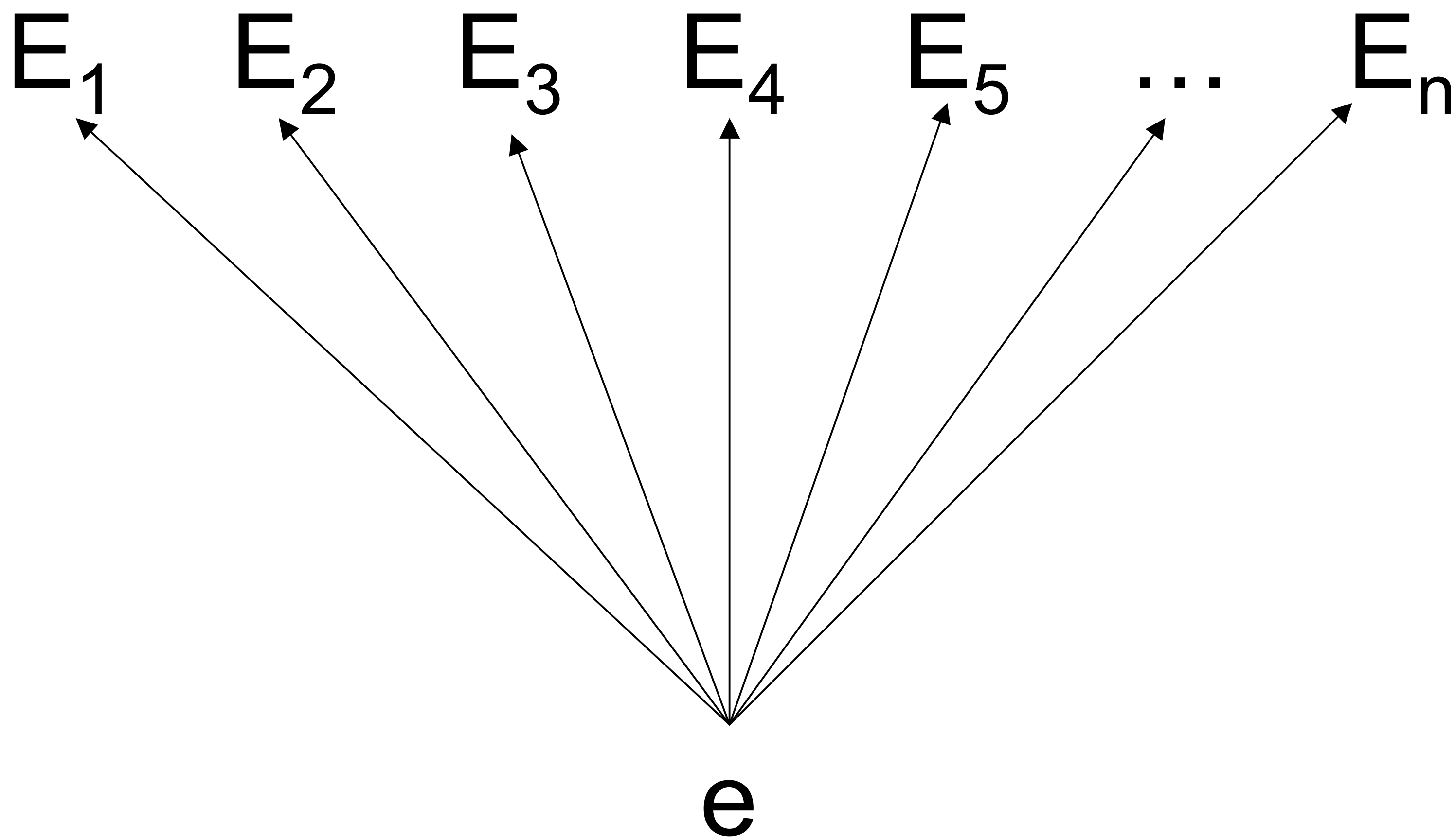
Another Representation Of Jeffrey Updating and the Uniformity Rule: Comments on Wagner on Commuting Probability Revisions

