Commentary on 'New Life for Carnap's *Aufbau*?' by Hannes Leitgeb

James Justus
University of Texas, Austin
justus.phil@mail.utexas.edu

1. **Introduction**. In another paper, Leitgeb (2005a) convincingly argues that the "companionship" and "imperfect community" problems Goodman (1977) raised for the method of quasianalysis outlined in the *Aufbau* are unavoidable if the *Aufbau*'s system basis is restricted to a relation of fixed finite adicity (*e.g*. 'recollection of similarity,' R_s) between elementary experiences (erlebs).[1] Leitgeb's (2005b) goal is to develop a revamped constructional system that avoids these problems, Goodman's "dimensionality" criticism, Quine's (1951) criticism of Carnap's construction of physical objects in terms of autopsychological ones, and that "shares several important properties with its predecessor" (p. 1).

According to the traditional and structuralist interpretations of the *Aufbau* (Leitgeb 2005b, p. 2), its goal was to establish necessary extensional equivalences between scientific sentences and sentences containing only logical terms and, for the traditional interpretation, terms referring to "the given" erlebs. Leitgeb's new system has less ambitious aims. It should ensure (2005b, p. 4):

(i) every scientific sentence is translatable into an empirically equivalent one containing only logico-mathematical terms and terms referring to a subject's experiences; and,
(ii) "the translation image expresses a *subject-invariant* constraint on experiences."

Leitgeb (2005b, p. 5) intends (ii) to capture the "structuralist" objectives of the *Aufbau* identified by the latter interpretation, but what its counterpart is in Leitgeb's system and how his system achieves it is unexplained in this paper. This issue probably needs a full exposition since, according to Richardson (1998, Ch. 3), the *Aufbau* failed to achieve its structuralist aims and the main reason it failed –the nondefinability of R_s by strictly logical operators– seems to be a problem that would also affect Leitgeb's system.

Since the strong goal and especially the parsimonious basis of the *Aufbau* are responsible for its failings, the new system's success depends primarily upon its stronger basis. For this reason, my comments focus primarily on Leitgeb's system basis. Specifically, I raise questions about its logical form and how it ensures the system avoids Goodman's criticisms. I also pose some potential problems for its interpretation.

2. **Leitgeb's System Basis**. Leitgeb (2005b, p. 17) believes three features of the *Aufbau*'s basis make it excessively "minimalistic":

(1) R_s is too weak, *i.e*. its extension is too large;
(2) R_s does not distinguish different kinds of similarity;

[1] Leitgeb's (2005a) argument is limited to the quasianalysis method presented in the *Aufbau*. Mormann (1994) showed that a more sophisticated quasianalysis method Carnap had developed in 1923 avoids Goodman's companionship and imperfect community criticisms. Leitgeb's (2005a) criticism of quasianalysis is not, therefore, applicable to this more sophisticated version. This version of quasianalysis does not, however, avoid Goodman's dimensionality criticism.

(3) R_s is dichotomous: it cannot distinguish degrees of similarity.

The basis of Leitgeb's system is intended to be much stronger. Its basic elements are "experiential tropes instantiated by the erlebs of a given and fixed subject *S* within a given interval of time," (p. 18) and its basic relations are membership (∈), an ordering relation (<), and a qualitative overlap relation (O_v).

Formally, Leitgeb (2005b, p. 19) suggests experiential tropes correspond to ordered pairs $\langle C_q, C_t \rangle$, where:

(i) C_q is a bounded, extended, closed convex set of quality points in a sensory quality space;

(ii) C_t is a bounded, extended, closed convex set of temporal instants on the "real" time axis, *i.e*. a compact segment of **R** with finite length (and a total ordering);

(iii) some *S* erleb instantiates some point in C_q within the interval C_t.

(i)-(iii) are only necessary conditions for being a basic element (Leitgeb 2005b, p. 19).[2] If they were jointly sufficient, the set of basic elements would necessarily be infinite, which Leitgeb rejects.

