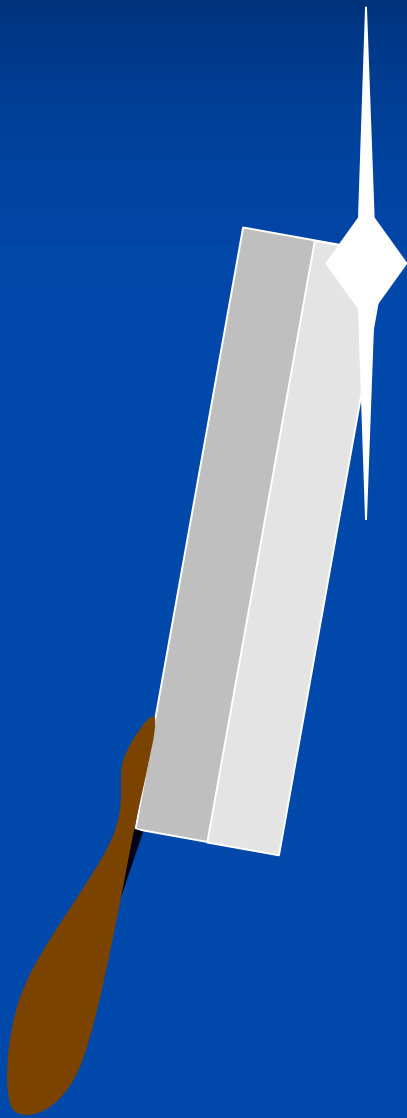


Ockham's Razor:

**What it is,
What it isn't,
How it works, and
How it doesn't**



Kevin T. Kelly
Department of Philosophy
Carnegie Mellon University
www.cmu.edu

Further Reading

“Efficient Convergence Implies Ockham's Razor”, *Proceedings of the 2002 International Workshop on Computational Models of Scientific Reasoning and Applications*, Las Vegas, USA, June 24-27, 2002.

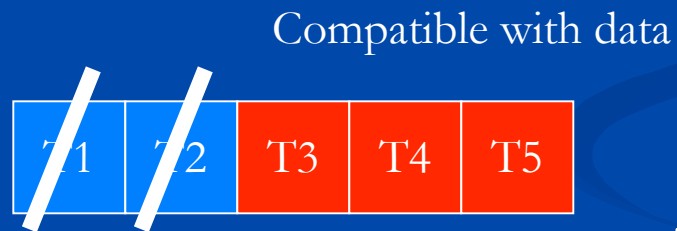
(with C. Glymour) **“Why Probability Does Not Capture the Logic of Scientific Justification”**, C. Hitchcock, ed., *Contemporary Debates in the Philosophy of Science*, Oxford: Blackwell, 2004.

“Justification as Truth-finding Efficiency: How Ockham's Razor Works”, *Minds and Machines* 14: 2004, pp. 485-505.

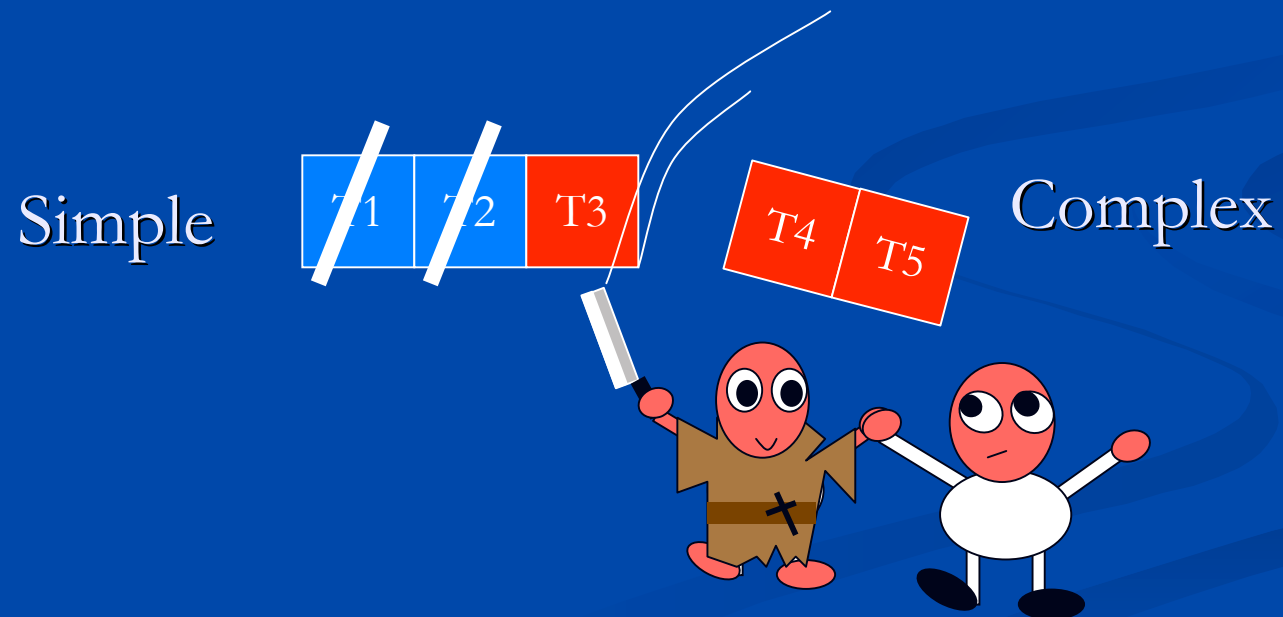
“Learning, Simplicity, Truth, and Misinformation”, *The Philosophy of Information*, under review.

“Ockham's Razor, Efficiency, and the Infinite Game of Science”, proceedings, *Foundations of the Formal Sciences 2004: Infinite Game Theory*, Springer, under review.

Which Theory to Choose?

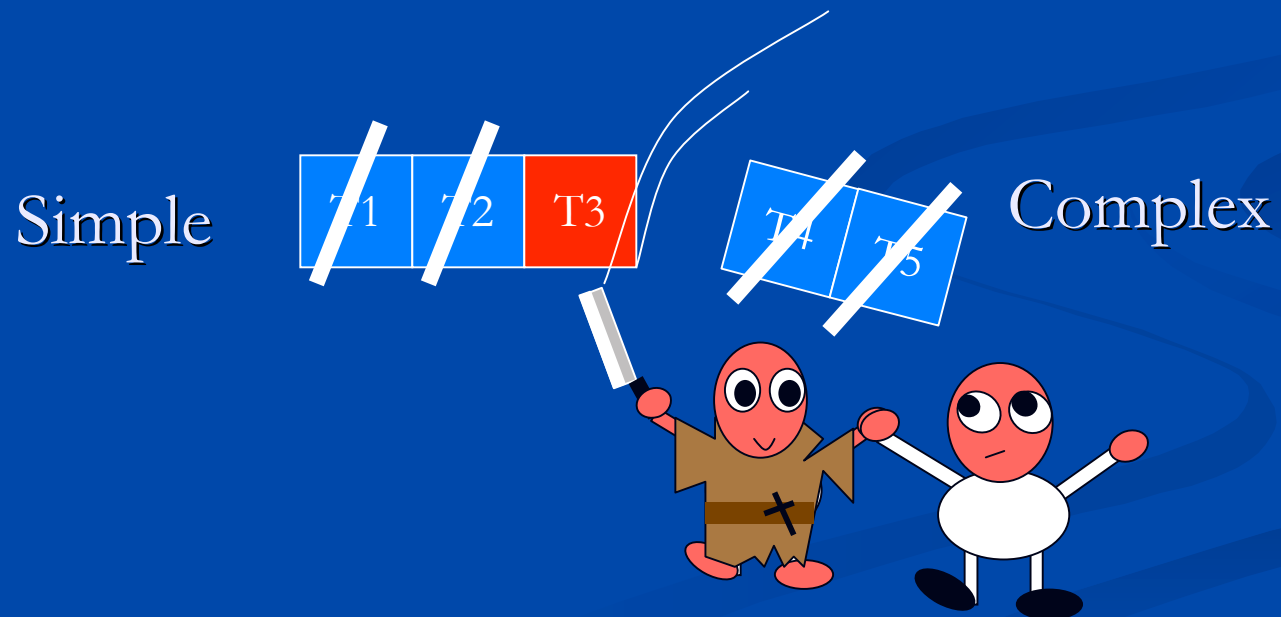


Use Ockham's Razor



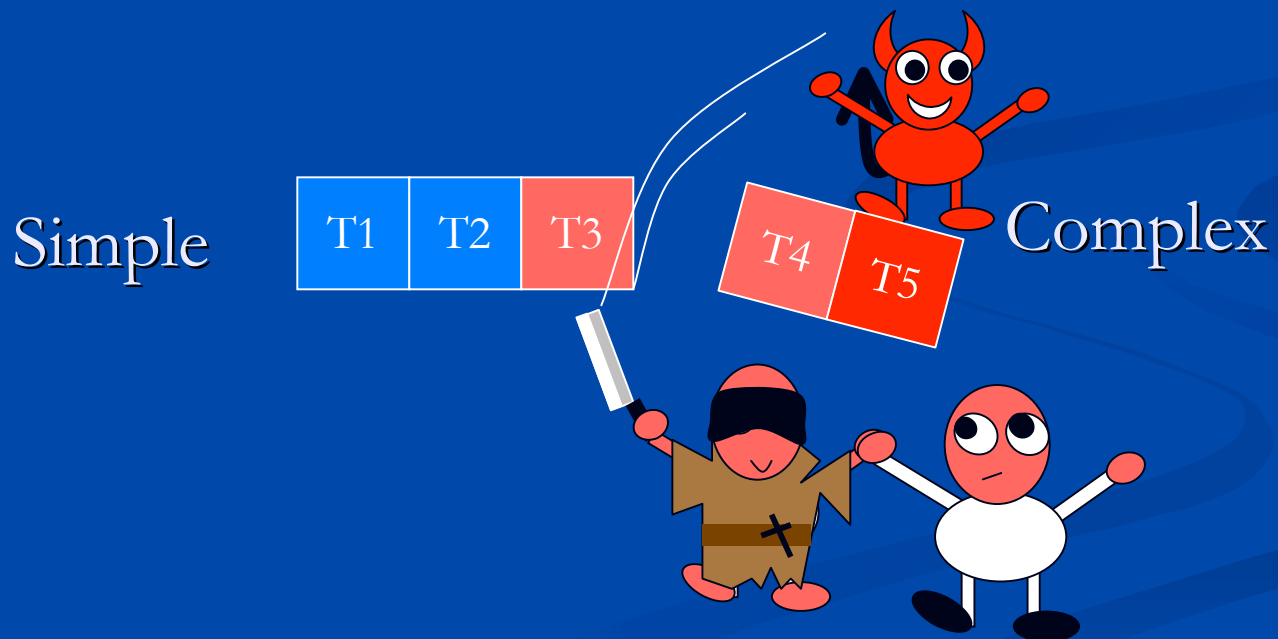
Dilemma

If you **know** the truth is simple,
then you **don't need** Ockham.



Dilemma

If you **don't know** the truth is simple,
then how could a **fixed simplicity bias** help you if the
truth is complex?



Puzzle

A **fixed** bias is like a **broken** thermometer.

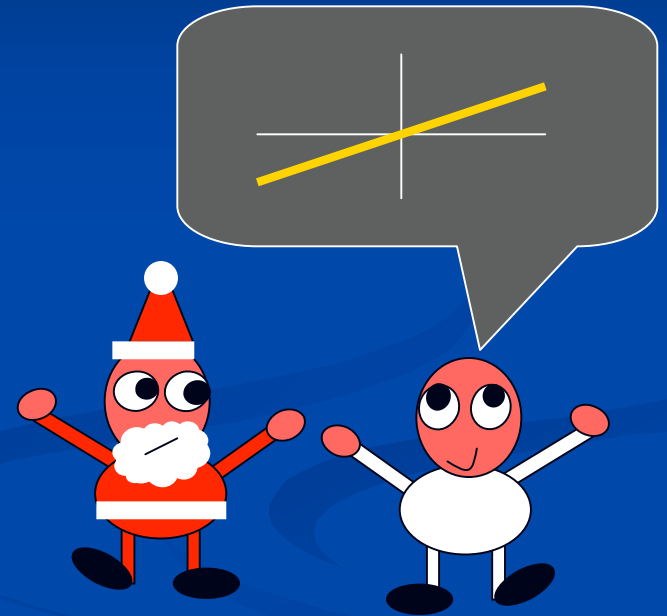
How could it possibly help you find **unknown** truth?



I. Ockham Apologists

Wishful Thinking

- Simple theories are nice if true:
 - Testability
 - Unity
 - Best explanation
 - Aesthetic appeal
 - Compress data
- So is believing that you are the emperor.

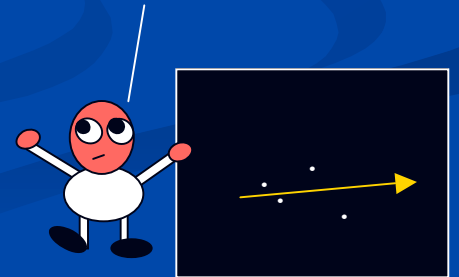


Overfitting

- Maximum likelihood estimates based on overly complex theories can have greater **predictive error** (AIC, Cross-validation, etc.).
 - Same is true even if you **know** the **true** model is **complex**.
 - Doesn't **converge** to true model.
 - Depends on **random data**.

The truth is complex.
-God-

Thanks, but a simpler
model still has lower
predictive error.



Ignorance = Knowledge

Messy worlds are **legion**

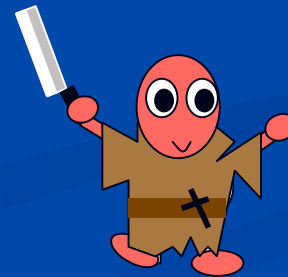
Tidy worlds are **few**.