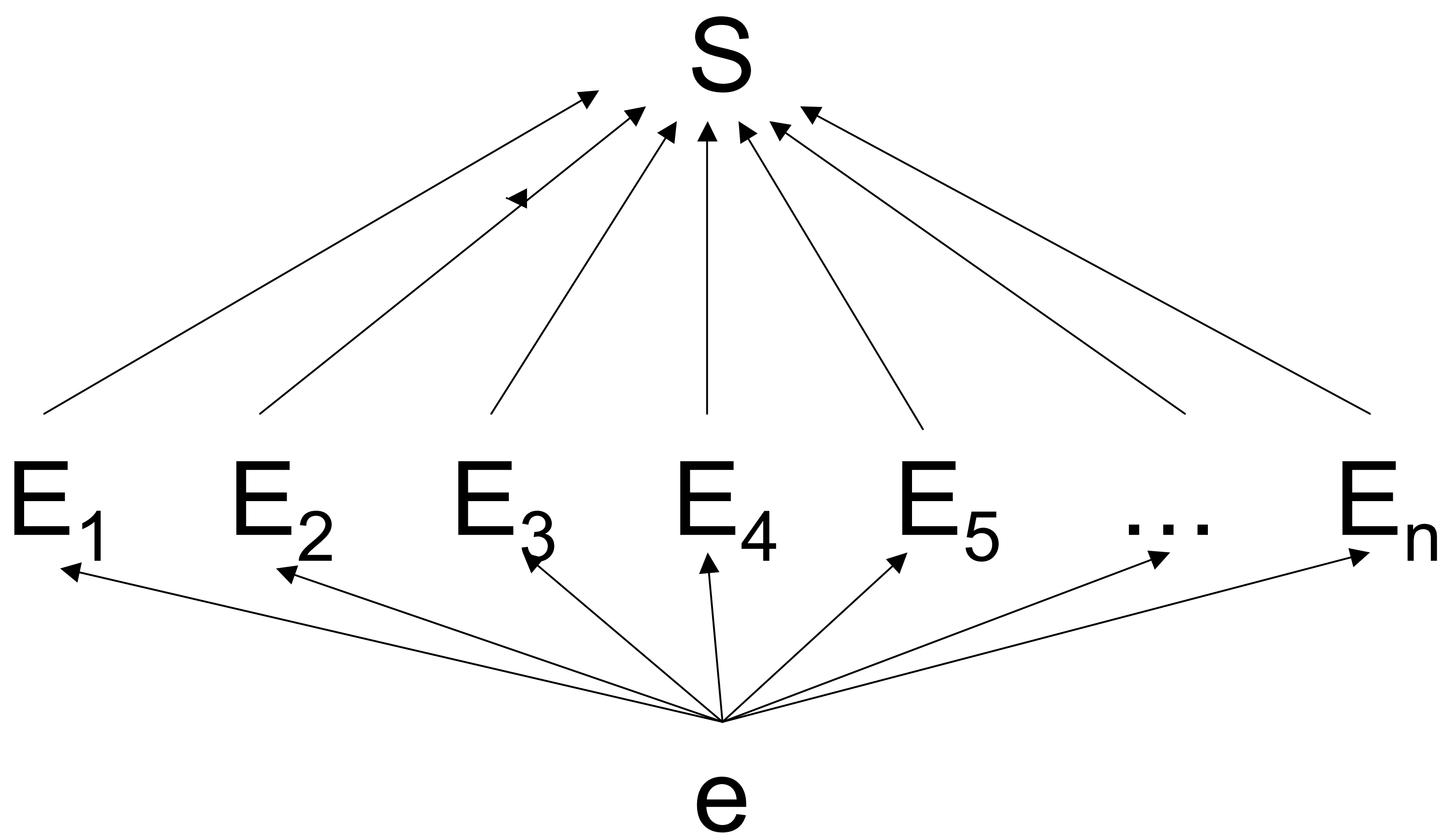
James Hawthorne

Univ. Of Oklahoma

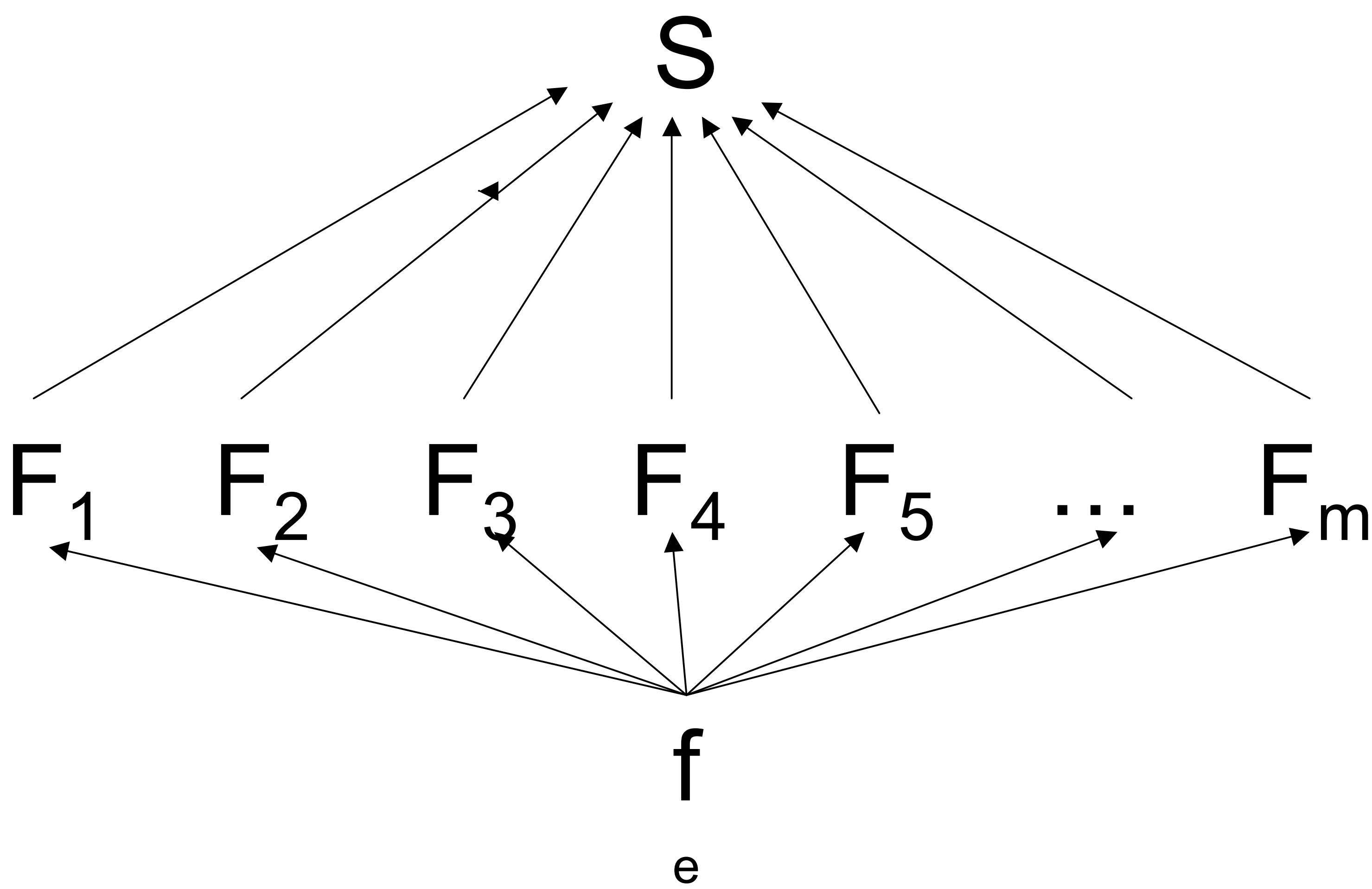
Nondoxastic State Directly Influences Doxastic State of Evidence Base



Doxastic State of Evidence Basis Influences Belief-Strengths for All Other Propositions



Doxastic State of Evidence Basis Influences Belief-Strengths for All Other Propositions



How does/should e influence the Evidence Basis ???

1. $e \rightarrow P_e[E_i]$ $P \rightarrow P_e$

---

2. $e \rightarrow \pi_P{}^e[E_i]$ $\pi_P{}^e : P \rightarrow P_e$

$$P_e[E_i] = \pi_P{}^e[E_i] \cdot P[E_i]$$

---

3. $e \rightarrow \beta_P{}^e[E_i : E_j]$ $\beta_P{}^e: (P[E_i]/P[E_j]) \rightarrow (P_e[E_i]/P_e[E_j])$

$$(P_e[E_i]/P_e[E_j]) = \beta_P{}^e[E_i : E_j] \cdot (P[E_i]/P[E_j])$$

If e were propositionally expressible

2. $e \rightarrow \pi_P{}^e[E_i]$ $\quad P[E_i \mid e] = P_e[E_i] = \pi_P{}^e[E_i] \cdot P[E_i]$

so we would have $\pi_P{}^e[E_i] = P[e \mid E_i] / P[e]$

---

3. $e \rightarrow \beta_P{}^e[E_i : E_j]$ $\quad P[E_i \mid e]/P[E_j \mid e] = (P_e[E_i]/P_e[E_j])$

