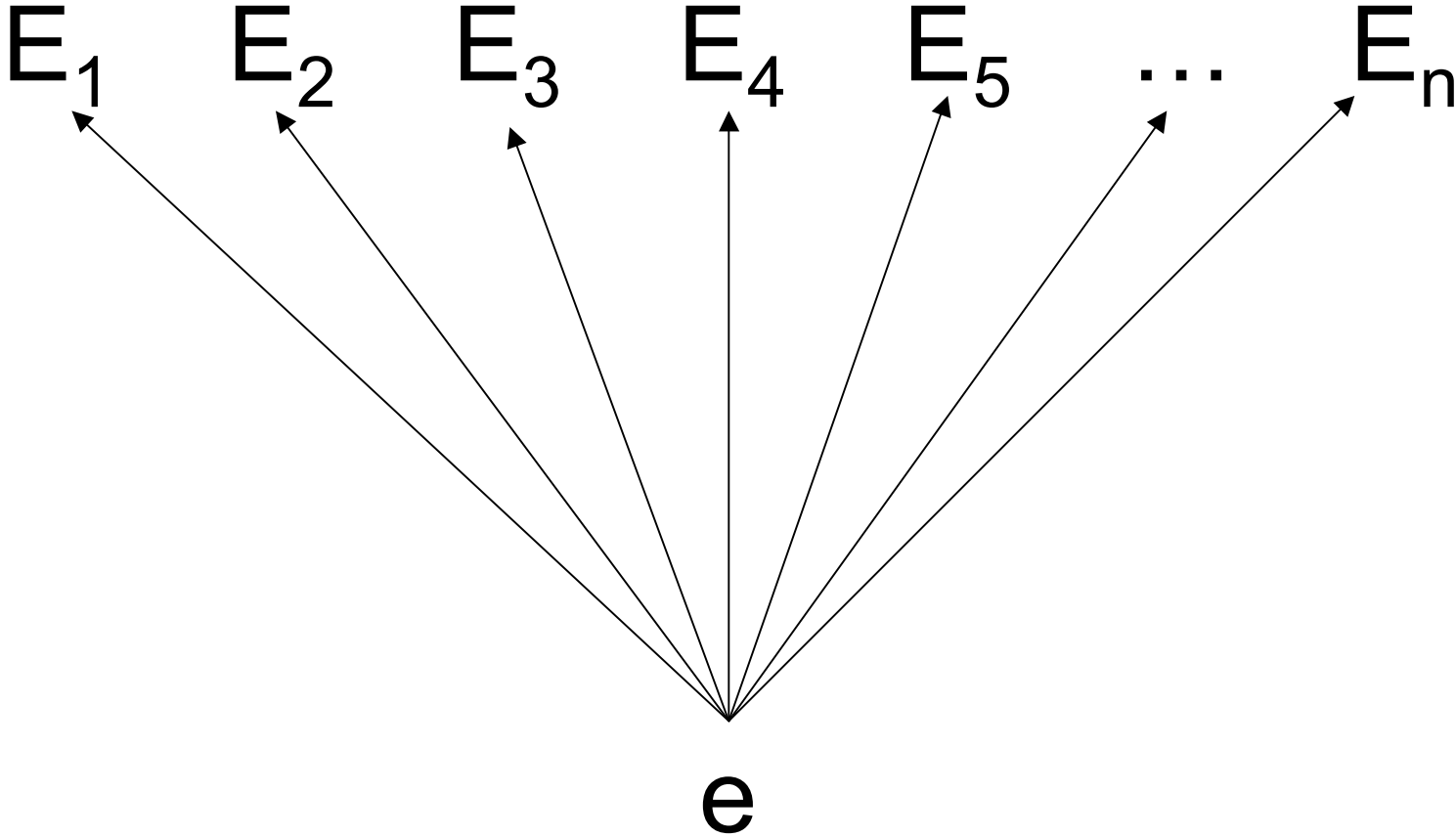


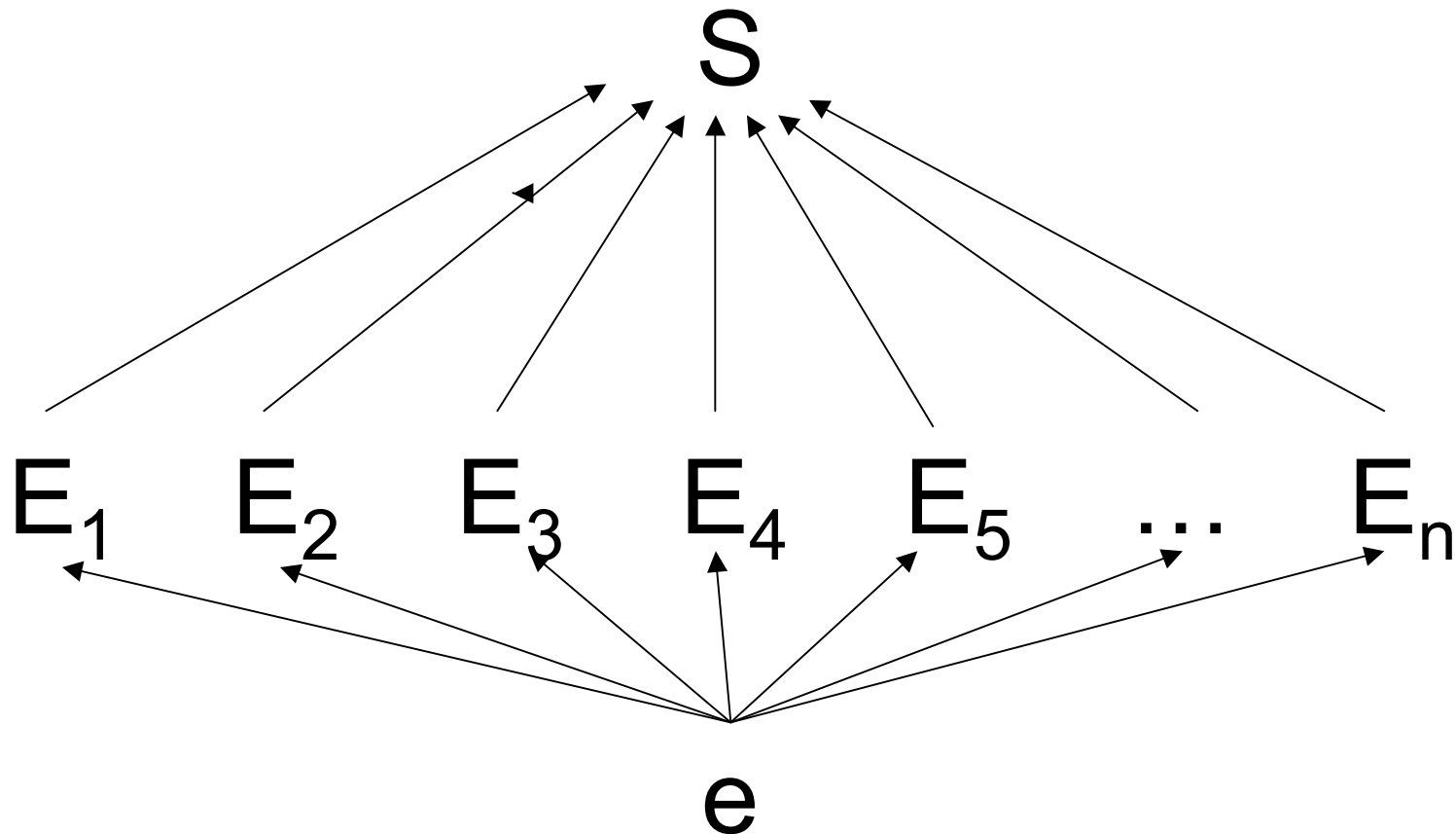
Another Representation Of
Jeffrey Updating and the
Uniformity Rule:
Comments on Wagner on
Commuting Probability Revisions