That is why the tidy worlds

Are those most likely **true**. (Carnap)



unity



Ignorance = Knowledge

Messy worlds are **legion**

Tidy worlds are **few**.

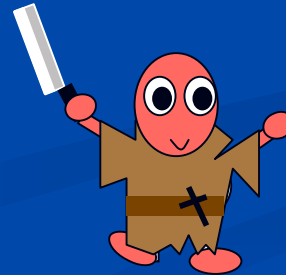
That is why the tidy worlds

Are those most likely **true**. (Carnap)

$1/3$



$1/3$



$1/3$



Ignorance = Knowledge

Messy worlds are **legion**

Tidy worlds are **few**.

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Are those most likely **true**. (Carnap)

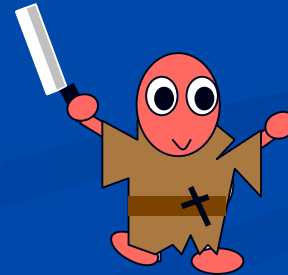
2/6



1/6



1/6



2/6



Depends on Notation

But mess depends on **coding**,
which Goodman noticed, too.

The picture is **inverted** if
we **translate** green to grue.

2/6



1/6

1/6



2/6

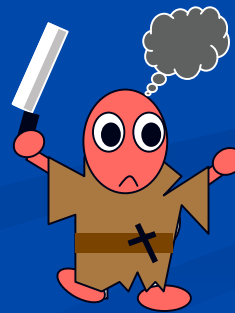


Notation
Indicates truth?



Same for Algorithmic Complexity

- **Goodman's problem** works against every **fixed** simplicity ranking (independent of the processes by which data are generated and coded prior to learning).
- **Extra problem:** any pair-wise ranking of theories can be **reversed** by choosing an **alternative computer language**.
- So how could simplicity help us find the true theory?



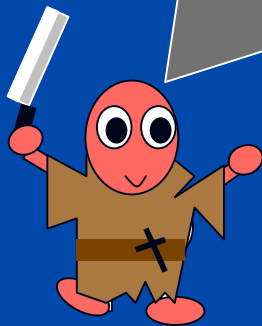
Notation
Indicates truth?



Just Beg the Question

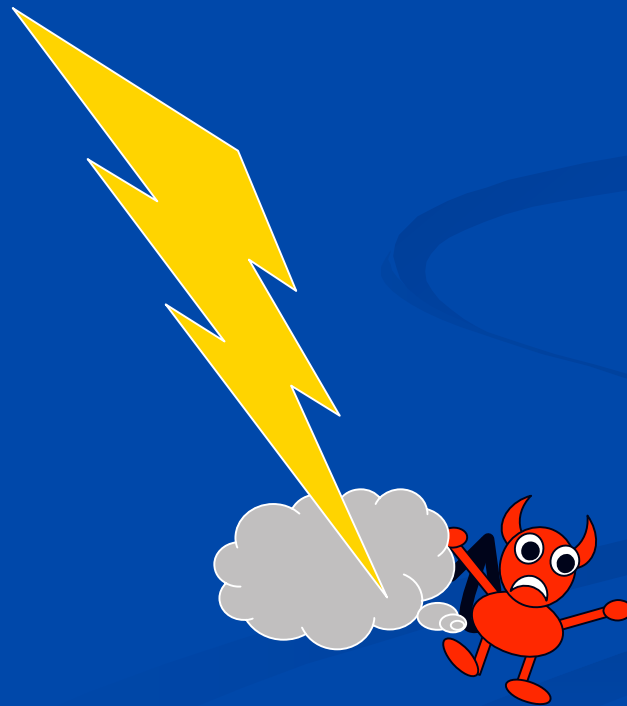
- **Assign** high prior probability to simple theories.
 - Why **should** you?
 - Preference for **complexity** has the same “explanation”.

You presume simplicity
Therefore you should presume simplicity!



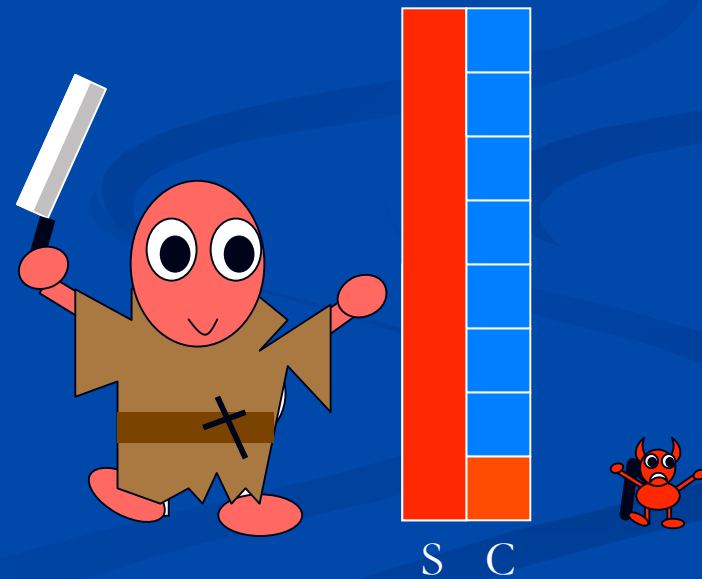
Miracle Argument

- Simple data would be a **miracle** if a complex theory were true (Bayes, BIC, Putnam).



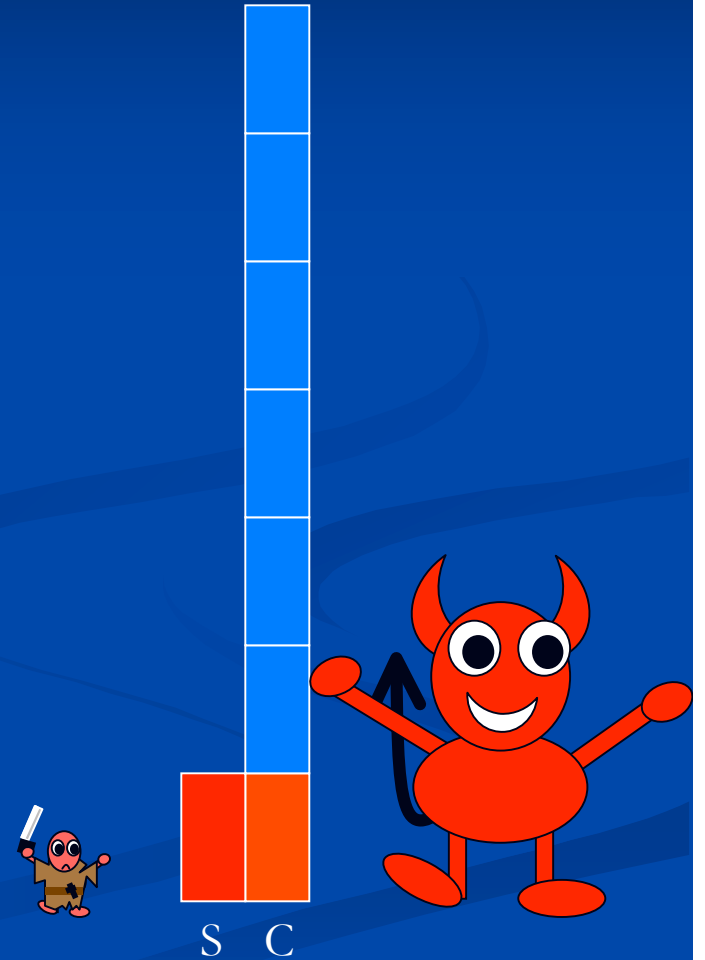
Begs the Question

- “Fairness” between theories →
bias against complex worlds.



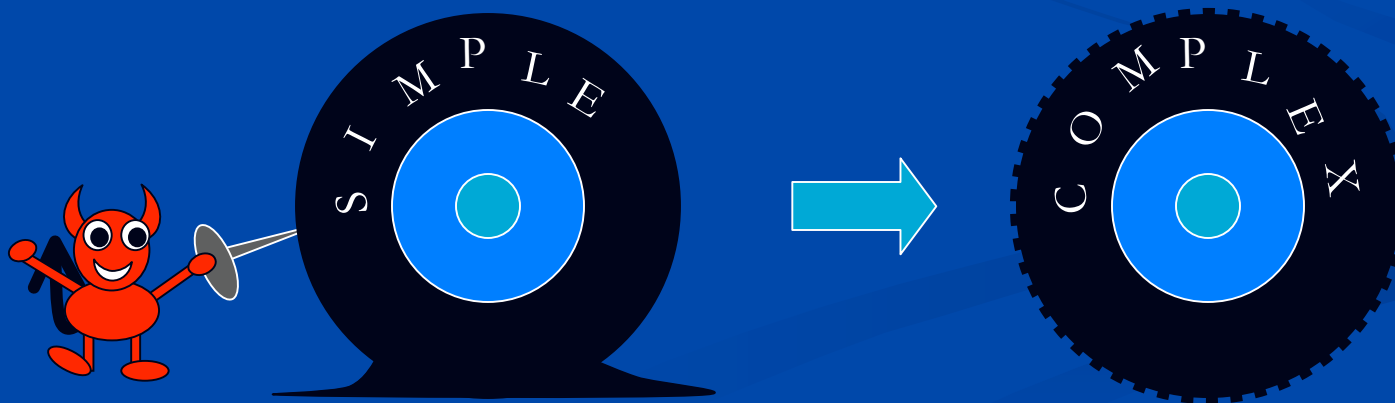
Two Can Play That Game

- “Fairness” between worlds →
bias against simple theory.



Convergence

- At least a simplicity bias **doesn't prevent convergence** to the truth (MDL, BIC, Bayes, SGS, etc.).
 - Neither do **other** biases.
 - May as well recommend **flat tires** since they can be fixed.



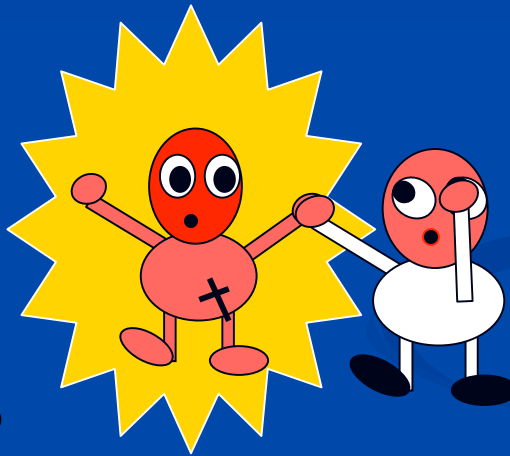
Does Ockham Have No Frock?

Ash Heap of History

Philosopher's stone,
Perpetual motion,
Free lunch

⋮

Ockham's Razor???



II. How Ockham Helps You Find the Truth

What is Guidance?

■ Indication or tracking

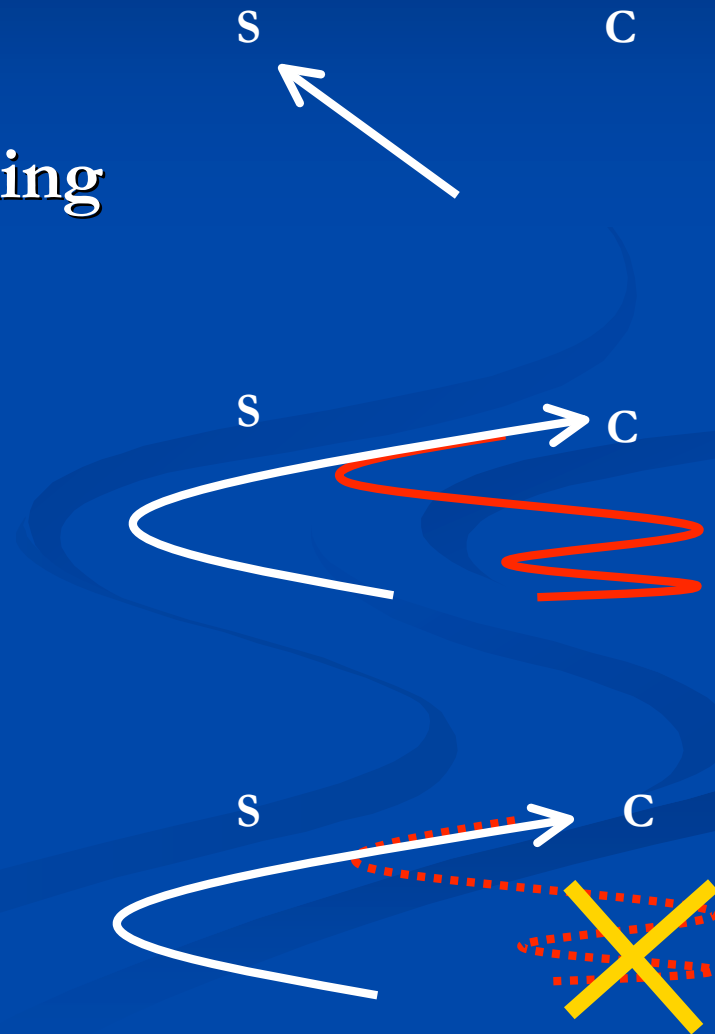
- Too *strong*
- *Fixed* bias can't *indicate* anything

■ Convergence

- Too *weak*
- True of other biases

■ “Straightest” convergence

- Just right?