$$= \beta_P{}^e[E_i : E_j]\cdot(P[E_i]/P[E_j])$$

so we would have

$$\beta_P{}^e[E_i : E_j] = P[e \mid E_i] / P[e \mid E_j]$$

Rigidity Condition: given evidence basis $\{E_i\}$,

$$P_e[S \mid E_i] = P[S \mid E_i]$$

If e were propositional, we'd write it as follows:

$$P[S \mid E_i \cdot e] = P[S \mid E_i]$$

Basic Jeffrey Updating

$$P_e[S] = \textstyle\sum_i P_e[S \mid E_i] \cdot P_e[E_i]$$

$$= \textstyle\sum_i P[S \mid E_i] \cdot P_e[E_i]$$

$$= \textstyle\sum_{\{i\,:\,P[E_i]>0\}} P[S \cdot E_i] \cdot (P_e[E_i]/P[E_i])$$

$$= \textstyle\sum_{\{i\,:\,P[E_i]>0\}} P[S \cdot E_i] \cdot \pi_P{}^e[E_i]$$

Basic Sequential Jeffrey Updating

$$P_{ef}[S] = \sum_j P_e[S \mid F_j] \cdot P_{ef}[F_j]$$

$$= \sum_{\{j:\, P_e[F_j]>0\}} P_e[S \cdot F_j] \cdot (P_{ef}[F_j]/P_e[F_j])$$

$$= \sum_{\{j:\, P_e[F_j]>0\}} \sum_{\{i:\, P[E_i]>0\}} P[S \cdot F_j \cdot E_i] \cdot (P_e[E_i]/P[E_i]) \cdot (P_{ef}[F_j]/P_e[F_j])$$

$$= \sum_{\{j:\, P_e[F_j]>0\}} \sum_{\{i:\, P[E_i]>0\}} P[S \cdot F_j \cdot E_i] \cdot \pi_P{}^e[E_i] \cdot \pi_{P_e}{}^{ef}[F_j]$$

Basic Jeffrey Updating – Order Effect

$$P_{ef}[S] = \sum_{\{j:\ P_e[F_j]>0\}} \sum_{\{i:\ P[E_i]>0\}} P[S \cdot F_j \cdot E_i]$$

-

$$(P_e[E_i]/P[E_i]) \cdot (P_{ef}[F_j]/P_e[F_j])$$

$$P_{fe}[S] = \sum_{\{j:\ P[F_j]>0\}} \sum_{\{i:\ P_f[Ei]>0\}} P[S \cdot F_j \cdot E_i]$$

-

$$(P_{fe}[E_i]/P_f[E_i]) \cdot (P_f[F_j]/P[F_j])$$

Basic Jeffrey Updating – Order Effect

$$P_{ef}[S] = \sum_{\{j:\ P_e[F_j]>0\}} \sum_{\{i:\ P[E_i]>0\}} P[S \cdot F_j \cdot E_i]$$

•

$$\pi_P{}^{e}[E_i] \cdot \pi_{Pe}{}^{ef}[F_j]$$

$$P_{fe}[S] = \sum_{\{j:\ P[F_j]>0\}} \sum_{\{i:\ P_f[Ei]>0\}} P[S \cdot F_j \cdot E_i]$$

•

$$\pi_{Pf}{}^{fe}[E_i] \cdot \pi_P{}^{f}[F_j]$$

Basic Jeffrey Updating for a Long Sequence

$$P_{e\ldots fg}[S]$$

$$= \sum_{\{k:\, P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i:\, P[E_i]>0\}} P[S\cdot G_k\cdot\ldots\cdot E_i]\cdot$$

$$(P_e[E_i]/P[E_i])\cdot\ldots\cdot(P_{e\ldots fg}[G_k]/P_{e\ldots f}[G_k])$$

$$= \sum_{\{k:\, P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i:\, P[E_i]>0\}} P[S\cdot G_k\cdot\ldots\cdot E_i]\cdot$$

$$\pi_P{}^{e}[E_i] \cdot\ldots\cdot \pi_{Pe\ldots f}{}^{e\ldots fg}[G_k]$$

How does/should e influence the Evidence Basis ???