James Hawthorne
Univ. Of Oklahoma

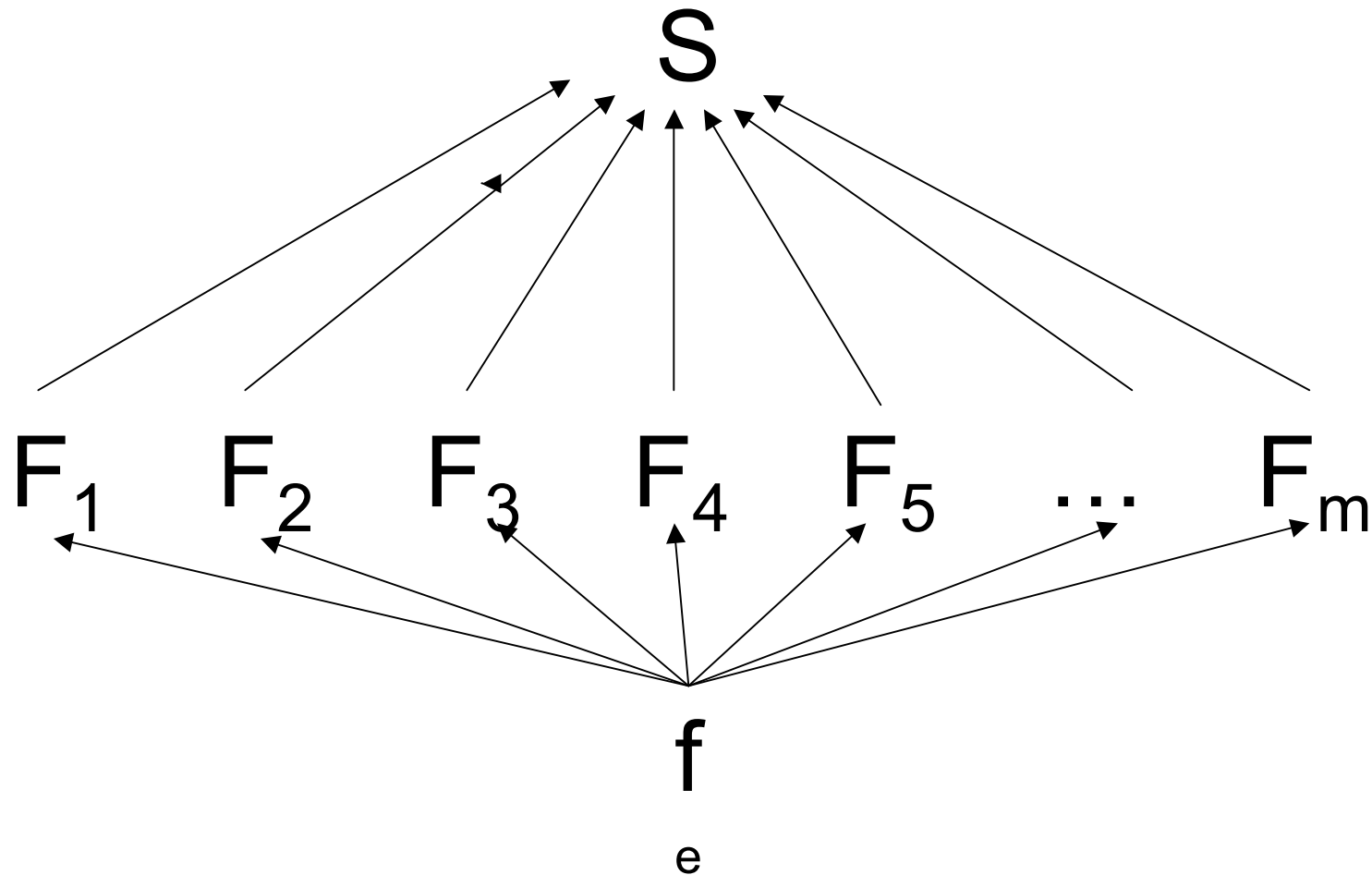
Nondoxastic State Directly Influences Doxastic State of Evidence Base



Doxastic State of Evidence Basis Influences Belief-Strengths for All Other Propositions



Doxastic State of Evidence Basis Influences Belief-Strengths for All Other Propositions



How does/should e influence the Evidence Basis ???