A True Story



A True Story



A True Story



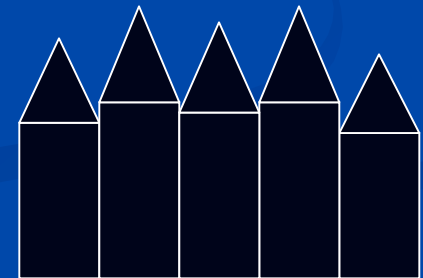
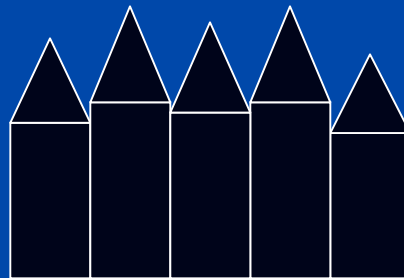
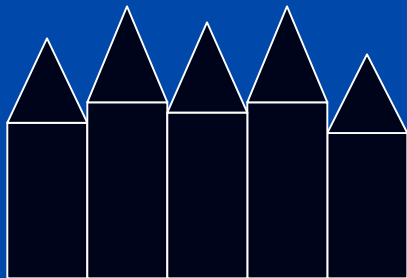
A True Story



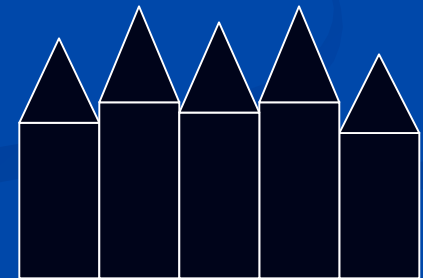
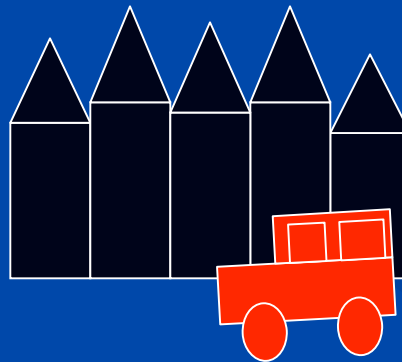
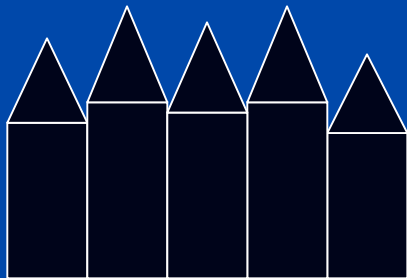
A True Story



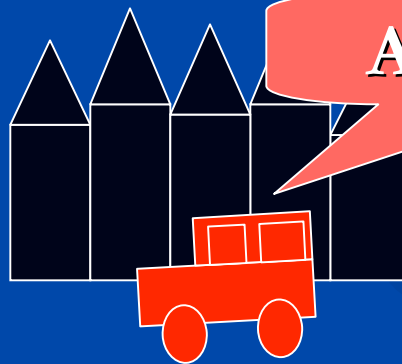
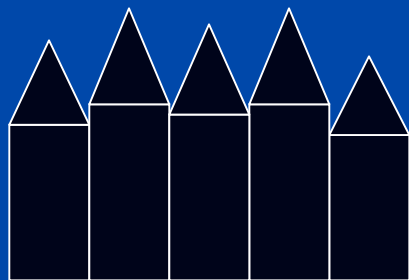
A True Story



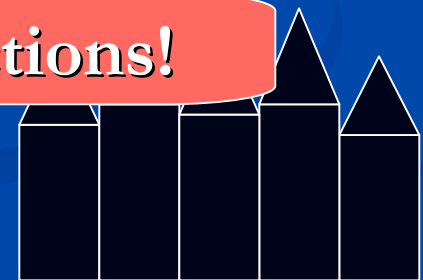
A True Story



A True Story



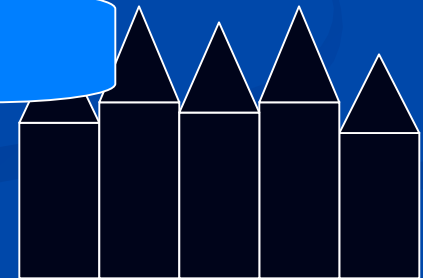
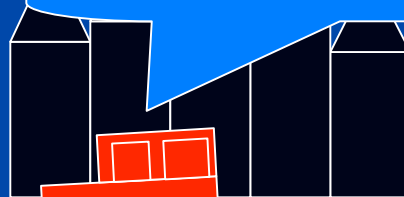
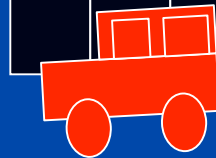
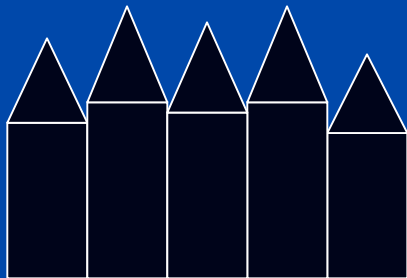
Ask directions!



A True Story

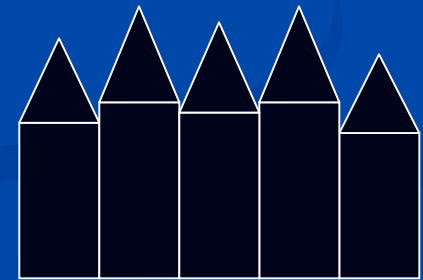
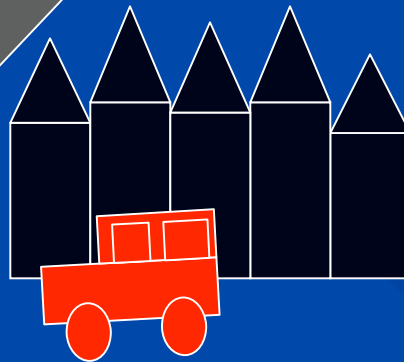
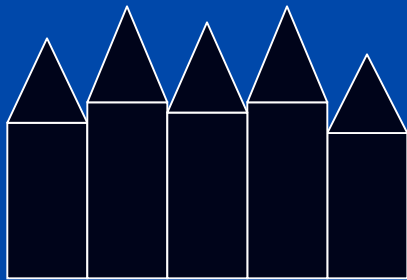


Where's ...



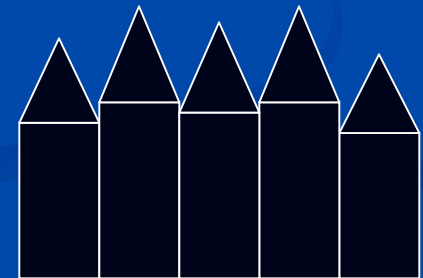
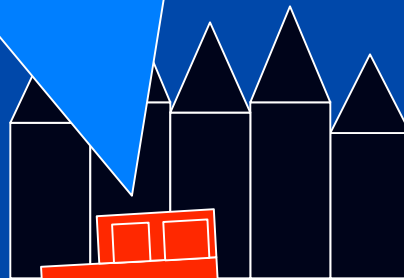
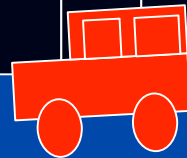
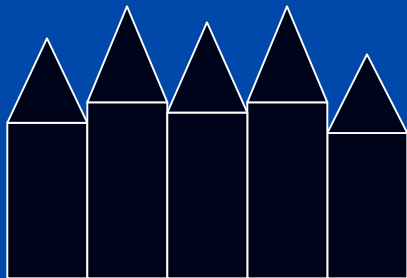
What Does She Say?

Turn around.
The freeway ramp
is on the left.

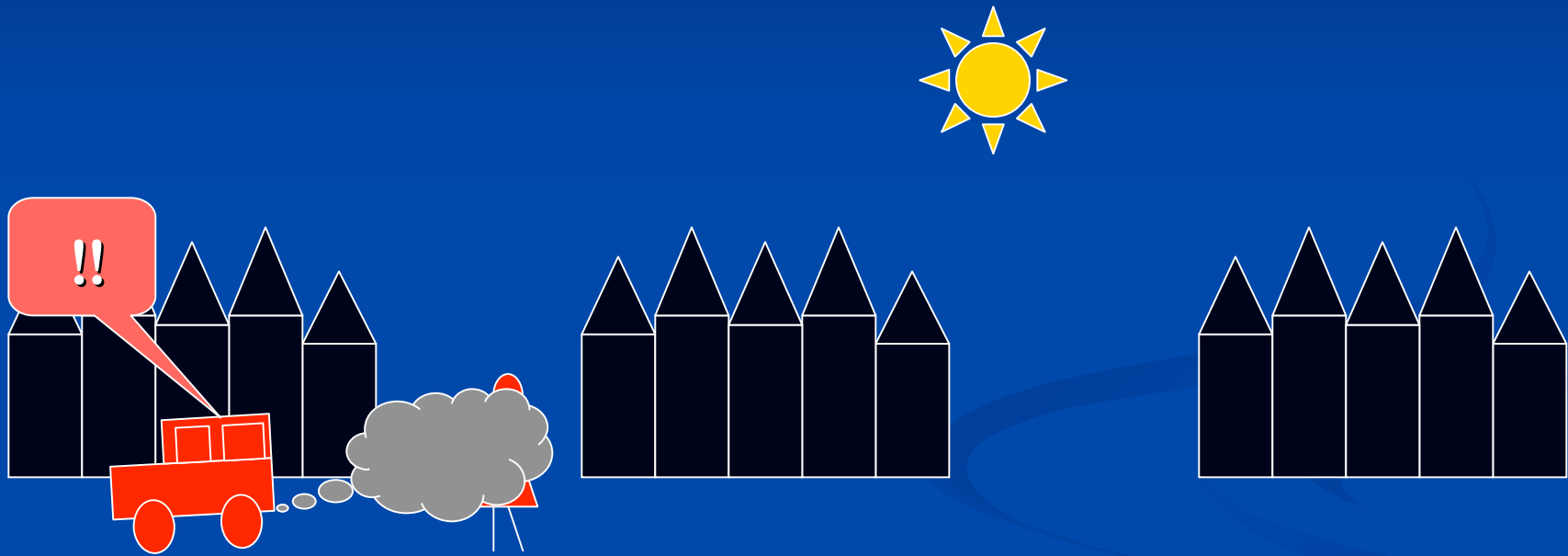


You Have Better Ideas

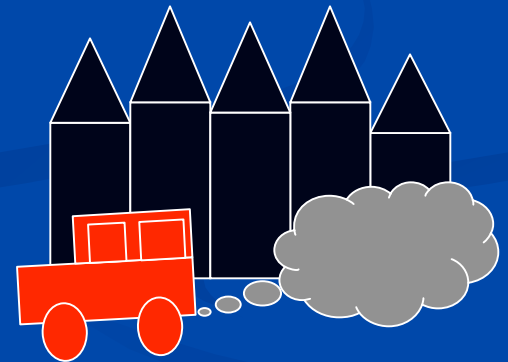
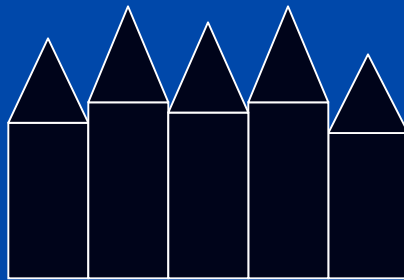
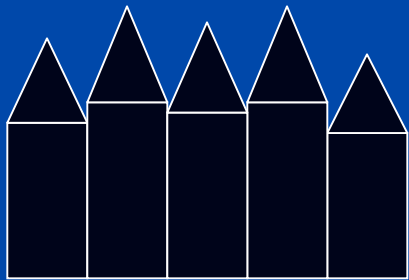
Phooey!
The Sun was on the right!