Tropes are "property bits" of some temporal interval. By *experiential* tropes, Leitgeb seems to be restricting the basic elements to tropes that are bits of phenomenal properties, which makes them instantiable by phenomenal objects. This is what allows them to be instantiated by erlebs, and Leitgeb (2005b, pp. 23 and 25) explicitly states that the quality points constituting the C_q of the basic elements are phenomenal. Leitgeb's system basis does not, therefore, seem to be, "open both to a phenomenalistic *and* a physicalistic interpretation" (2005b, p. 5).

Leitgeb (2005b, p. 19) suggests the C_t correspond to subsets of the formal model of *physical* time. Two consequences of this should be recognized. First, the ordering on C_t may not adequately represent the phenomenal sense of time duration of a subject *S*. Second, since the C_t formally represent subsets of *physical* time and the C_q are sets of *phenomenal* quality points, the basic elements of Leitgeb's system somewhat awkwardly contain phenomenal and physical components. Even if "higher" levels of Leitgeb's constructional system appear to have a clear phenomenal or physical interpretation, the mix of different types of components in the basis from which all other objects are constructed obscures the proper interpretation of the system as a whole (see §3).

As in the *Aufbau*, '∈' is the usual set membership relation. '<' orders the pairs such that $\langle C_q^1, C_t^1 \rangle < \langle C_q^2, C_t^2 \rangle$ iff:

$$(\forall x \in C_t^1)(\forall y \in C_t^2)(x <_t y);$$

where '$<_t$' designates the usual ordering relation on **R** interpreted as the axis of physical time.[3]

---

[2] Unless otherwise clear from the context, in the following terms like 'basic element' and 'system basis' designate the formal features of Leitgeb's constructional system, not what they are intended to represent.

[3] '$<_t$' should probably be explicitly included among the basic relations of Leitgeb's system.

'O_v' is a unary predicate on sets of basic elements. Formally, for $X=\{\langle C_q^i, C_t^i\rangle: i\in I\}\subseteq B_{as}$, $O_v(X)$ iff $\bigcap_{i\in I} C_q^i \neq \varnothing$. Informally, $O_v(X)$ measures whether the C_q^i of X share at least one quality point. The set of basic elements of Leitgeb's system (B_{as}) can be defined in terms of O_v: $B_{as} = \bigcup O_v(b)$ (Leitgeb 2005b, p. 24). Note that O_v is dichotomous; it does not measure the size of the intersection.

Leitgeb's system basis overcomes weaknesses (1)-(3) of the *Aufbau*'s basis. Convexity, Leitgeb (2005b, p. 21) suggests, overcomes (2) by formalizing the idea of different kinds of similarity: "if *p* is similar to *r* in a particular respect (say, *Q*) and *q* is qualitatively between *p* and *r*, then it seems to be necessary that *p* and *r* are similar to *q* in the same respect *Q*. But this is just the closure condition for convex sets." Thus, different intersections of C_q, which are also convex, may correspond to different kinds of similarity (*e.g*. different sensory similarities: visual, auditory, *etc*.). With respect to (3), Leitgeb (2005b, p. 27) is able to define the set of phenomenal visual quality points and a metric on it. Whether Leitgeb's system overcomes (1) cannot be determined without specification of sufficient conditions for his basic elements. They would indicate whether the relationship between Leitgeb's basic elements and erlebs is one-one, one-many, many-one, *etc*., and clearly could be specified so that (1) is overcome.

3. **The Adequacy of a Constructional System Basis**. The adequacy of a constructional system basis depends on the system's intended interpretation and its objectives. Although the intended interpretation and objectives of the *Aufbau* have proved to be underdetermined by its actual text, Carnap (1928) makes several explicit claims about the goals and proper understanding of its basis. For instance, Carnap chose an "autopsychological" rather than a physical or "general psychological" basis so that, "the constructional system reflect not only the logical-constructional order of the objects, but also their epistemic order," and because, "an autopsychological basis still has the advantage that the totality of all objects is constructed from a considerably smaller basis" (1928, §64). The former condition underlies Carnap's frequent insistence that the *Aufbau* is not intended to describe the actual process of human cognition but rather to pursue the normative project of "rationally reconstructing" it (1928, §§54 and 100). The latter is presumably a requirement for the structuralist aspects of the *Aufbau*.