1. $e \rightarrow P_e[E_i]$ $P \rightarrow P_e$

2. $e \rightarrow \pi_P^e[E_i]$ $\pi_P^e : P \rightarrow P_e$

$$P_e[E_i] = \pi_P^e[E_i] \cdot P[E_i]$$

3. $e \rightarrow \beta_P^e[E_i : E_j]$ $\beta_P^e : (P[E_i]/P[E_j]) \rightarrow (P_e[E_i]/P_e[E_j])$

$$(P_e[E_i]/P_e[E_j]) = \beta_P^e[E_i : E_j] \cdot (P[E_i]/P[E_j])$$

If e were propositionally expressible

$$2. e \rightarrow \pi_P^e[E_i] \quad P[E_i | e] = P_e[E_i] = \pi_P^e[E_i] \cdot P[E_i]$$

so we would have $\pi_P^e[E_i] = P[e | E_i] / P[e]$

$$3. e \rightarrow \beta_P^e[E_i : E_j] \quad \begin{aligned} P[E_i | e] / P[E_j | e] &= (P_e[E_i] / P_e[E_j]) \\ &= \beta_P^e[E_i : E_j] \cdot (P[E_i] / P[E_j]) \end{aligned}$$

so we would have

$$\beta_P^e[E_i : E_j] = P[e | E_i] / P[e | E_j]$$

Rigidity Condition: given evidence basis $\{E_i\}$,

$$P_e[S | E_i] = P[S | E_i]$$

If e were propositional, we'd write it as follows:

$$P[S | E_i \cdot e] = P[S | E_i]$$

Basic Jeffrey Updating

$$\begin{aligned} P_e[S] &= \sum_i P_e[S | E_i] \cdot P_e[E_i] \\ &= \sum_i P[S | E_i] \cdot P_e[E_i] \\ &= \sum_{\{i : P[E_i] > 0\}} P[S \cdot E_i] \cdot (P_e[E_i] / P[E_i]) \\ &= \sum_{\{i : P[E_i] > 0\}} P[S \cdot E_i] \cdot \pi_P^e[E_i] \end{aligned}$$

Basic Sequential Jeffrey Updating

$$\begin{aligned} P_{\text{ef}}[S] &= \sum_j P_e[S | F_j] \cdot P_{\text{ef}}[F_j] \\ &= \sum_{\{j: P_e[F_j] > 0\}} P_e[S \cdot F_j] \cdot (P_{\text{ef}}[F_j] / P_e[F_j]) \\ &= \sum_{\{j: P_e[F_j] > 0\}} \sum_{\{i: P[E_i] > 0\}} P[S \cdot F_j \cdot E_i] \cdot \\ &\quad (P_e[E_i] / P[E_i]) \cdot (P_{\text{ef}}[F_j] / P_e[F_j]) \\ &= \sum_{\{j: P_e[F_j] > 0\}} \sum_{\{i: P[E_i] > 0\}} P[S \cdot F_j \cdot E_i] \cdot \\ &\quad \pi_P^e[E_i] \cdot \pi_{P_e^{\text{ef}}}[F_j] \end{aligned}$$

Basic Jeffrey Updating – Order Effect

$$P_{ef}[S] = \sum_{\{j: P_e[F_j]>0\}} \sum_{\{i: P[E_i]>0\}} P[S \cdot F_j \cdot E_i]$$