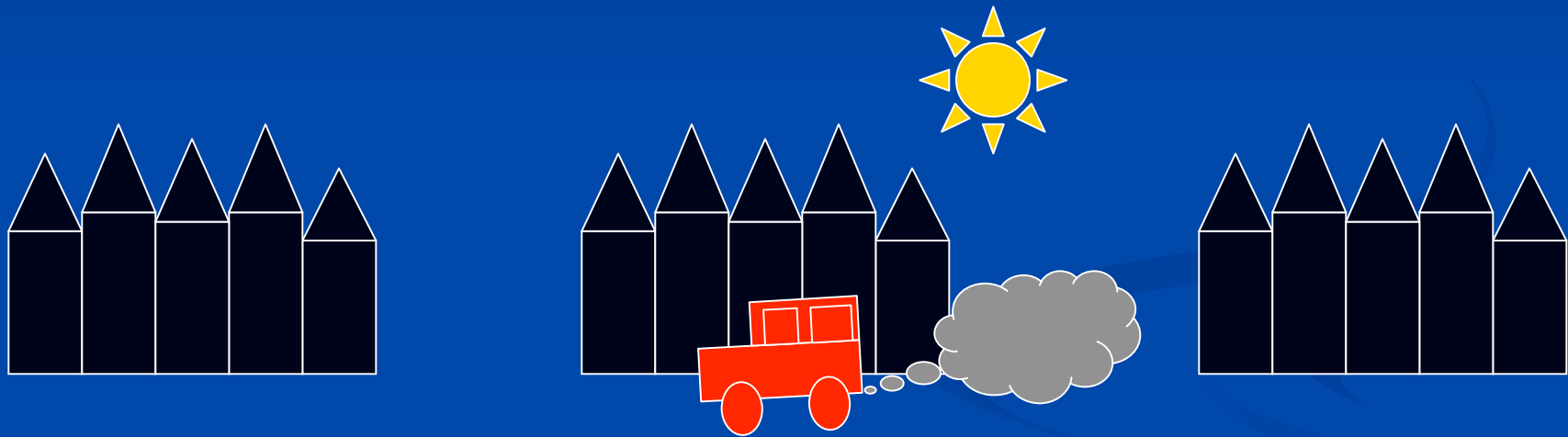
You Have Better Ideas



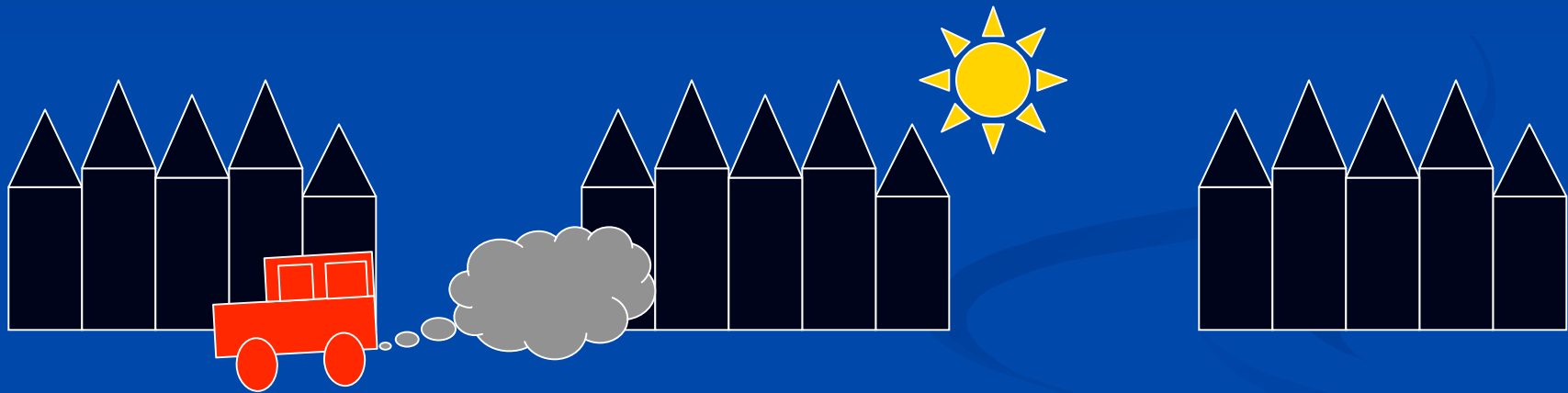
You Have Better Ideas



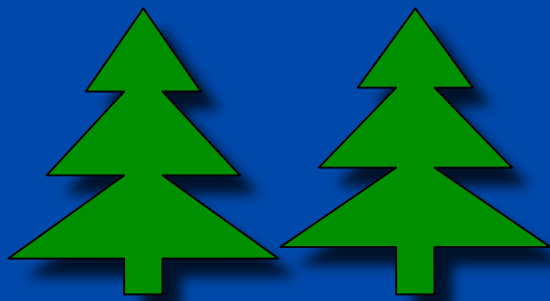
You Have Better Ideas



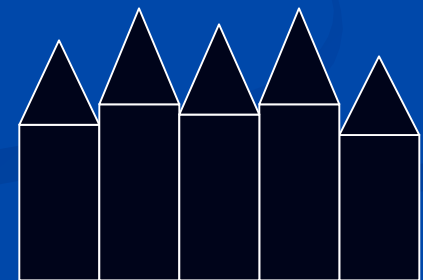
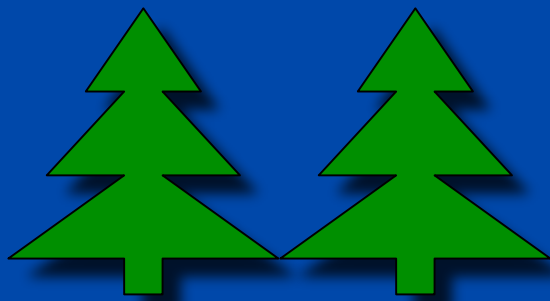
You Have Better Ideas



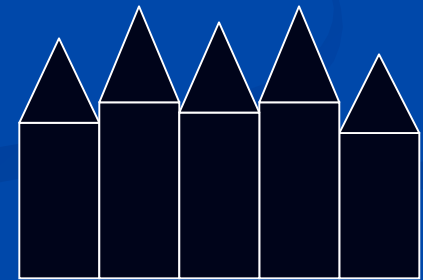
Stay the Course!



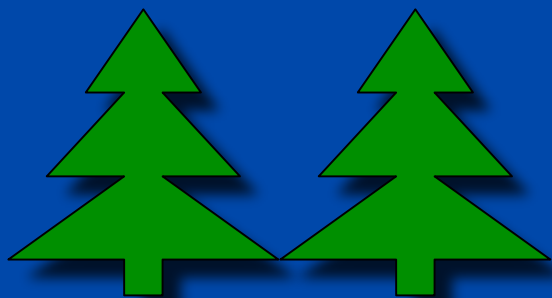
Stay the Course!



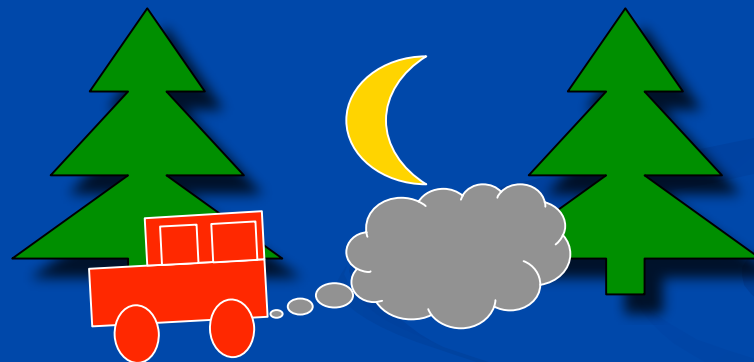
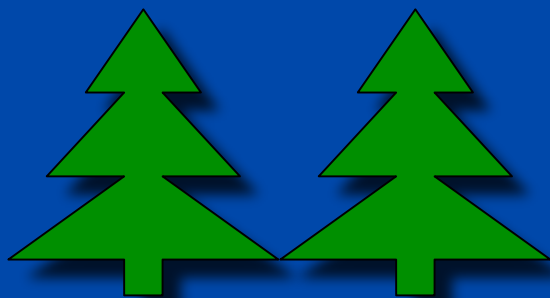
Stay the Course!



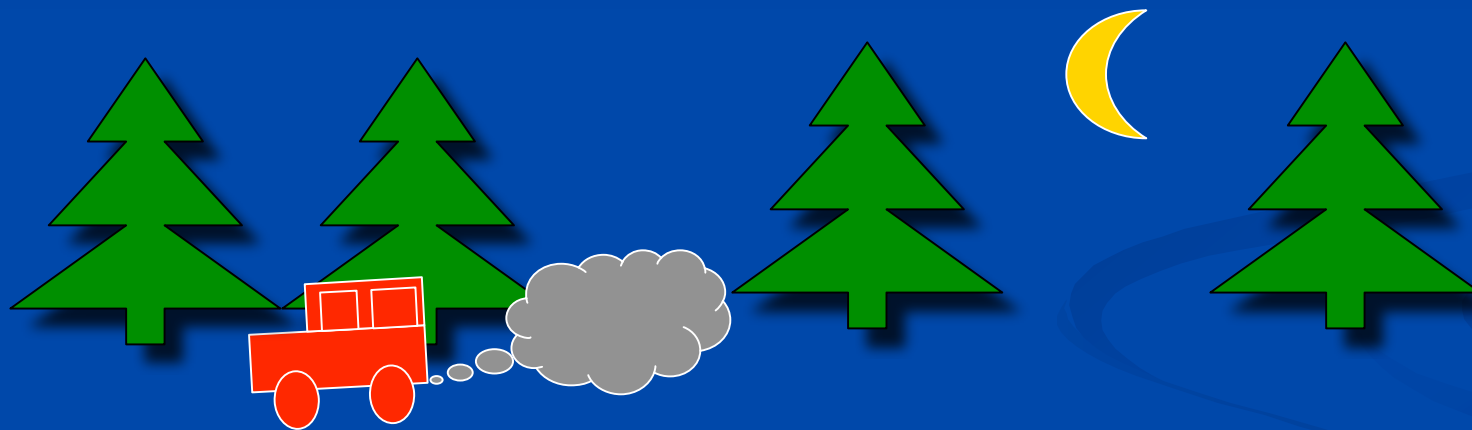
Don't Flip-flop!



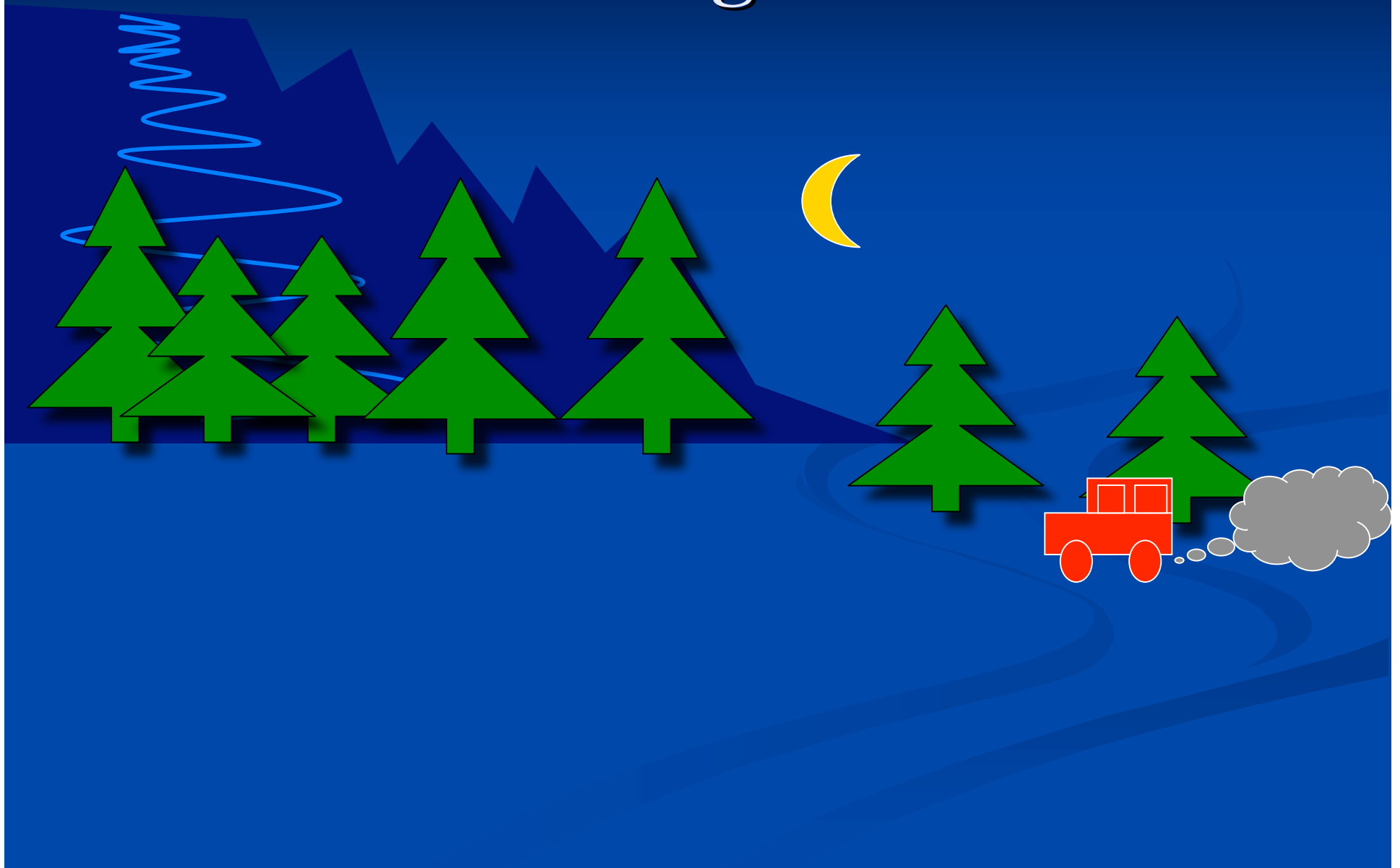
Don't Flip-flop!



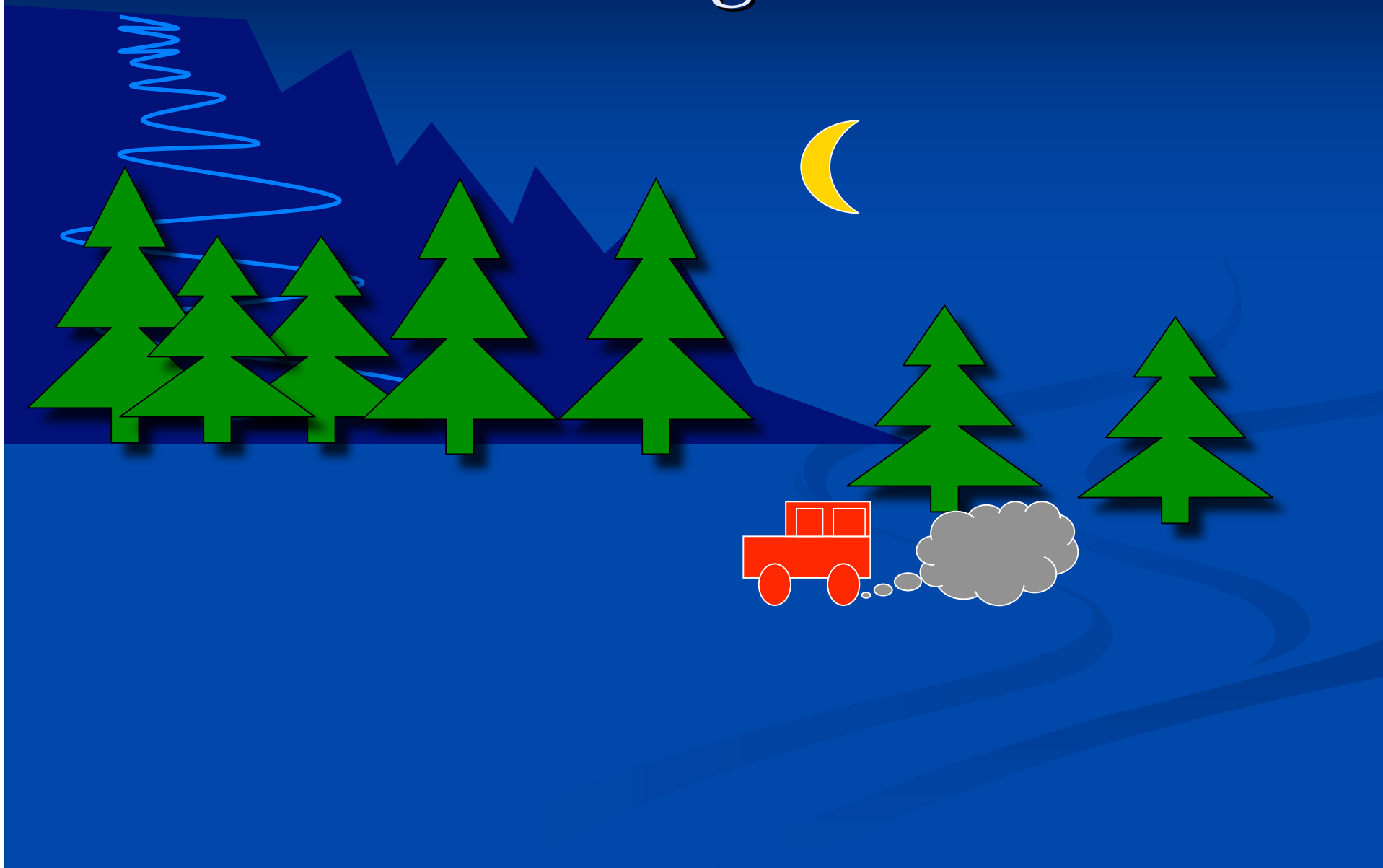
Don't Flip-flop!



