Comments on *Judgment Aggregation without Paradox*

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Main Aim

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  - Instability: there are different non-equivalent, but *prima facie* equally good, ways of applying PBP (Bovens and Rabinowicz (2004)). [Discussed in section 1]
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  - Connection: applying majority-voting on isolated premises blinds the procedure to the logical connections between them. [Discussed in section 2]
- One more common drawback of PBP & Fusion
  Manipulability: aggregated outcome can be manipulated by strategic voting. [Discussed in section 3]

An Example

The San Francisco Art School is looking for models (of any sex) for their painting classes. Jerry shows up for the auditions. A seven-member committee votes on the following:

- A: Jerry is attractive
- B: Jerry is poor in social skills
- C: Jerry is accepted

- Integrity Constraint: $((A \,\&\, B) \vee (\sim A \,\&\, \sim B)) \equiv C$

	A	B	C	$A \& B$	$\sim A \& \sim B$	$A \equiv B$
Voter 1	1	1	1	1	0	1
Voter 2	1	1	1	1	0	1
Voter 3	1	1	1	1	0	1
Voter 4	1	0	0	0	0	0
Voter 5	1	0	0	0	0	0
Voter 6	0	1	0	0	0	0
Voter 7	0	1	0	0	0	0
Majority	1	1	0	0	0	0

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- Example of *instability* of PBP: if we identify A and B as premises, PBP's outcome on C is 1.
- If we collect majority on the complex propositions $A \,\&\, B$ and $\sim A \,\&\, \sim B$ as premises, PBP's outcome on C is 0.

Applying the Fusion Procedure

- $Mod(\mathcal{K}_1) = Mod(\mathcal{K}_2) = Mod(\mathcal{K}_3) = \{(1,1,1)\}$
- $Mod(\mathcal{K}_4) = Mod(\mathcal{K}_5) = \{(1,0,0)\}$
- $Mod(\mathcal{K}_6) = Mod(\mathcal{K}_7) = \{(0,1,0)\}$
- $Mod(IC) = \{(1,1,1),(0,0,1),(1,0,0),(0,1,0)\}$

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A	B	C	$\mathcal{K}_1$	$\mathcal{K}_2$	$\mathcal{K}_3$	$\mathcal{K}_4$	$\mathcal{K}_5$	$\mathcal{K}_6$	$\mathcal{K}_7$	$d_\Sigma(I,E)$
1	1	1	0	0	0	2	2	2	2	**8**
1	0	0	2	2	2	0	0	2	2	10
0	1	0	2	2	2	2	2	0	0	10
0	0	1	2	2	2	2	2	2	2	14

A Language Variation

- Let us now move to a language that does not contain A, and B but where there are atomic sentences $D :\approx (A \,\&\, B)$ and $E :\approx ({\sim}A \,\&\, {\sim}B)$.

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- Same vote as before, but now the integrity constraints are expressed in terms of D, E, and C.
- IC: $(D \vee E) \equiv C$ and $D \rightarrow \sim E$.
    - Note the extra-IC: it takes care of the connection between $(A \,\&\, B)$ and $(\sim A \,\&\, \sim B)$

Reversal!

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D	E	C	$\mathcal{K}_1$	$\mathcal{K}_2$	$\mathcal{K}_3$	$\mathcal{K}_4$	$\mathcal{K}_5$	$\mathcal{K}_6$	$\mathcal{K}_7$	$d_\Sigma(I,E)$
1	0	1	0	0	0	2	2	2	2	8
0	1	1	2	2	2	2	2	2	2	14
0	0	0	2	2	2	0	0	0	0	**6**

Instability and Linguistic Variance Compared

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  - (Proper) instability of PBP: different ways of applying the same procedure yield different outcomes–holding fixed the identification of the atoms.
  - In the current case, we need to carry out the fusion procedure exactly in the same way, but look at two different languages. (*linguistic instability*).
- The common feature between linguistic instability and instability proper is that in both cases the aggregation procedure sees a difference where there should be none.

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- Let $\mathcal{L}_2$ have three sentences X, Y, Z, such that: $Z :\approx C$, $Y :\approx B \equiv C$, $X :\approx A \equiv B$.

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  - in virtue of the expressive equivalence, there is never a need for extra-integrity constraints.
- Let $\mathcal{L}_2$ have three sentences X, Y, Z, such that: $Z :\approx C$, $Y :\approx B \equiv C$, $X :\approx A \equiv B$.
- E.g.: A vote on $(A \,\&\, B) \equiv C$ becomes a vote on $[(X \equiv (Z \equiv Y)) \,\&\, (Z \equiv Y)] \equiv Z$.