A similar (and ideally more detailed) specification of the intended interpretation and objectives of Leitgeb's system basis is needed. Leitgeb provides some information in this regard, for example:

> [a] Sentences which involve our basic predicates may be used to describe which sense experiences *S* has. These descriptions of *S*'s experiences in terms of basic predicates are not necessarily *S*'s "first person" descriptions, but they might just as well be a neuroscientist's "third-person" descriptions. *S* is also not assumed to be consciously aware of her sense experiences, i.e., our basis is open to the existence of unconscious sense experiences (p. 21)

> [b] We do not subscribe to any sort of epistemological foundationalism: sentences involving our basic terms are not necessarily certain or self-justifying; we might think that they are true but in fact they are false. As far as the topic of justification is concerned, their status differs only gradually from the status of sentences about the physical world. Neither is it our goal to justify sentences about the physical world on the basis of sentences that can be formulated in the language of our new constitution system (p. 21)
>
> [c] The basis of our system has both an "enlightened" phenomenalistic interpretation (as Carnap's in the old *Aufbau* had) and a physicalistic interpretation...One physicalistic way of viewing our basic elements is to think of them in terms of neural activation patterns of perceptual detector units: a pattern that corresponds formally to a pair $\langle C_q, C_t \rangle$ is generated by a detector if and only if an external stimulus is detected that overlaps qualitatively with the range C_q while overlapping temporally with the range C_t (p. 22)
>
> [d] Our own choice is inspired and –hopefully– also somewhat justified by findings in cognitive science and neuroscience, although we will not say much about these background theories in this paper. However, it should be clear that every attempt of rational reconstruction such as Carnap's or the present one presupposes some amount of idealization. In this respect, it is helpful to think of the given subject *S* not as a human being but rather as an artificial cognitive agent (p. 23)

Although these comments are helpful, they do not address some crucial issues about the constructional system discussed below.

3.1. **A Phenomenalistic vs. Physicalistic System Basis**. For Leitgeb's system basis to have a plausible phenomenalistic *and* a plausible physicalistic interpretation ([c]), its formalism must adequately represent the cognitive life of a subject from a "first person" and a "third person" perspective ([a]). [c] provides some specific information about the latter interpretation and [d] provides information about the former, but Leitgeb gives no argument that the new basis, or more generally that any constructional system basis can accommodate both perspectives. Given the different character of the phenomenalistic "first person" perspective and the physicalistic "third person" perspective, attempts to develop such an argument may be futile. Descriptions based on one perspective will usually not, and perhaps never, be equivalent to descriptions based on the other. The *Aufbau*'s basis, for instance, was specifically tailored to be interpreted phenomenalistically, and Carnap (1928, §§59 and 62) was clear that a physicalistic system basis would be different, and would not achieve his epistemological objective.

3.2. **The Logical Form of the System Basis**. Besides clarifying what Leitgeb's system is intended to represent, a more thorough defense of the specific logical form of the system basis is needed, especially where it differs from the *Aufbau*'s.

Consider some of their significant differences. First, the basic elements of Leitgeb's system are not unanalyzable, as was the case for the formal representation of the erlebs of the *Aufbau* (Carnap 1928, §68).[4] As an ordered pair, each Leitgeb basic element is a set (*e.g.* $\{C_q, \{C_t\}\}$) whose elements are sets of quality points (for C_q) and sets of sets of temporal instants (for C_t). Whether the ordered pairs $\langle C_q, C_t \rangle$ or their constituent elements should be considered the "real" basic elements of Leitgeb's system may be merely

[4] Leitgeb (2005b, p. 19) seems to have overlooked this fact: "Carnap's erlebs correspond formally to sets of quality points."

terminological, but that the "basic" ordered pairs of the system have analyzable structure is nontrivial. It is, in fact, essential to avoiding Goodman's criticisms (see below and §3.3) and should therefore be justified explicitly.