Then Again...



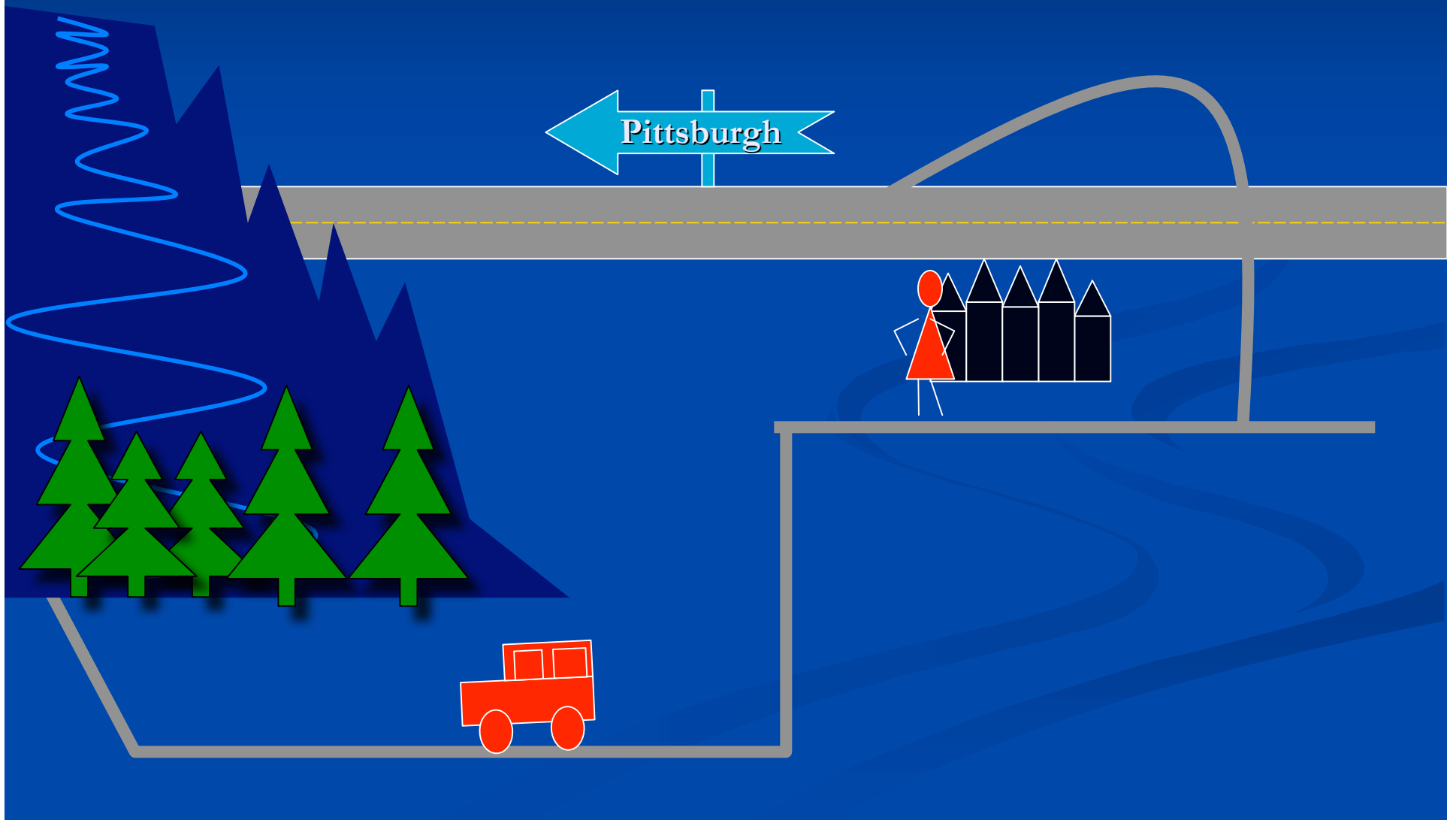
Then Again...



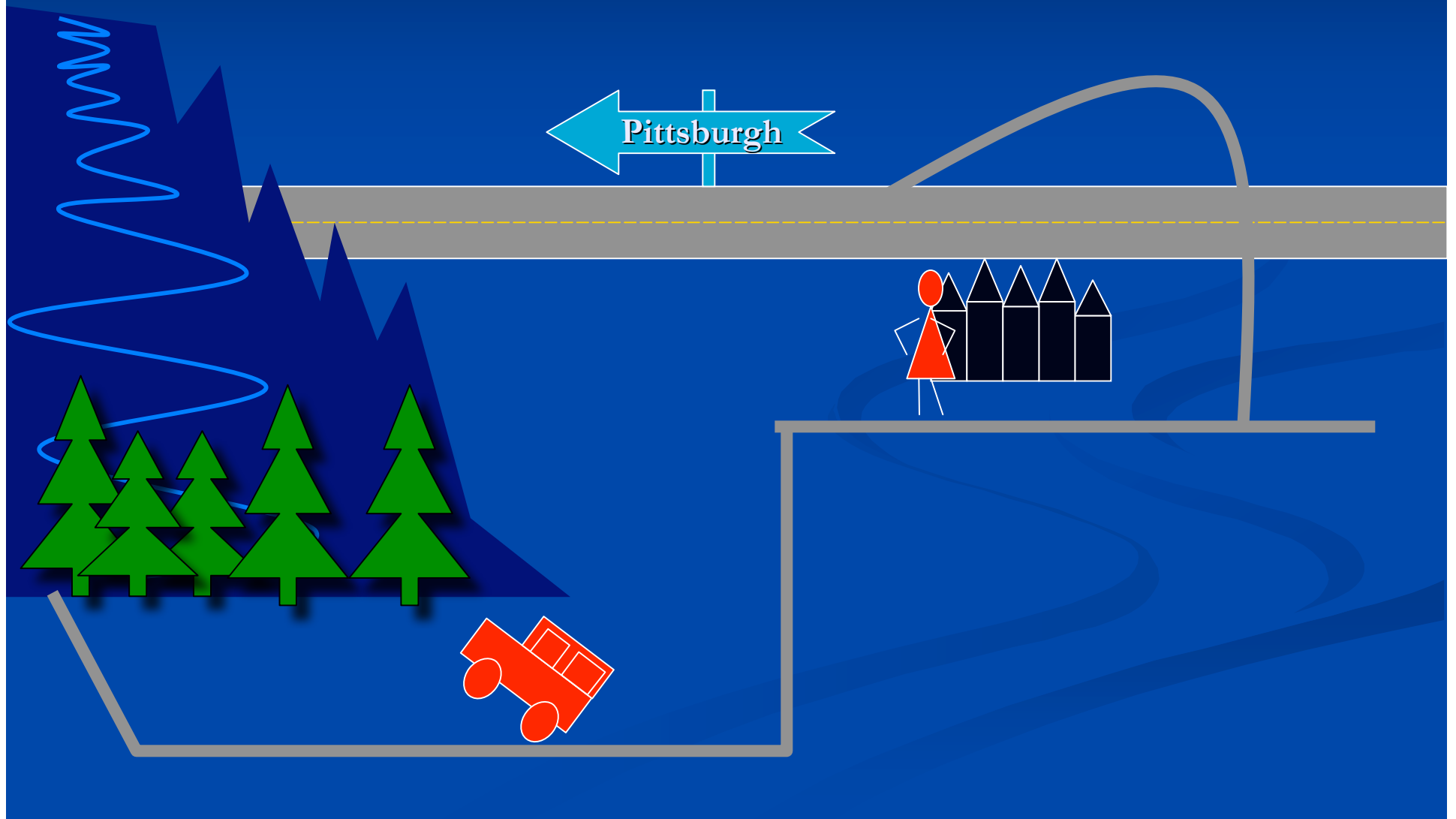
Then Again...



