What might be the case after a change in view

(An)Thony Gillies

Department of Philosophy
University of Michigan

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The Problem

Fuhrmann Triviality Result

Belief revision cannot be "preservative" for reflective agents

This is usually put **AGM-wise**:

- Epistemic states are **belief sets**—sets of sentences of our favorite language
- An agent in state K believes φ iff $\varphi \in K$
- Rationality constraints on revision are constraints on the K's

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My Soap-Box

I don't go that way.

1. Rationality constraints are only as good as the relations of "epistemic commitment"—consequence relations!—they are built on
2. So the consequence relations should be an explicit part of our modeling, not hidden in the background. It's prettier to do that model-theoretically.
3. When we do this for modals, it will be a **dynamic** consequence relation that I will push for

A Not-Very-Diplomatic Subtitle

What formal epistemology can learn from formal semantics

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Outline

Preservation

The Conservative's Credo

Information is not gratuitous! Belief change should minimize information loss

We are dealing here with coarse-grained qualitative models of belief change, so this is naturally codified as

Preservation

If you don't already believe $\neg\varphi$ in a prior state, then revising that state with φ should land you in a posterior state that is stronger—carries more commitments—than the prior state

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Two truisms about *might*

You have two marbles (red, yellow) and a box. You put one of the marbles in the box without showing me which one. Then I ought to believe

(1) The yellow marble might (in view of what else I believe) be in the box.

Conversely: if I believe something like (1), then I **ought not** believe the yellow marble **isn't** in the box.

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Well, they're truisms if we assume . . .

1. This *might* is **epistemic** and **solpisistic**
   - Intuitively: a consistency check on I believe
2. That 'belief'-talk is suitably permissive
   - Maybe things like *might p* aren't truth-bearing, and so maybe strictly speaking **belief** isn't quite the attitude we have toward them

In Other Words: *might* is a reflective modal

- My epistemic state commits me to *might p* iff it doesn't commit me to $\neg p$.
- Dually for *must p*.

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Preliminaries

- A revision model specifies 4 things: what states are; what the language of beliefs is, and what the language of inputs is; a revision function from states × inputs to states; and a "commitment" relation between states and beliefs
- Fix a language L^+—the smallest that contains CPL and is such that:
  - For all $\varphi \in$ CPL: $\Diamond\varphi, \neg\Diamond\varphi \in L^+$
  - Introduce $\Box$ as the dual to $\Diamond$
- We will assume that **beliefs** are (expressed by) formulas in L^+, but **inputs** by formulas in CPL

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Basic AGM Revision

- States: $K, K', \cdots \in \mathbf{K}$
- (More about **K** in a minute)

Two Constraints Not Up For Grabs

S_{AGM} $\varphi \in K \star \varphi$

C_{AGM} If $\neg\varphi \notin \mathrm{Cn}(\emptyset)$ then $K \star \varphi$ is consistent

A belief set K is **consistent** (w.r.t. L^+) iff for no $\varphi \in L^+$ is it the case that $\varphi, \neg\varphi \in K$

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Our Two Players

AGM Preservation

$\mathsf{P}_{\mathsf{AGM}}$ If $\neg\varphi \notin K$, then $K \subseteq K \star \varphi$

A model is **basic** iff it satisfies the two non-negotiable constraints plus $\mathsf{P}_{\mathsf{AGM}}$

Take a $\varphi \in$ CPL and belief set K. Poss(K) is the smallest set s.t.

- if $\varphi \in K$, then $\Box\varphi \in K$
- if $\varphi \notin K$, then $\Diamond\varphi \in K$

Closure Under Poss

All belief sets $K \in \mathbf{K}$ are closed under Poss—i.e., Poss(K) $\subseteq K$

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The Fuhrmann Result

An (AGM-wise) model is **non-trivial** iff:
For Some $\varphi \in$ CPL and K: $\varphi \notin K$ & $\neg\varphi \notin K$

Proposition (Fuhrmann, Levi)

If a model $\langle \mathsf{K}, \star \rangle$ is basic, it is trivial

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Revision Models

Worlds, states Fix a set W of worlds. States $s, s', \ldots$ are subsets of W. I is the set of such s's.

Revision function $\circ : I \times \text{CPL} \rightarrow I$

Consequence relation $\models \subseteq I \times L^+$

Revision model $M = \langle I, \circ, \models \rangle$

We'll say that state s is **consistent** (w.r.t. a choice for $\models$) iff for no $\varphi \in L^+$ is it the case that $s \models \varphi$ and $s \models \neg\varphi$. We'll write it this way: $s \neq \bot$

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C If $[\![\neg\varphi]\!] \neq W$ then $s \circ \varphi \neq \bot$

Preservation

P If $s \not\models \neg\varphi$, then $\{\psi : s \models \psi\} \subseteq \{\psi : s \circ \varphi \models \psi\}$

A model $M = \langle I, \circ, \models \rangle$ is basic iff it satisfies S, C, and P

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Basically Reflective Consequence

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$\models \subseteq I \times L^+$ is basically reflective iff:

- $s \models \varphi$ iff $s \subseteq [\![\varphi]\!]$, for $\varphi \in$ CPL
- if $s \models \varphi$, then $s \models \Box\varphi$
- if $s \not\models \neg\varphi$, then $s \models \Diamond\varphi$
- (truth-functionally equivalent subformulas can be swapped inside the scope of the modals)

- s is **consistent** w.r.t. $\models$ iff for no $\varphi \in L^+$ is it the case that $s \models \varphi$ and $s \models \neg\varphi$—i.e., not both are in $\{\psi : s \models \psi\}$
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A Contrived Example

Fix a state s, and form the autoepistemic closure (in L^+) of it. Then you've got yourself a basically reflective consequence relation.