Conclusion on Instability

My conclusion in this section is that, to defend her approach, and at the same time be able to use *instability* as an objection to PBP, Gabriella owes both of the following:

1. An explanation of why *linguistic* instability is harmless for the Fusion procedure.

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1. An explanation of why *linguistic* instability is harmless for the Fusion procedure.
2. An account of why the explanation in (1) cannot be used by a supporter of PBP against the instability criticism.

Reasons and Connections

- Distinction: (i) collective judgments in which the outcome must be supported by collectively accepted reasons vs. (ii) judgments in which it is enough to reach some collective conclusion.

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- Standard view:
  - (i) is best dealt by procedures which, like PBP and fusion, give a complete outcome
  - (ii) is better dealt by the Conclusion Based Procedure (CPB).
- In the art-school case, we can imagine contexts in which the reasons for acceptance of C are important (e.g. if Jerry is accepted because attractive and unapproachable ($A \,\&\, B$) then he gets a worse contract than he would get if he were accepted because unattractive and easygoing ($\sim A \,\&\, \sim B$).

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- But here, the fusion procedure recommends the same outcome as PBP, even though only a minority supports $(A \& B)$ (and thus $(A \equiv B)$).
- Question: in what sense does the Fusion procedure do better when it comes to keeping track of logical connections between premises?

Independence on a Set of Propositions

- F. Dietrich and C. List, *Strategy-proof judgment aggregation*, forthcoming.

(Ind_Y) An aggregation function F is independent (on a set of propositions Y) iff for every proposition p ($\in Y$), and $n \in \omega$ there is a function $\phi : \{0,1\}^n \rightarrow \{0,1\}$ such that $p \in F(\mathcal{K}_1, ..., \mathcal{K}_m)$ iff $\phi(v_1(p), ..., v_n(p)) = 1$ where $v_i(p)$ is $\mathcal{K}_i$'s vote on p .

- For each member p of Y the aggregated outcome on p is a function of the pattern of individual judgments on p.

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- For each member p of Y the aggregated outcome on p is a function of the pattern of individual judgments on p.
- In our example, whenever $Y \supset \{C\}$, both *PBP* and *Fusion* are not Independent on Y.

	A	B	C		A	B	C
Voter 1	1	1	**1**		1	1	**1**
Voter 2	1	1	**1**		1	1	**1**
Voter 3	1	1	**1**		1	1	**1**
Voter 4	1	0	**0**		1	0	**0**
Voter 5	1	0	**0**		1	0	**0**
Voter 6	0	1	**0**		1	0	**0**
Voter 7	0	1	**0**		1	0	**0**
PBP + Fusion	1	1	1		1	0	0

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- Strategic voting on p is voting untruthfully on other propositions so as to change the collective outcome on p into the desired outcome.
- More formally, let F be an aggregation procedure.

(Man_Y) F is *manipulable* at $(\mathcal{K}_1, ..., \mathcal{K}_n)$ by individual i on p iff $\mathcal{K}_i$ disagrees with $F(\mathcal{K}_1, ..., \mathcal{K}_i, ..., \mathcal{K}_n)$ on p, but $\mathcal{K}_i$ agrees with $F(\mathcal{K}_1, ..., \mathcal{K}_i^*, ..., \mathcal{K}_n)$ on p for some alternative knowledge-base $\mathcal{K}_i^*$.

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- Hence both PBP and Fusion are manipulable on $\{C\}$. This seems bad.
- A possible defense (for both PBP and Fusion) is to say that the central cases for the application of these procedure are cases in which we need collective outcome + collective reasons.
- One could claim: in such cases strategic voting will not be attractive.

Response, Upshot

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- What is the supporter of Fusion to say in these cases? Does Fusion apply only when the need for collective reasons does not depend on any particular outcome on C?

Response, Upshot

- However, in cases like the one I described we only need collective reasons given certain collective outcomes. (We need collective reasons if Jerry is accepted, but not if Jerry is rejected).
- What is the supporter of Fusion to say in these cases? Does Fusion apply only when the need for collective reasons does not depend on any particular outcome on C?
- If so, how are we aggregate judgments in cases where the need for collective reasons *does* depend on a specific outcome?
- If not, in what sense is manipulability on (sets containing) the conclusion not a problem for the Fusion approach?