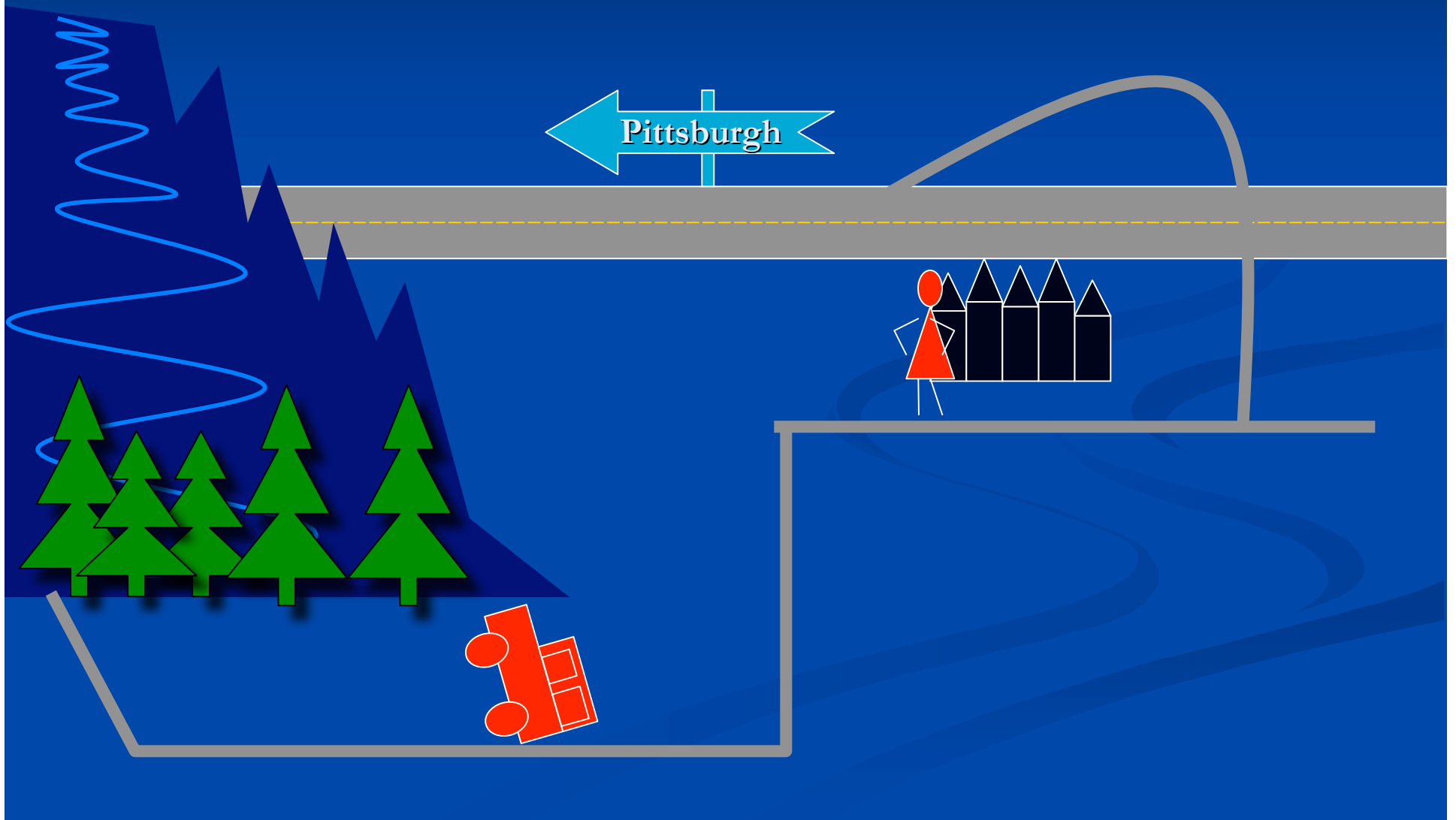
One Good Flip Can Save a Lot of Flop



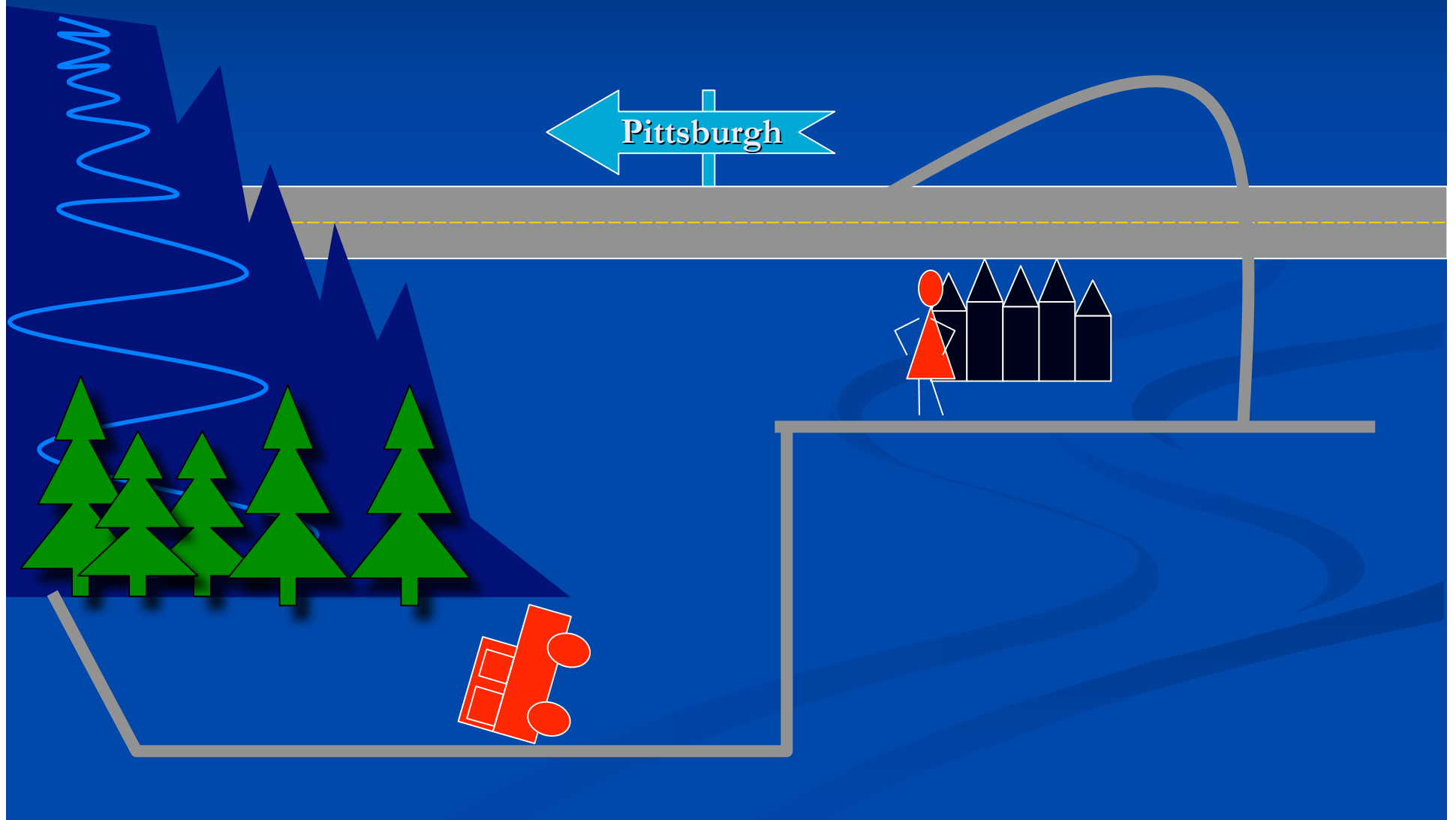
The U-Turn



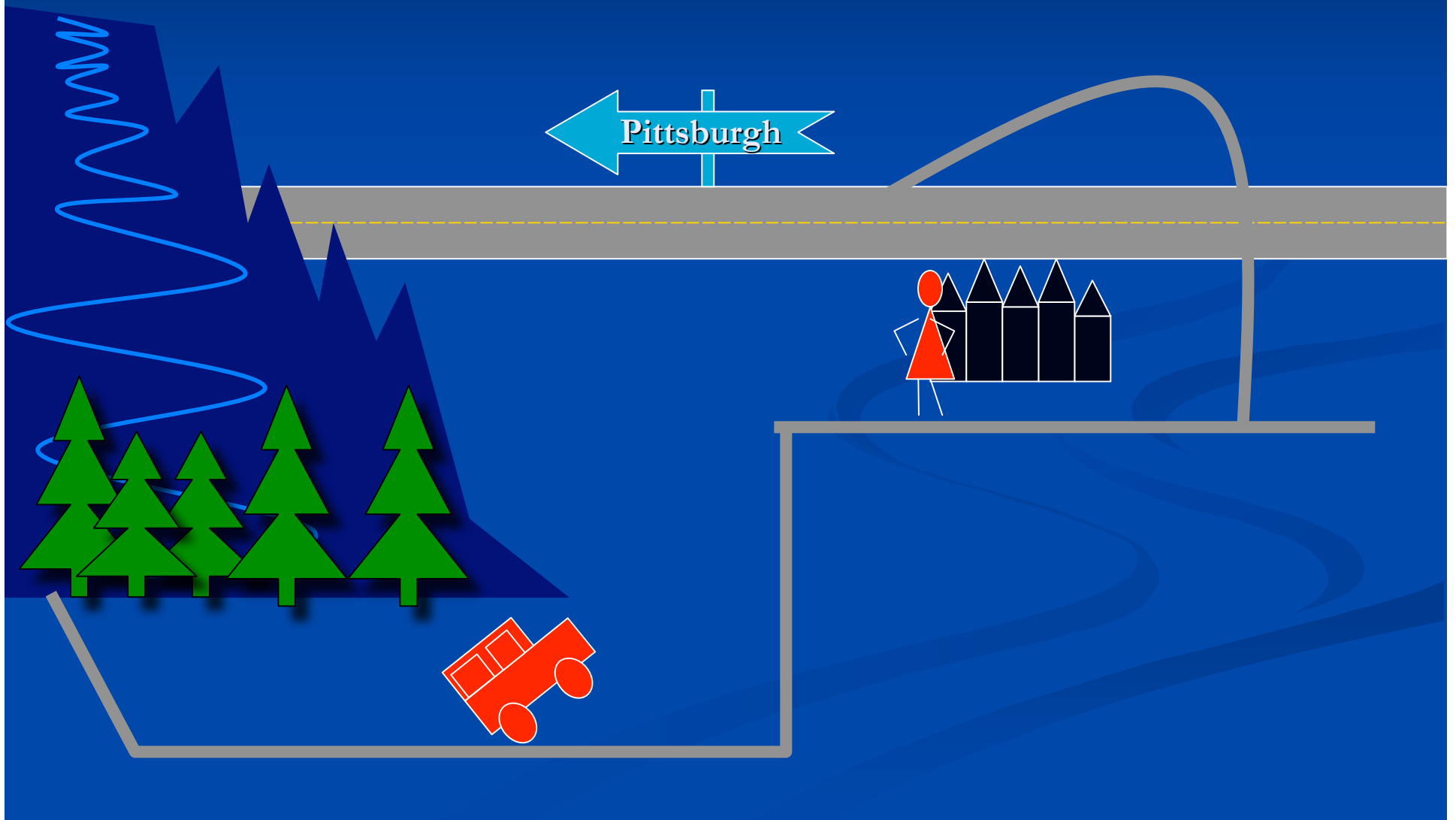
The U-Turn



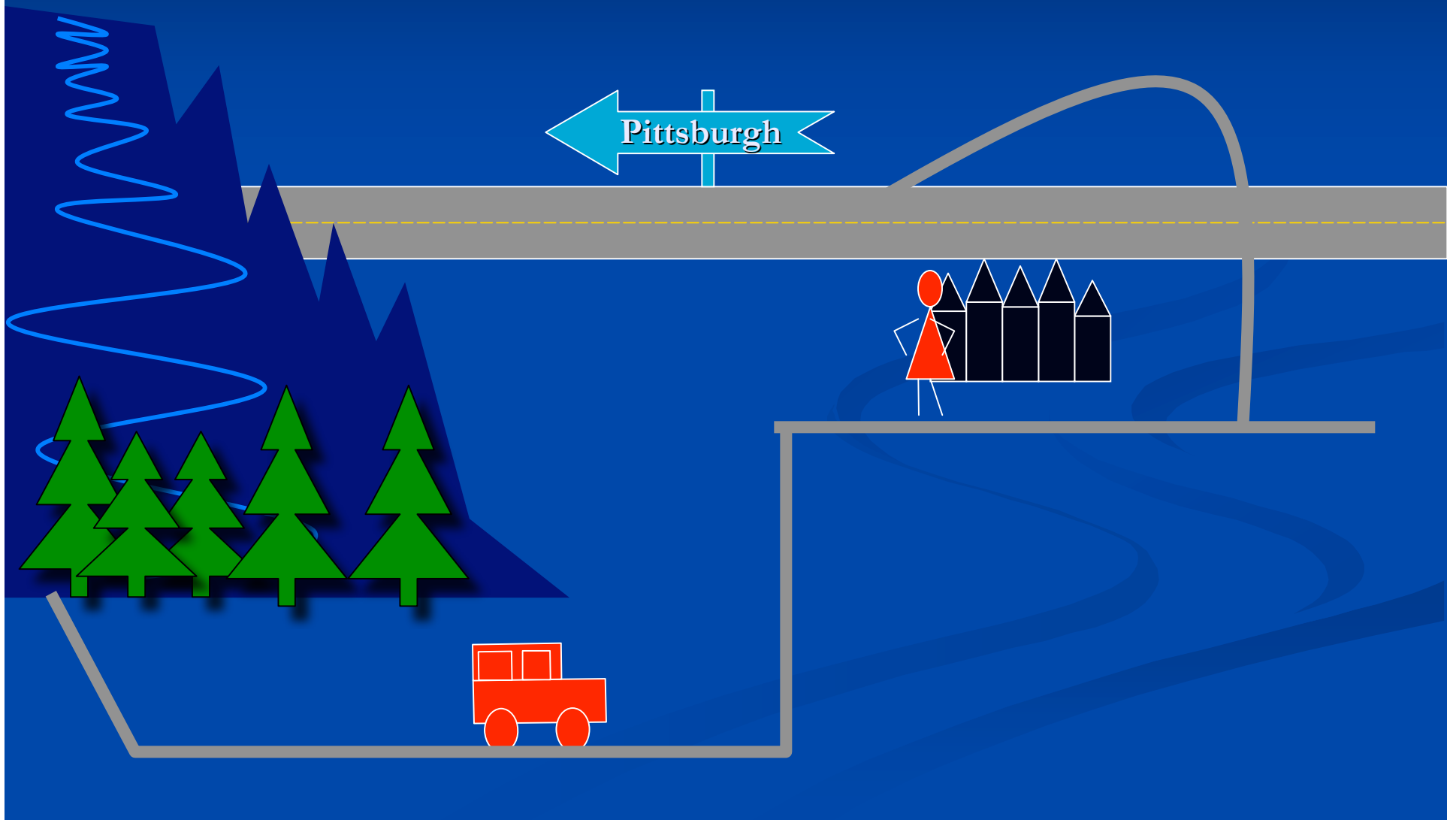