•

$$(P_e[E_i]/P[E_i]) \cdot (P_{ef}[F_j]/P_e[F_j])$$

$$P_{fe}[S] = \sum_{\{j: P[F_j]>0\}} \sum_{\{i: P_f[E_i]>0\}} P[S \cdot F_j \cdot E_i]$$

•

$$(P_{fe}[E_i]/P_f[E_i]) \cdot (P_f[F_j]/P[F_j])$$

Basic Jeffrey Updating – Order Effect

$$P_{\text{ef}}[S] = \sum_{\{j: P_e[F_j] > 0\}} \sum_{\{i: P[E_i] > 0\}} P[S \cdot F_j \cdot E_i]$$

$$\pi_P^e[E_i] \cdot \pi_{P_e}^{\text{ef}}[F_j]$$

$$P_{\text{fe}}[S] = \sum_{\{j: P[F_j] > 0\}} \sum_{\{i: P_f[E_i] > 0\}} P[S \cdot F_j \cdot E_i]$$

$$\pi_{P_f}^{\text{fe}}[E_i] \cdot \pi_P^f[F_j]$$

Basic Jeffrey Updating for a Long Sequence

$$P_{e\dots fg}[S]$$

$$= \sum_{\{k: P_{e\dots g}[G_k] > 0\}} \dots \sum_{\{i: P[E_i] > 0\}} P[S \cdot G_k \cdot \dots \cdot E_i] \cdot$$

$$(P_e[E_i]/P[E_i]) \cdot \dots \cdot (P_{e\dots fg}[G_k]/P_{e\dots f}[G_k])$$

$$= \sum_{\{k: P_{e\dots g}[G_k] > 0\}} \dots \sum_{\{i: P[E_i] > 0\}} P[S \cdot G_k \cdot \dots \cdot E_i] \cdot$$

$$\pi_{P^e}[E_i] \cdot \dots \cdot \pi_{P_{e\dots f}^{e\dots fg}}[G_k]$$

How does/should e influence the Evidence Basis ???

1. $e \rightarrow P_e[E_i]$ $P \rightarrow P_e$: P_e is **ORDER-DEPENDENT**

2. $e \rightarrow \pi_P^e[E_i]$ $\pi_P^e : P \rightarrow P_e$: π_P^e is **P-value-DEPENDENT**

$$P_e[E_i] = \pi_P^e[E_i] \cdot P[E_i] \quad \text{i.e., Fails Modularity}$$

$$\pi_P^e[E_i] = P[e | E_i] / P[e]$$

3. $e \rightarrow \beta_P^e[E_i]$ **What about $\beta_P^e[E_i]$ as an Update Factor**

$$(P_e[E_i]/P_e[E_j]) = \beta_P^e[E_i : E_j] \cdot (P[E_i]/P[E_j])$$

$$\beta_P^e[E_i : E_j] = P[e | E_i] / P[e | E_j]$$

Basic Jeffrey Updating

Notice that

$$1 = P_{e\dots fg}[\text{tautology}]$$

$$= \sum_{\{k: P_{e\dots g}[G_k] > 0\}} \dots \sum_{\{i: P[E_i] > 0\}} P[G_k \cdot \dots \cdot E_i] \cdot$$

$$(P_e[E_i]/P[E_i]) \cdot \dots \cdot (P_{e\dots fg}[G_k]/P_{e\dots f}[G_k])$$

(useful for the denominator of the next equation)

