collectively accepted reasons vs. (ii) judgments in which it is enough to reach some collective conclusion.

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- Standard view:
  - (i) is best dealt by procedures which, like PBP and fusion, give a complete outcome
  - (ii) is better dealt by the Conclusion Based Procedure (CPB).
- In the art-school case, we can imagine contexts in which the reasons for acceptance of C are important (e.g. if Jerry is accepted because attractive and unapproachable ($A \,\&\, B$) then he gets a worse contract than he would get if he were accepted because unattractive and easygoing ($\sim A \,\&\, \sim B$).

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- But here, the fusion procedure recommends the same outcome as PBP, even though only a minority supports $(A \& B)$ (and thus $(A \equiv B)$).
- Question: in what sense does the Fusion procedure do better when it comes to keeping track of logical connections between premises?

Independence on a Set of Propositions

- F. Dietrich and C. List, *Strategy-proof judgment aggregation*, forthcoming.

(Ind_Y) An aggregation function F is independent (on a set of propositions Y) iff for every proposition p ($\in Y$), and $n \in \omega$ there is a function $\phi : \{0,1\}^n \rightarrow \{0,1\}$ such that $p \in F(\mathcal{K}_1, ..., \mathcal{K}_m)$ iff $\phi(v_1(p), ..., v_n(p)) = 1$ where $v_i(p)$ is $\mathcal{K}_i$'s vote on p .

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- For each member p of Y the aggregated outcome on p is a function of the pattern of individual judgments on p.
- In our example, whenever $Y \supset \{C\}$, both *PBP* and *Fusion* are not Independent on Y.

	A	B	C		A	B	C
Voter 1	1	1	**1**		1	1	**1**
Voter 2	1	1	**1**		1	1	**1**
Voter 3	1	1	**1**		1	1	**1**
Voter 4	1	0	**0**		1	0	**0**
Voter 5	1	0	**0**		1	0	**0**
Voter 6	0	1	**0**		1	0	**0**
Voter 7	0	1	**0**		1	0	**0**
PBP + Fusion	1	1	1		1	0	0

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- Strategic voting on p is voting untruthfully on other propositions so as to change the collective outcome on p into the desired outcome.
- More formally, let F be an aggregation procedure.

(Man_Y) F is *manipulable* at $(\mathcal{K}_1, ..., \mathcal{K}_n)$ by individual i on p iff $\mathcal{K}_i$ disagrees with $F(\mathcal{K}_1, ..., \mathcal{K}_i, ..., \mathcal{K}_n)$ on p, but $\mathcal{K}_i$ agrees with $F(\mathcal{K}_1, ..., \mathcal{K}_i^*, ..., \mathcal{K}_n)$ on p for some alternative knowledge-base $\mathcal{K}_i^*$.

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- Hence both PBP and Fusion are manipulable on $\{C\}$. This seems bad.
- A possible defense (for both PBP and Fusion) is to say that the central cases for the application of these procedure are cases in which we need collective outcome + collective reasons.
- One could claim: in such cases strategic voting will not be attractive.

Response, Upshot

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- However, in cases like the one I described we only need collective reasons given certain collective outcomes. (We need collective reasons if Jerry is accepted, but not if Jerry is rejected).
- What is the supporter of Fusion to say in these cases? Does Fusion apply only when the need for collective reasons does not depend on any particular outcome on C?
- If so, how are we aggregate judgments in cases where the need for collective reasons *does* depend on a specific outcome?
- If not, in what sense is manipulability on (sets containing) the conclusion not a problem for the Fusion approach?