The U-Turn



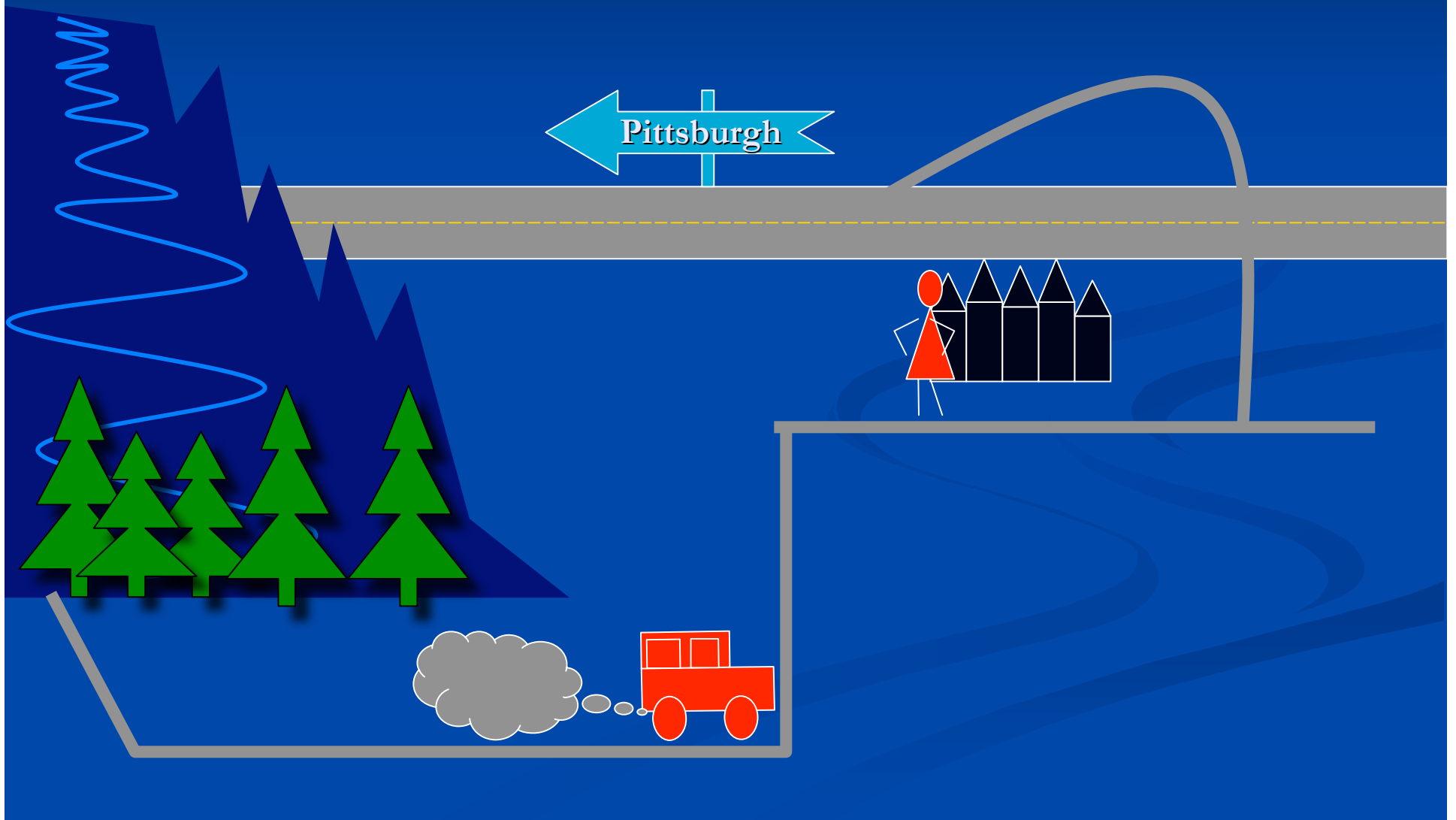
The U-Turn



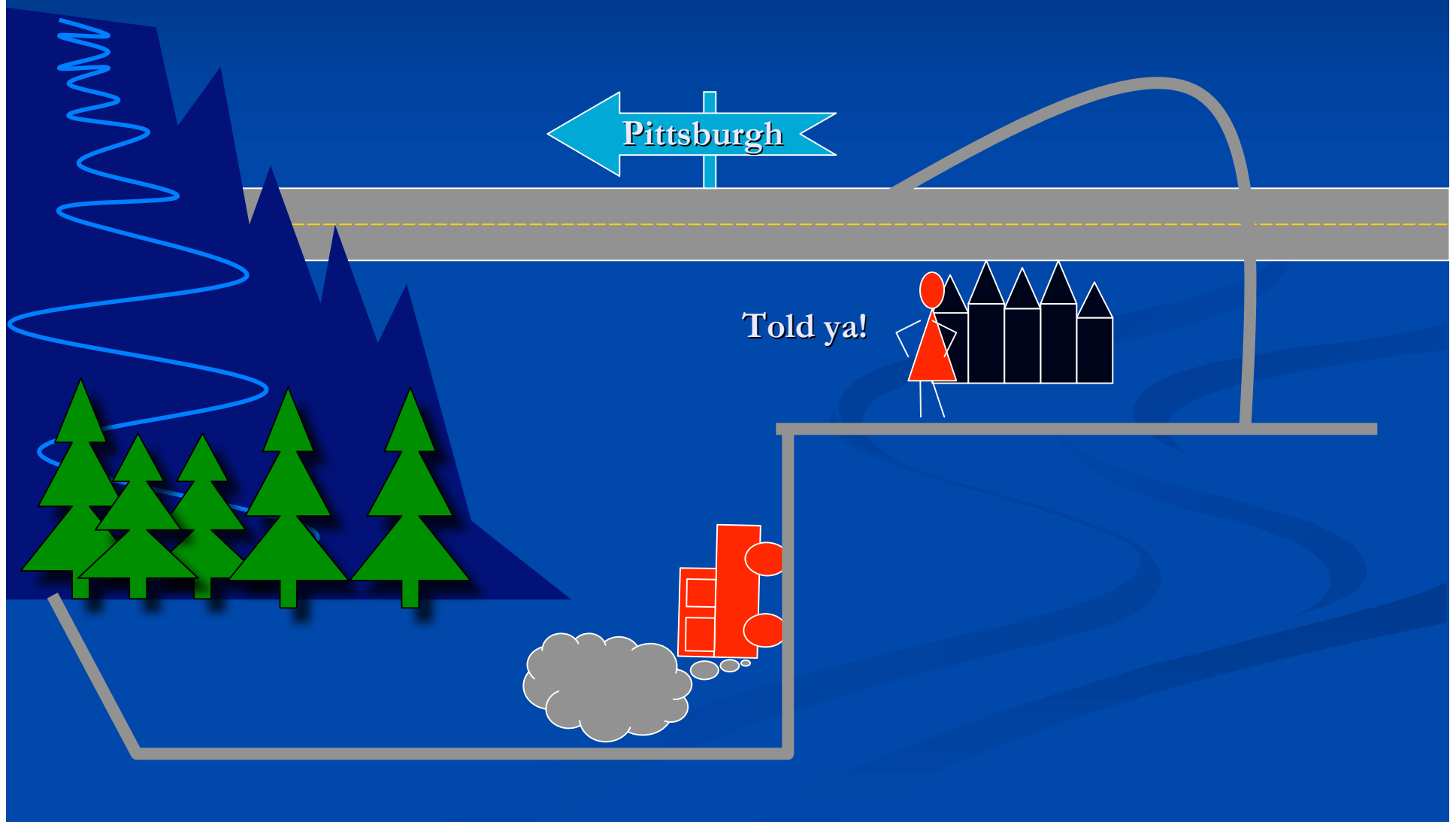
The U-Turn



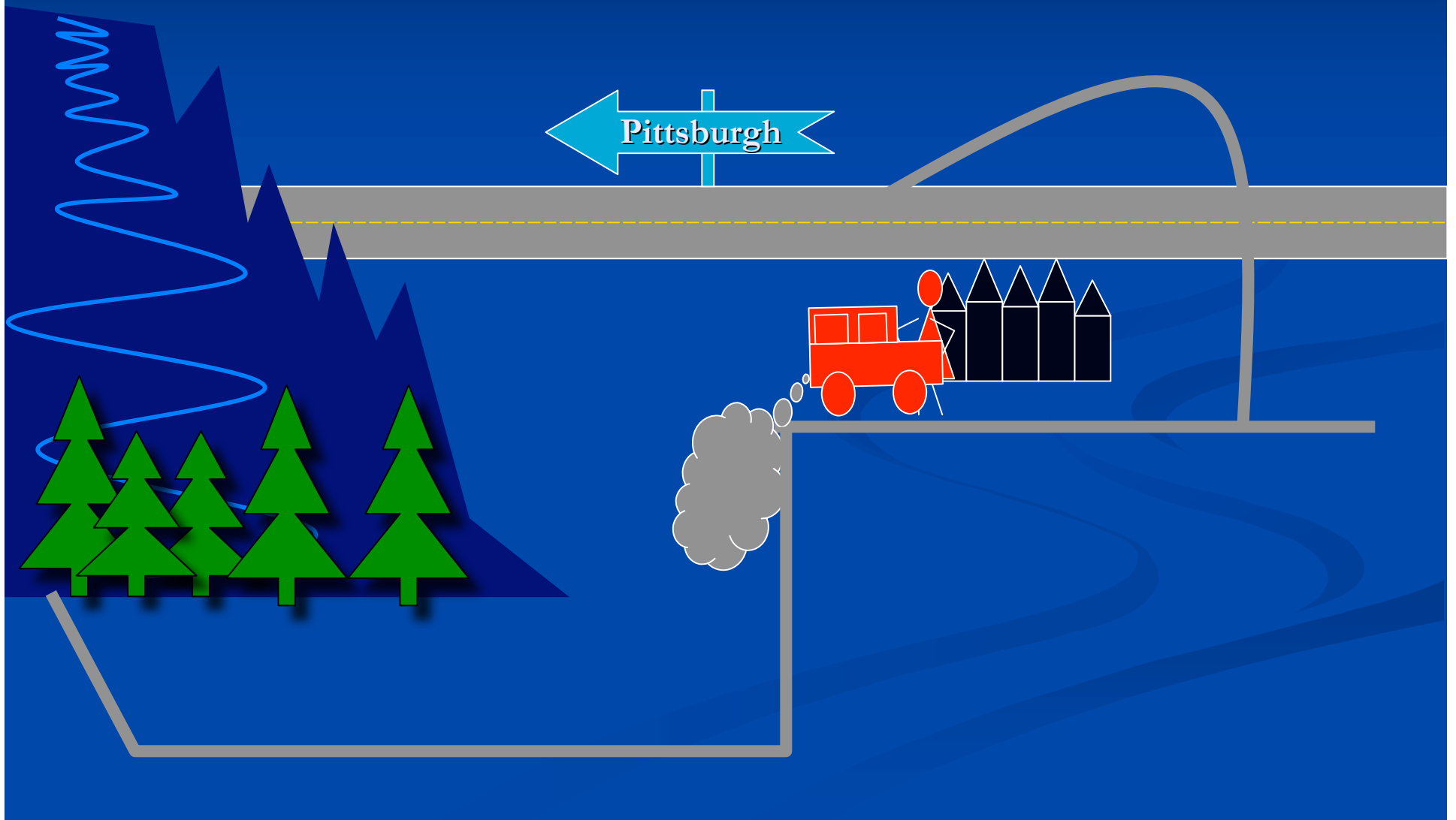
The U-Turn



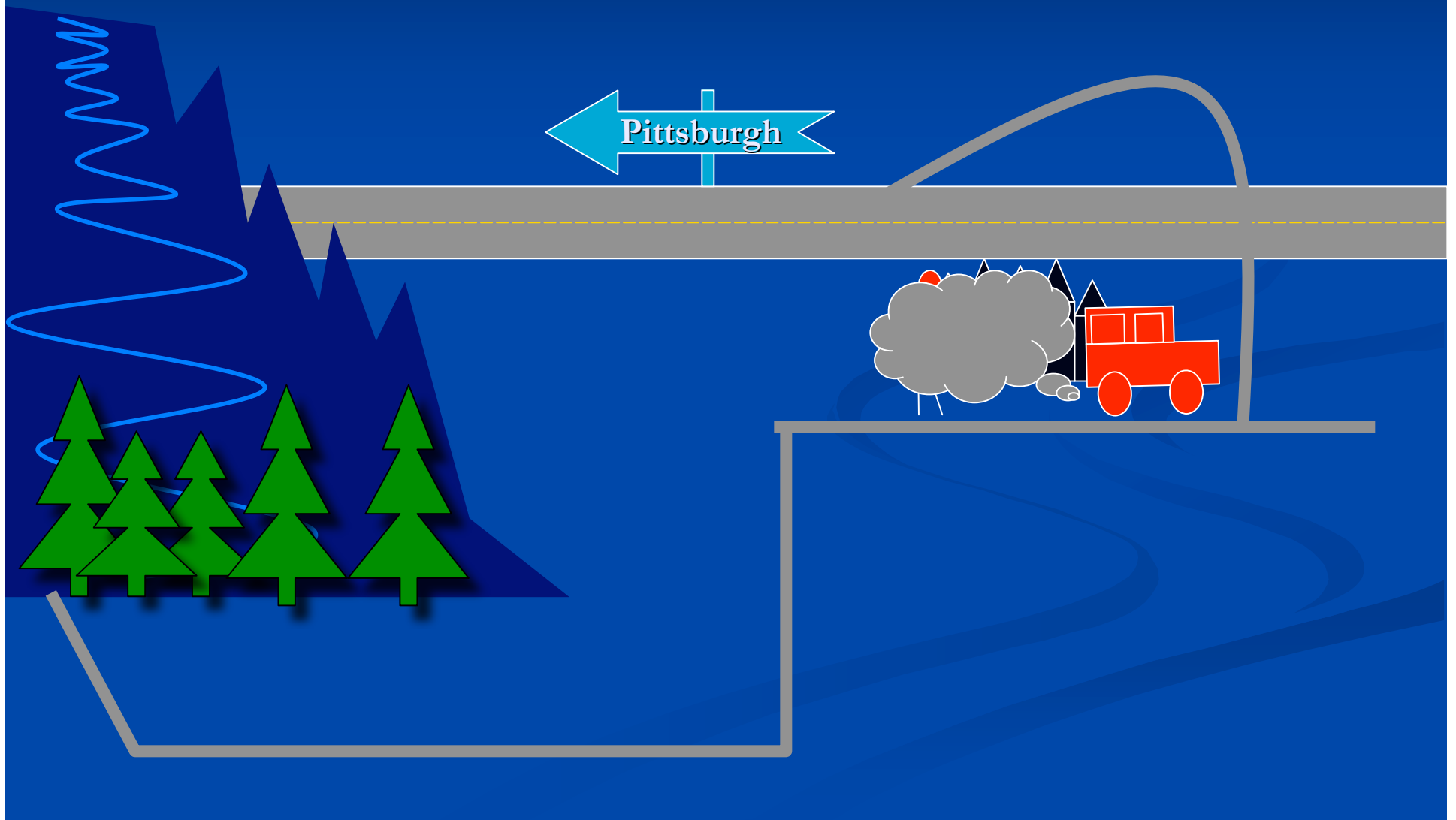