Another significant difference concerns the cardinality of components of the basic elements. The set of erlebs of the *Aufbau* is finite, and since their formal representations are also unanalyzable, they contain no elements. Leitgeb (2005b, p. 15) permits the cardinality of B_{as} to be finite or infinite, but the basic ordered pairs contain elements of infinite (in fact, uncountable) cardinality. As compact segments of $\mathbf{R}$, each C_t is uncountable, and Leitgeb's (2005b, p. 26) use of Helly's theorem to avoid Goodman's dimensionality criticism requires each C_p be an uncountable subset of $\mathbf{R}^n$ for some n (see §3.3). This property of the system basis' formal structure therefore plays a crucial role and should be justified explicitly. A more detailed explanation of what this aspect of the system is intended to represent would also allay worries that the infinite cardinality of these sets seems to mischaracterize the discrete nature of human cognition, even significantly idealized, which were the type of cognitive systems the *Aufbau* was intended to describe.

Overcoming the weaknesses of the *Aufbau*'s basis partly justifies the logical form of Leitgeb's system basis, but not fully. First, it obviously does not uniquely determine its logical form. Second, it does not indicate why this specific logical form is required to represent how a subject experiences. To illustrate what would constitute such a justification, consider one reason for the logical form of the *Aufbau*'s basis. A, if not the primary goal of the *Aufbau* was to demonstrate, "how science can arrive at intersubjectively valid assertions if all its objects are to be constructed from the standpoint of the individual subject" (Carnap 1928, §66). Thus, an account of how an individual subject experiences –more precisely for the *Aufbau*, an idealized subject– *and how the logical form of the system basis should represent this* was required. The finitude of the set of erlebs and, more specifically, the unanalyzability of their formal representation were features of the logical form of the *Aufbau*'s basis Carnap (1928, §67 and 68) thought Gestalt psychology required.

A similar justification of the logical form of Leitgeb's system basis is necessary because it is what ensures the constructional system avoids Goodman's criticisms. Consider the companionship and imperfect community problems first. Leitgeb (2005b, p. 28) states:

> why is it that we were able to avoid Goodman's problems in our new setting? Our basic elements are already situated on the level of Carnap's quality spheres, so we did not have to take the first step of quasianalysis; the difficulties of companionship and imperfect community simply do not arise.

Thus, quasianalysis is unnecessary within Leitgeb's system: the objects and relations that had to be constructed through quasianalysis in the *Aufbau* are built into the logical form of his system's basis. Criticisms that target quasianalysis, such as Goodman's companionship and imperfect community criticisms, are obviously inapplicable. The worry, however, is that, without a justification for the specific logical form of the system basis, the epistemological significance of the new system's avoidance of these criticisms, though formally transparent and unproblematic, is unclear. Goodman's (1977) own constructional system, for instance, avoids problems of companionship and imperfect community by making its basic elements phenomenal quality points, and quasianalysis thereby unnecessary. Partly for this reason, however, Moulinas (1991) questions its relevance for the kind of epistemological project Carnap (1928) was attempting.

3.3. *k*-Hellyness and Helly's Theorem. There are similar worries about Leitgeb's novel strategy to avoid Goodman's dimensionality criticism. Within the new system, Leitgeb utilizes the concept of *k*-Hellyness to define the dimensionality of sense classes. Formally, for a "connectivity component" $X \subseteq \mathrm{B}_{\mathrm{as}}$ and $k \in \mathbf{N}$ (2005b, pp. 25-26):

$$X \text{ is } k\text{-Helly} =_{\mathrm{df}} (\forall Y \subseteq X)[\mathrm{O}_{\mathrm{v}}(Y) \equiv (\forall Z \subseteq Y)(|Z| \leq k \rightarrow \mathrm{O}_{\mathrm{v}}(Z))];$$

where '$|\cdot|$' designates the cardinality of $\cdot$. X is *k*-dimensional if it is (k+1)-Helly but not *k*-Helly. This concept of dimension has the opportune property that the dimensionality of $X \subseteq \mathrm{B}_{\mathrm{as}}$ does not depend upon its cardinality as Carnap's topological conception of dimension problematically did.

Helly's theorem entails there are subsets of B_{as} satisfying this definition.

> **Helly's Theorem**: Every class of closed, bounded, convex subsets of $\mathbf{R}^n$ is (n+1)-Helly (Leitgeb 2005b, p. 26; *cf*. Matoušek 2002).