Let K_s be the smallest set s.t.

- $\varphi \in K_s$ iff $s \subseteq [\![\varphi]\!]$ (for $\varphi \in$ CPL)
- if $\varphi \in K_s$, then $\Box\varphi \in K_s$;
- if $\neg\varphi \notin K_s$, then $\Diamond\varphi \in K_s$;
- if $[\![\alpha]\!] = [\![\beta]\!]$, then $\psi \in K_s$ iff $\psi[\alpha/\beta] \in K_s$.

Then define:

- $s \models^+ \varphi$ iff $\varphi \in K_s$

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Basic Commitment Is Well-Behaved

Observation

Where $\models$ is a basically reflective relation, and s, s' any states:

- If $s \neq \bot$, then $s \models \Box\varphi$ iff $s \models \varphi$
- For any $\varphi \in \text{CPL}$, either $s \models \Diamond\varphi$ or $s \models \neg\Diamond\varphi$
- If $\{\varphi \in \text{CPL} : s \models \varphi\} = \{\varphi \in \text{CPL} : s' \models \varphi\}$, then $\{\varphi \in L^+ : s \models \varphi\} = \{\varphi \in L^+ : s' \models \varphi\}$ (if s, s' are consistent)

The Fuhrmann Result, Again

- **M** is the class of revision models with a basically reflective consequence relation
- $\langle I, \circ, \models \rangle$ is **non-trivial** iff there $s \in I$, φ such that $s \not\models \varphi$ and $s \not\models \neg\varphi$

Proposition

If $M \in$ **M** is basic, it is trivial.

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Give Up On Reflective Agents

- Things like $\Diamond\varphi$ don't express propositions of the normal sort, and so aren't really the kinds of things that can be the object of belief
- And so they don't really enter into our constraints on revision models at all

Give Up On Preservation

- Suppose our revision operator is governed by the Levi Identity—revising by φ decomposes into a contraction/downdate/weakening w.r.t. $\neg\varphi$ followed by an expansion/update w.r.t. φ
- Suppose contraction/downdate/weakening **idles** on non-belief (Easy Contraction)
- These entail Preservation
- So we have to get rid of either the Levi Identity or the vacuity constraint on contraction/downdate/weakening

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Levi Identity And Easy Downdates

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downdate: $s \subseteq s \downarrow \varphi$

M satisfies the Levi Identity (LI) iff: $s \circ \varphi = (s \downarrow \neg\varphi) \cap [\![\varphi]\!]$

Easy Weakening

EW If $s \not\models \varphi$, then $s \downarrow \varphi = s$

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LI + EW Do Not Entail P

LI + EW amounts to

Easy Revision

ER if $s \not\models \neg\varphi$ then $s \circ \varphi = s \cap [\![\varphi]\!]$

- Assume LI and EW for a model $M = \langle I, \circ, \models^{+} \rangle$
  - $\models^{+}$ is our contrived example of a basically reflective consequence relation
- Suppose $s = \{w_1, w_2\}$, where $w_1(p) = 1$ and $w_2(p) = 0$
  - So $s \not\models^{+} \neg p$
- Consider $s \circ p$. By LI + EW (= ER) $s \circ p = s \cap [\![p]\!] = \{w_1\}$
- But then $s \models^{+} \Diamond\neg p$ and $s \circ p \not\models^{+} \Diamond\neg p$, violating P

LI + EW Do Not Entail P

LI + EW amounts to

Easy Revision

ER if $s \not\models \neg\varphi$ then $s \circ \varphi = s \cap [\![\varphi]\!]$

- Assume LI and EW for a model $M = \langle I, \circ, \models^{+} \rangle$
  - $\models^{+}$ is our contrived example of a basically reflective consequence relation
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So What's The Deal?

To get the entailment to go through, we also need to assume something about the consequence relation

Persistence

$\models \subseteq I \times L$ is **persistent** iff for all $\varphi \in L$:
if $s \models \varphi$ and $s' \subseteq s$, then $s' \models \varphi$

Observation

Consider a model $M = \langle I, \circ, \models \rangle$. If $\models$ is persistent, then if M satisfies ER it satisfies P

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Persistence = Bad

In the context of these modals persistence is just a **bad idea**

if $w_1(p) = 1$ and $w_2(p) = 0$

- $\{w_1, w_2\}$ should have $\Diamond\neg p$ as a consequence
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Consequence From Updates

- Suppose $\varphi \in$ CPL is true/supported/etc. w.r.t. s
- That means that s is a fixed-point of updating s with the information that φ carries
  - $s \models_{CL} \varphi$ iff $s \cap [\![\varphi]\!] = s$
- To extend this picture to the modal fragment, we have to either generalize the updating function or $[\![\cdot]\!]$
- Let's do the former

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Updating

Let $L^\Diamond$ be the smallest set including CPL closed under $\neg, \wedge, \Diamond$

Updates

1. $s \uparrow p = \{w \in s : w(p) = 1\}$
2. $s \uparrow \neg\varphi = s \setminus (s \uparrow \varphi)$
3. $s \uparrow (\varphi \wedge \psi) = (s \uparrow \varphi) \uparrow \psi$
4. $s \uparrow \Diamond\varphi = \{w \in s : s \uparrow \varphi \neq \emptyset\}$

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Three Fun Facts

Observation

It is not in general the case that $s \uparrow \varphi = \bigcup_{w \in s} \{w\} \uparrow \varphi$

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Off The Shelf Revision

- Persistence is the problem
- So we've got ourselves a non-persistent consequence relation
- Take any old "broadly conditional" revision model off the shelf
- Swap out the consequence relation in it, and put $\Vdash$ in its place
- That's it: the resulting (non-trivial) model will satisfy S, C, and ER—and we have an easy prediction for why it will not satisfy P

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