Basic Jeffrey Updating

$$P_{e\dots g}[S] =$$

$$\frac{\sum_{\{k:P_{e\dots g}[G_k]>0\}} \dots \sum_{\{i:P[E_i]>0\}} P[S \cdot G_k \cdot \dots \cdot E_i] \cdot \frac{(P_e[E_i]/P[E_i]) \dots (P_{e\dots fg}[G_k]/P_{e\dots f}[G_k])}{(P_e[E_1]/P[E_1]) \dots (P_{e\dots fg}[G_1]/P_{e\dots f}[G_1])}}{\dots}$$

$$\frac{\sum_{\{k:P_{e\dots g}[G_k]>0\}} \dots \sum_{\{i:P[E_i]>0\}} P[G_k \cdot \dots \cdot E_i] \cdot \frac{(P_e[E_i]/P[E_i]) \dots (P_{e\dots fg}[G_k]/P_{e\dots f}[G_k])}{(P_e[E_1]/P[E_1]) \dots (P_{e\dots fg}[G_1]/P_{e\dots f}[G_1])}}{\dots}$$

Basic Jeffrey Updating with Bayes-Factors

$$P_{e\dots g}[S] =$$

$$\sum_{\{k:P_{e\dots g}[G_k]>0\}} \dots \sum_{\{i:P[E_i]>0\}} P[S \cdot G_k \cdot \dots \cdot E_i] \cdot \\ \beta_P^e[E_i : E_1] \cdot \dots \cdot \beta_{Pe\dots f}^{e\dots fg}[G_k : G_1]$$

$$\sum_{\{k:P_{e\dots g}[G_k]>0\}} \dots \sum_{\{i:P[E_i]>0\}} P[G_k \cdot \dots \cdot E_i] \cdot \\ \beta_P^e[E_i : E_1] \cdot \dots \cdot \beta_{Pe\dots f}^{e\dots fg}[G_k : G_1]$$

Extended Jeffrey Updating General Update Factors

Uniformity Rule:

$$\beta_{Pa\dots ef}^{a\dots efg}[G_k:G_1] = \beta_{Pa\dots e}^{a\dots eg}[G_k:G_1]$$

whenever G_k is in the basis of \mathbf{g} but
not in the basis of \mathbf{f}

Uniformity = Extended Rigidity

If f and g were propositional, the usual Jeffrey account of Rigidity would give $P[f | G_k \cdot g] = P[f | G_k]$.

From this the Uniformity Rule would be derivable:

$$\begin{aligned}
 \beta_{P^{fg}}[G_k : G_1] &= (P_{fg}[G_k]/P_{fg}[G_1]) / (P_f[G_k]/P_f[G_1]) \\
 &= (P[G_k | f \cdot g]/P[G_1 | f \cdot g]) / (P[G_k | f]/P[G_1 | f]) \\
 &= (P[f \cdot g | G_k]/P[f \cdot g | G_1]) / (P[f | G_k]/P[f | G_1]) \\
 &= \frac{(P[f | G_k \cdot g] \cdot P[g | G_k]) / (P[f | G_1 \cdot g] \cdot P[g | G_1])}{(P[f | G_k] / P[f | G_1])} \\
 &= P[g | G_k] / P[g | G_1] = (P[G_k | g]/P[G_1 | g]) / (P[G_k]/P[G_1]) \\
 &= \beta_P^g[G_k : G_1]
 \end{aligned}$$

Jeffrey Updating with Bayes-Factors that satisfy the Uniformity Rule

$$P_{e\dots g}[S] =$$

$$\sum_{\{k:P_{e\dots g}[G_k]>0\}} \dots \sum_{\{i:P[E_i]>0\}} P[S \cdot G_k \cdot \dots \cdot E_i] \cdot \beta_P^e[E_i : E_1] \cdot \dots \cdot \beta_P^g[G_k : G_1]$$

$$\sum_{\{k:P_{e\dots g}[G_k]>0\}} \dots \sum_{\{i:P[E_i]>0\}} P[G_k \cdot \dots \cdot E_i] \cdot \beta_P^e[E_i : E_1] \cdot \dots \cdot \beta_P^g[G_k : G_1]$$

Extended Jeffrey Updating General Update Factors

Suppose we apply the Uniformity Rule even
when g and f affect the same basis G_k :

$$\beta_P^f[G_k:G_1] \cdot \beta_{Pf}^{fg}[G_k:G_1] =$$
$$\beta_P^f[G_k:G_1] \cdot \beta_P^g[G_k:G_1]$$

which on iteration blows up for $\beta_P^f[G_k:G_1]$, etc.

larger than 1 and goes to 0 when smaller than 1.