1. $e \rightarrow P_e[E_i]$ $P \rightarrow P_e$: P_e is ORDER-DEPENDENT

----

2. $e \rightarrow \pi_P{}^e[E_i]$ $\pi_P{}^e : P \rightarrow P_e$: $\pi_P{}^e$ is P-value-DEPENDENT

$P_e[E_i] = \pi_P{}^e[E_i] \cdot P[E_i]$ i.e., Fails Modularity

$\pi_P{}^e[E_i] = P[e \mid E_i] / P[e]$

----

3. $e \rightarrow \beta_P{}^e[E_i]$ What about $\beta_P{}^e[E_i]$ as an Update Factor

$(P_e[E_i]/P_e[E_j]) = \beta_P{}^e[E_i : E_j] \cdot (P[E_i]/P[E_j])$

$\beta_P{}^e[E_i : E_j] = P[e \mid E_i] / P[e \mid E_j]$

Basic Jeffrey Updating

Notice that

$$1 = P_{e\ldots fg}[\text{tautology}]$$

$$= \sum_{\{k:\ P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i:\ P[E_i]>0\}} P[G_k\cdot\ldots\cdot E_i]\cdot$$

$$(P_e[E_i]/P[E_i])\cdot\ldots\cdot(P_{e\ldots fg}[G_k]/P_{e\ldots f}[G_k])$$

(useful for the denominator of the next equation)

Basic Jeffrey Updating

$$P_{e\ldots g}[S] =$$

$$\frac{\sum_{\{k:P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i\,:\,P[E_i]>0\}} P[S\cdot G_k\cdot\ldots\cdot E_i]\cdot \frac{(P_e[E_i]/P[E_i])}{(P_e[E_1]/P[E_1])}\cdot\ldots\cdot\frac{(P_{e\ldots fg}[G_k]/P_{e\ldots f}[G_k])}{(P_{e\ldots fg}[G_1]/P_{e\ldots f}[G_1])}}{\sum_{\{k:P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i\,:\,P[E_i]>0\}} P[G_k\cdot\ldots\cdot E_i]\cdot \frac{(P_e[E_i]/P[E_i])}{(P_e[E_1]/P[E_1])}\cdot\ldots\cdot\frac{(P_{e\ldots fg}[G_k]/P_{e\ldots f}[G_k])}{(P_{e\ldots fg}[G_1]/P_{e\ldots f}[G_1])}}$$

Basic Jeffrey Updating with Bayes-Factors

$$P_{e\ldots g}[S] =$$

$$\frac{\sum_{\{k:P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i\,:\,P[E_i]>0\}} P[S\cdot G_k\cdot \ldots \cdot E_i] \cdot \beta_P{}^{e}[E_i : E_1] \cdot\ldots\cdot \beta_{Pe\ldots f}{}^{e\ldots fg}[G_k : G_1]}{\sum_{\{k:P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i\,:\,P[E_i]>0\}} P[G_k\cdot \ldots \cdot E_i] \cdot \beta_P{}^{e}[E_i : E_1] \cdot\ldots\cdot \beta_{Pe\ldots f}{}^{e\ldots fg}[G_k : G_1]}$$

Extended Jeffrey Updating
General Update Factors

Uniformity Rule:

$$\beta_{Pa...ef}{}^{a...efg}[G_k:G_1] = \beta_{Pa...e}{}^{a...eg}[G_k:G_1]$$

whenever G_k is in the basis of **g** but **not** in the basis of **f**

Uniformity = Extended Rigidity

If f and g were propositional, the usual Jeffrey account of Rigidity would give $P[f \mid G_k \cdot g] = P[f \mid G_k]$.

From this the Uniformity Rule would be derivable:

$$\beta_{Pf}^{fg}[G_k:G_1] = (P_{fg}[G_k]/P_{fg}[G_1]) / (P_f[G_k]/P_f[G_1])$$

$$= (P[G_k \mid f \cdot g]/P[G_1 \mid f \cdot g]) / (P[G_k \mid f]/P[G_1 \mid f])$$

$$= (P[f \cdot g \mid G_k]/P[f \cdot g \mid G_1]) / (P[f \mid G_k]/P[f \mid G_1])$$

$$= \frac{(P[f \mid G_k \cdot g] \cdot P[g \mid G_k]) / (P[f \mid G_1 \cdot g] \cdot P[g \mid G_1])}{(P[f \mid G_k] / P[f \mid G_1])}$$

$$= P[g \mid G_k] / P[g \mid G_1] = (P[G_k \mid g]/P[G_1 \mid g]) / (P[G_k]/P[G_1])$$

$$= \beta_P^{g}[G_k:G_1]$$

Jeffrey Updating with Bayes-Factors that satisfy the Uniformity Rule

$$P_{e\ldots g}[S] =$$

$$\frac{\sum_{\{k:P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i\,:\,P[E_i]>0\}} P[S\cdot G_k\cdot \ldots \cdot E_i] \cdot \beta_P{}^{e}[E_i : E_1] \cdot\ldots\cdot \beta_P{}^{g}[G_k : G_1]}{\sum_{\{k:P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i\,:\,P[E_i]>0\}} P[G_k\cdot \ldots \cdot E_i] \cdot \beta_P{}^{e}[E_i : E_1] \cdot\ldots\cdot \beta_P{}^{g}[G_k : G_1]}$$