The rational behind the specific logical form of Leitgeb's basic elements is now clear: it is required for Helly's theorem and the applicability of the concept of *k*-Hellyness. Specifically, the C_q must be convex subsets of $\mathbf{R}^n$ for some n (Leitgeb [2005b] does not state explicitly that $C_q \subseteq \mathbf{R}^n$).

Avoiding the dimensionality criticism is a clear merit of the logical form of Leitgeb's system basis but, given its intended interpretation (see [a]), the principal issue is whether it adequately represents a subject *S*'s experiences. With respect to the convexity requirement, Leitgeb (2005b, p. 20) mentions the work of Gärdenfors (1990, 2000). Besides some general concerns about this part of his work, however, Gärdenfors' main argument for convexity specifically concerns the "projectibility" of predicates required for successful inductive inferences, not how a subject experiences. The latter requires a detailed defense based on cognitive and neurosciences (see [d]). If Leitgeb's system is to have a plausible phenomenalistic interpretation, a defense of the claim that $C_q \subseteq \mathbf{R}^n$ also seems necessary.

4. Conclusion. Considered as a formal structure, whether a constructional system avoids the companionship, imperfect community, dimensionality, and similar technical problems depends upon its formal structure. If the method of quasianalysis a system with a parsimonious basis utilizes is too weak, it will succumb to these kinds of criticisms, as Mormann (1994) and Leitgeb (2005a) have shown. If the system basis is sufficiently strong, however, it will avoid these problems, as Leitgeb's (2005b) system shows. The general philosophical significance of such systems can only be gauged with a clear understanding of exactly what avoidance of these problems requires of system structure. Ideally, a minimal set of requirements, or a manageable number of different sets that are minimal in different respects could be found. One set might specify the weakest form of quasianalysis with the *Aufbau*'s basis sufficient to avoid these problems; another might specify the weakest system basis that avoids them. These sets of requirements could then be evaluated with respect to different philosophical goals. With respect to epistemological goals, systems satisfying some sets of requirements might plausibly represent human cognition, while others might adequately represent the hypothetical cognition of an ideal subject, as in the *Aufbau*. Leitgeb's (2005b) system is an innovative example of a constructional system with a basis sufficient to avoid the companionship, imperfect community, and dimensionality criticisms, but other systems with weaker bases, or bases that may more adequately represent actual or idealized human cognition, may be similarly sufficient. There are, for instance, mathematical concepts weaker than convexity, such as simply-connected-and-compact, for which Helly-type theorems exist (Danzer *et al*. 1963). Systems with basic elements that exhibit these properties will also avoid the dimensionality objection.

References

Carnap, Rudolf ([1928], 1967), *The Logical Structure of the World*. Berkeley: University of California Press.

Danzer, L., Grünbaum, B., and Klee, V. (1963), "Helly's Theorem and its Relatives." In *Convexity*: *Proceedings of Symposia in Pure Mathematics*. Vol. 7 Providence: American Mathematical Society pp. 101-180.

Gärdenfors, Peter (1990), "Induction, Conceptual Spaces and AI." *Philosophy of Science* **57**: 78-95.

-------. (2000), *Conceptual Spaces*. Cambridge: MIT Press.

Goodman, Nelson (1977), *The Structure of Appearance*. 3rd Edition. Boston: D. Reidel.

Leitgeb, Hannes (2005a), "Under What Conditions Does Quasianalysis Succeed?" In review.

-------. (2005b), "New Life for Carnap's *Aufbau*?"

Matoušek, Jiří (2002), *Lectures on Discrete Geometry*. New York: Springer.

Mormann, Thomas (1994), "A Representational Reconstruction of Carnap's Quasianalysis." *PSA* Vol. 1. pp. 96-104.

Moulinas, C. Ulises (1991), "Making Sense of Carnap's '*Aufbau*'." *Erkenntnis* **35**: 263-285.

Quine, W. V. O. (1951), "Two Dogmas of Empiricism." *Philosophical Review* **60**: 20-43.

Richardson, Alan (1998), *Carnap's Construction of the World: The* Aufbau *and the Emergence of Logical Empiricism*. Cambridge: Cambridge University Press.