This is Garbers Problem

for the Uniformity Rule

Jeffrey Updating with Bayes-Factors that satisfy the Uniformity Rule on a Common Evidence Basis – Garber’s Problem

$$P_{e\dots g}[S] =$$

$$\sum_{\{k:P_{e\dots g}[G_k]>0\}} \dots \sum_{\{i:P[E_i]>0\}} P[S \cdot G_k \cdot \dots \cdot E_i] \cdot$$

$$\beta_P^e[E_i : E_j] \cdot \dots \cdot \beta_P^f[G_k : G_j] \cdot \beta_P^g[G_k : G_j]$$

$$\sum_{\{k:P_{e\dots g}[G_k]>0\}} \dots \sum_{\{i:P[E_i]>0\}} P[G_k \cdot \dots \cdot E_i] \cdot$$

$$\beta_P^e[E_i : E_j] \cdot \dots \cdot \beta_P^f[G_k : G_j] \cdot \beta_P^g[G_k : G_j]$$

Extended Jeffrey Updating General Update Factors

Notice: if g and f share an evidence basis and G_k is in it, even without the Uniformity Rule we have

$$\beta_{Pa\dots e}^{a\dots ef}[G_k:G_1] \cdot \beta_{Pa\dots ef}^{a\dots efg}[G_k:G_1] =$$

$$\frac{(P_{a\dots ef}[G_k]/P_{a\dots ef}[G_1]) \quad (P_{a\dots efg}[G_k]/P_{a\dots efg}[G_1])}{(P_{a\dots e}[G_1]/P_{a\dots e}[G_1]) \quad (P_{a\dots ef}[G_1]/P_{a\dots ef}[G_1])}$$

$$= \beta_{Pa\dots e}^{e(gf)}[G_k:G_1]$$

So even without the Uniformity Rule, update factors that share the same evidence basis “accumulate”

Jeffrey Updating with Bayes-Factors that satisfy the Uniformity Rule for Distinct Evidence Bases. and with Cumulative Update Factors ($\varepsilon, \dots, \gamma$) for Shared Evidence Bases

$$P_{e\dots g}[S] =$$

$$\sum_{\{k:P_{e\dots g}[G_k]>0\}} \dots \sum_{\{i:P[E_i]>0\}} P[S \cdot G_k \cdot \dots \cdot E_i] \cdot \beta_P^\varepsilon[E_i : E_1] \cdot \dots \cdot \beta_P^\gamma[G_k : G_1]$$

$$\sum_{\{k:P_{e\dots g}[G_k]>0\}} \dots \sum_{\{i:P[E_i]>0\}} P[G_k \cdot \dots \cdot E_i] \cdot \beta_P^\varepsilon[E_i : E_1] \cdot \dots \cdot \beta_P^\gamma[G_k : G_1]$$

Bayes' Theorem for Jeffrey Updating and the Uniformity Rule

$$P_{e\dots g}[H] = P[H] \cdot$$

$$\sum_{\{k:P_{e\dots g}[G_k]>0\}} \dots \sum_{\{i:P[E_i]>0\}} P[G_k \cdot \dots \cdot E_i | H] \cdot \\ \beta_P^\varepsilon[E_i : E_1] \cdot \dots \cdot \beta_P^\gamma[G_k : G_1]$$

$$\sum_{\{k:P_{e\dots g}[G_k]>0\}} \dots \sum_{\{i:P[E_i]>0\}} P[G_k \cdot \dots \cdot E_i] \cdot \\ \beta_P^\varepsilon[E_i : E_1] \cdot \dots \cdot \beta_P^\gamma[G_k : G_1]$$

END