Extended Jeffrey Updating
General Update Factors

Suppose we apply the Uniformity Rule even when g and f affect the same basis G_k:

$$\beta_P{}^f[G_k:G_1] \cdot \beta_{Pf}{}^{fg}[G_k:G_1] =$$

$$\beta_P{}^f[G_k:G_1] \cdot \beta_P{}^g[G_k:G_1]$$

which on iteration blows up for $\beta_P{}^f[G_k:G_1]$, etc. larger than 1 and goes to 0 when smaller than 1.

This is Garbers Problem

for the Uniformity Rule

Jeffrey Updating with Bayes-Factors that satisfy the Uniformity Rule on a Common Evidence Basis – Garber's Problem

$$P_{e\ldots g}[S] =$$

$$\frac{\sum_{\{k:P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i\,:\,P[E_i]>0\}} P[S\cdot G_k\cdot\ldots\cdot E_i] \cdot \beta_P{}^e[E_i : E_j] \cdot\ldots\cdot \beta_P{}^f[G_k : G_j] \cdot \beta_P{}^g[G_k : G_j]}{\sum_{\{k:P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i\,:\,P[E_i]>0\}} P[G_k\cdot\ldots\cdot E_i] \cdot \beta_P{}^e[E_i : E_j] \cdot\ldots\cdot \beta_P{}^f[G_k : G_j] \cdot \beta_P{}^g[G_k : G_j]}$$

Extended Jeffrey Updating
General Update Factors

Notice: if g and f share an evidence basis and G_k is in it, even without the Uniformity Rule we have

$$\beta_{Pa...e}^{a...ef}[G_k:G_1] \cdot \beta_{Pa...ef}^{a...efg}[G_k:G_1] =$$

$$\frac{(P_{a\ldots ef}[G_k]/P_{a\ldots ef}[G_1]}{(P_{a\ldots e}[G_1]/P_{a...e}[G_1])} \cdot \frac{(P_{a\ldots efg}[G_k]/P_{a\ldots efg}[G_1]}{(P_{a\ldots ef}[G_1]/P_{a...ef}[G_1])}$$

$$= \beta_{Pa...e}^{e(gf)}[G_k:G_1]$$

So even without the Uniformity Rule, update factors that share the same evidence basis “accumulate”

Jeffrey Updating with Bayes-Factors that satisfy the Uniformity Rule for Distinct Evidence Bases. and with Cumulative Update Factors (ε, ..., γ) for Shared Evidence Bases

$$P_{e\ldots g}[S] =$$

$$\frac{\sum_{\{k:P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i\,:\,P[E_i]>0\}} P[S\cdot G_k\cdot\ldots\cdot E_i] \cdot \beta_P{}^{\varepsilon}[E_i : E_1] \cdot\ldots\cdot \beta_P{}^{\gamma}[G_k : G_1]}{\sum_{\{k:P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i\,:\,P[E_i]>0\}} P[G_k\cdot\ldots\cdot E_i] \cdot \beta_P{}^{\varepsilon}[E_i : E_1] \cdot\ldots\cdot \beta_P{}^{\gamma}[G_k : G_1]}$$

Bayes' Theorem for Jeffrey Updating and the Uniformity Rule

$$P_{e\ldots g}[H] = P[H] \cdot \frac{\sum_{\{k:P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i\,:\,P[E_i]>0\}} P[G_k\cdot\ldots\cdot E_i \mid H] \cdot \beta_P{}^{\varepsilon}[E_i : E_1] \cdot\ldots\cdot \beta_P{}^{\gamma}[G_k : G_1]}{\sum_{\{k:P_{e\ldots g}[G_k]>0\}} \ldots \sum_{\{i\,:\,P[E_i]>0\}} P[G_k\cdot\ldots\cdot E_i] \cdot \beta_P{}^{\varepsilon}[E_i : E_1] \cdot\ldots\cdot \beta_P{}^{\gamma}[G_k : G_1]}$$

END