The U-Turn



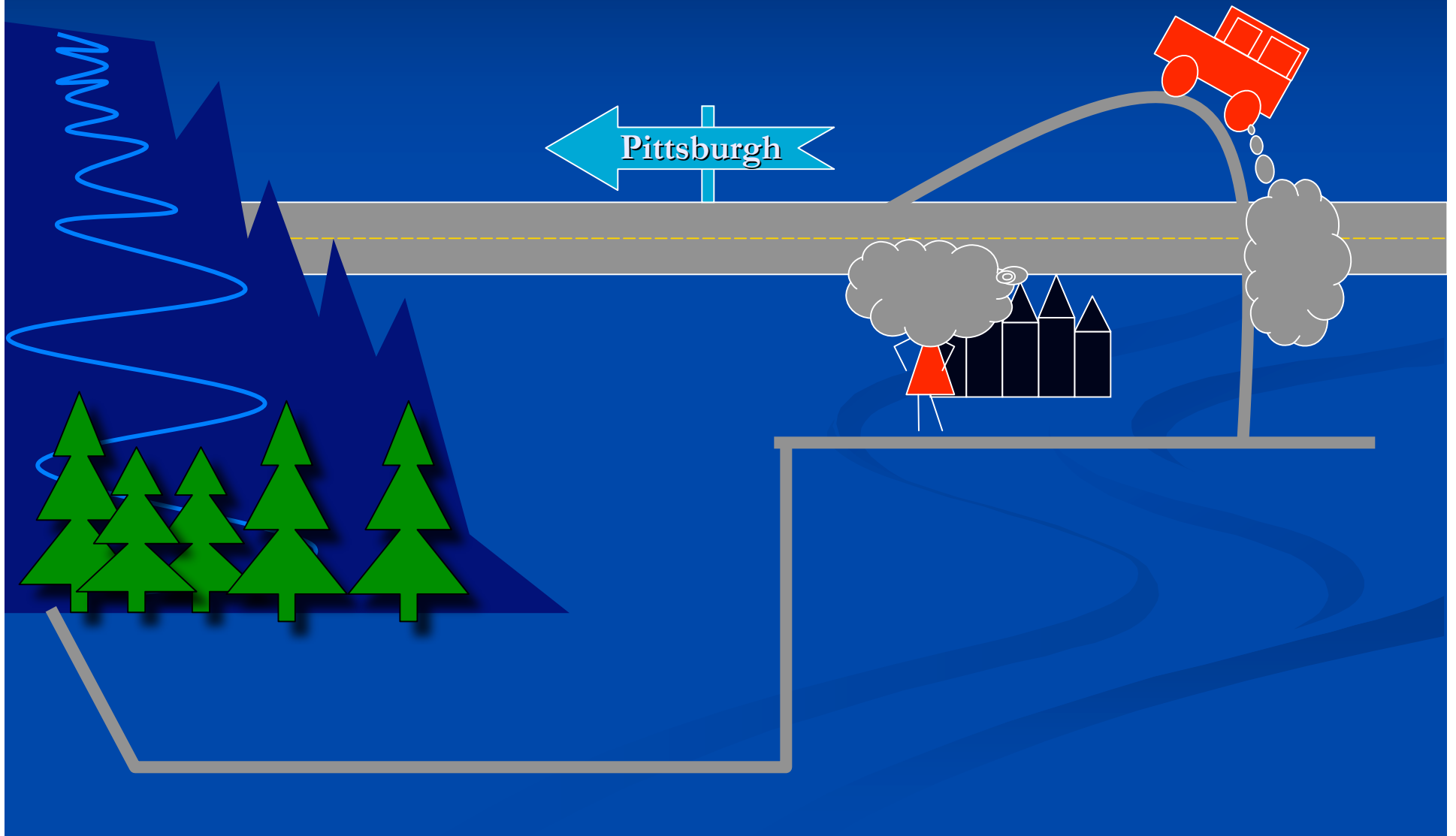
The U-Turn



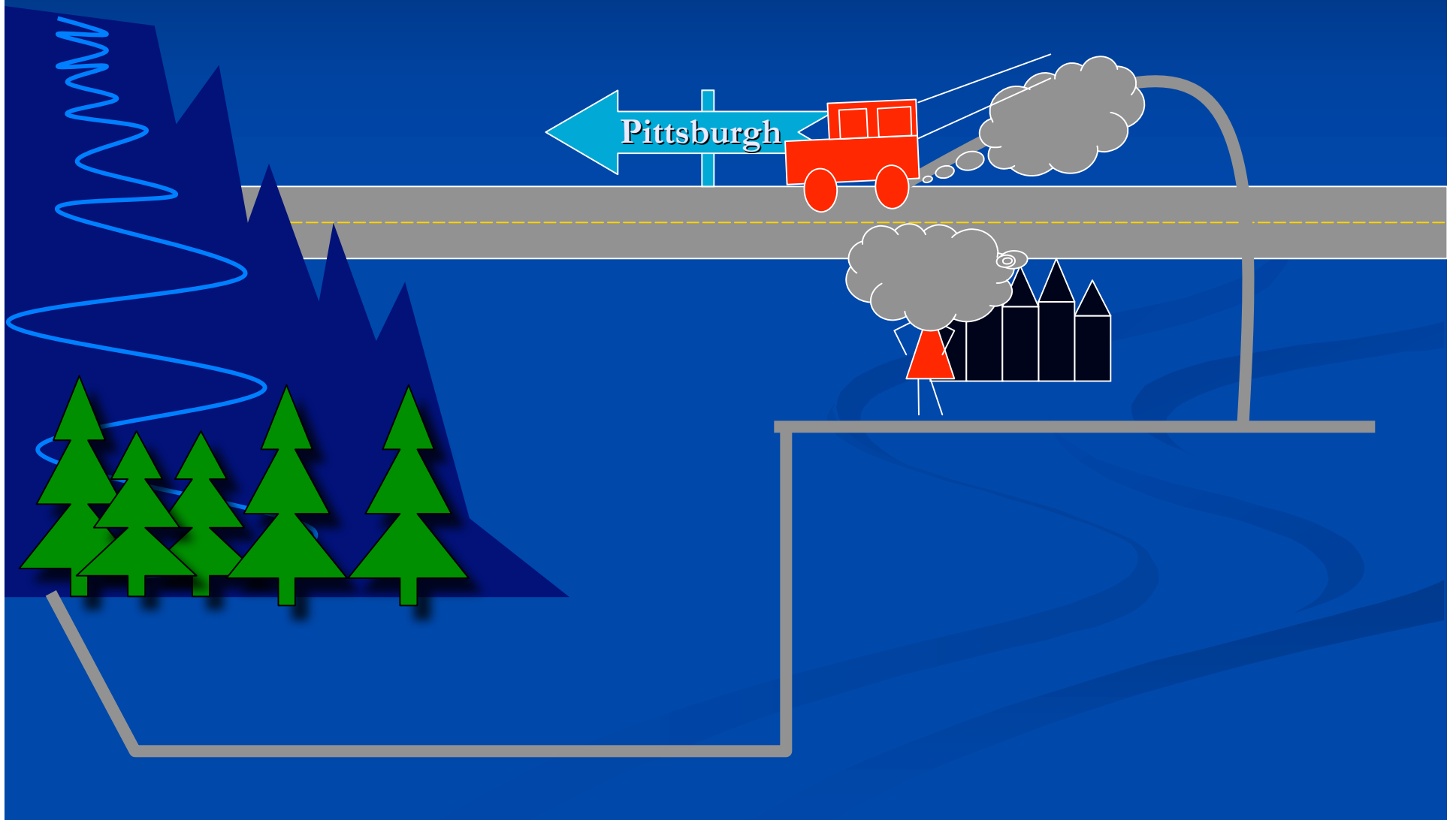
The U-Turn



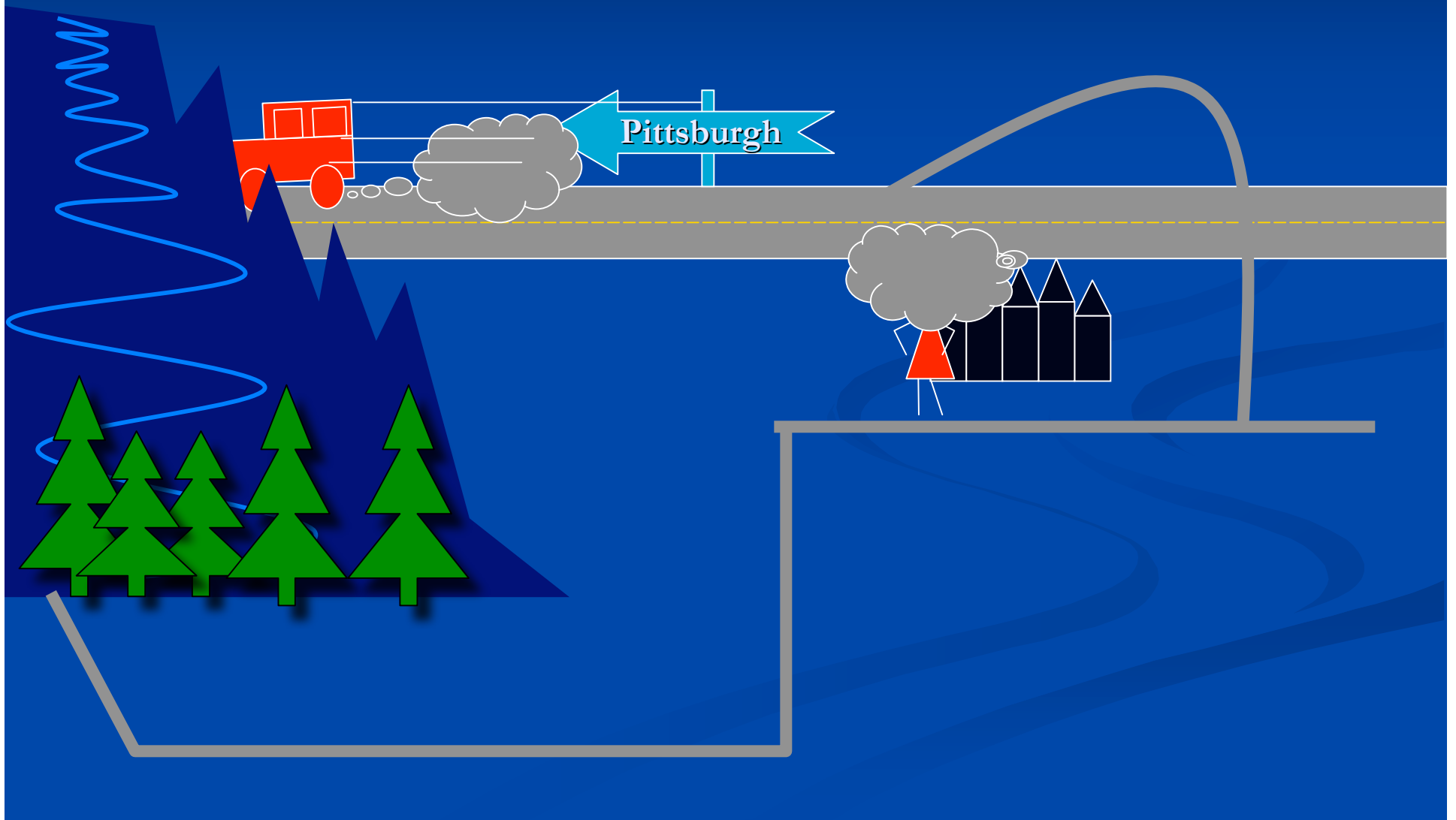
The U-Turn



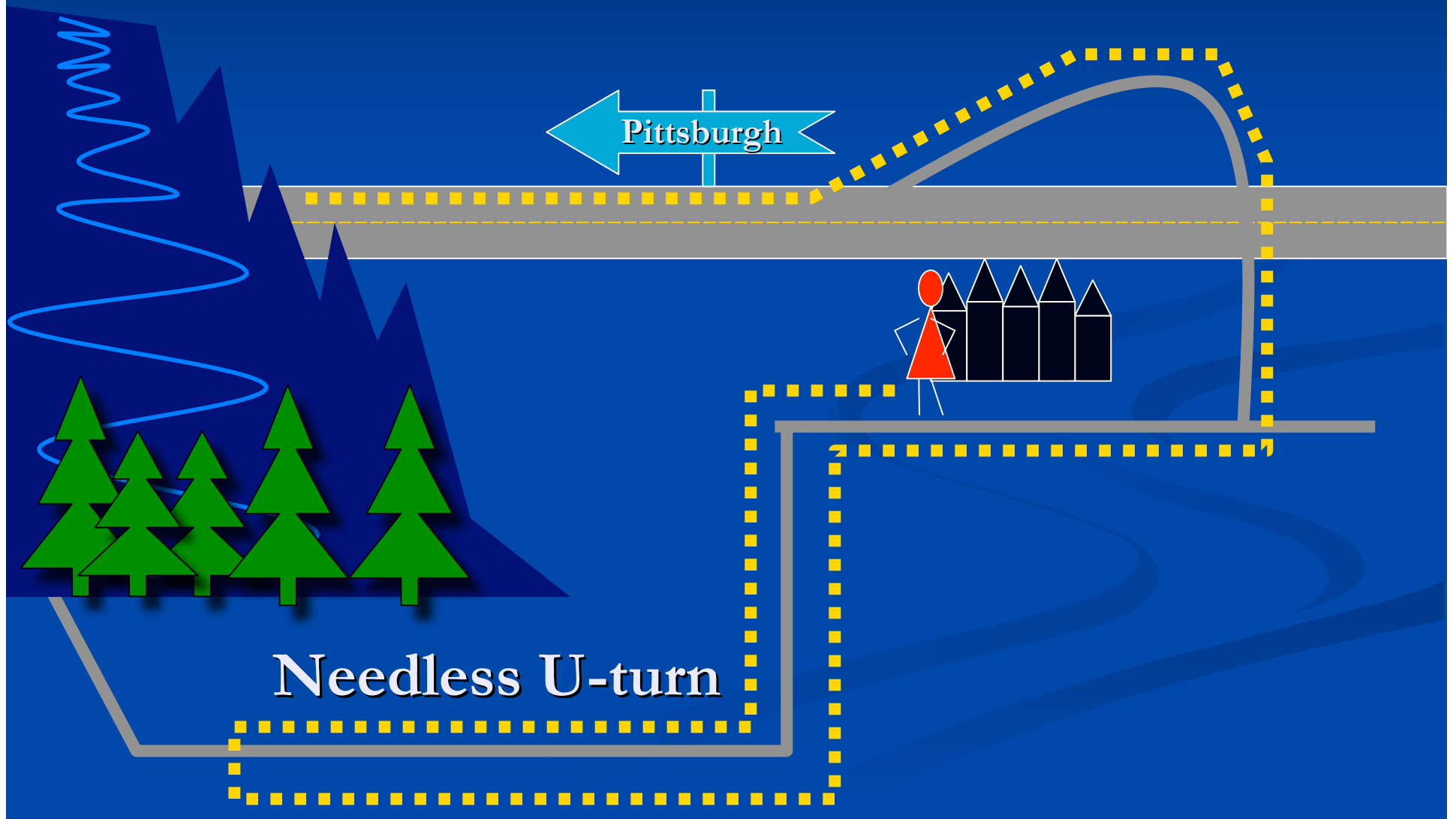
The U-Turn



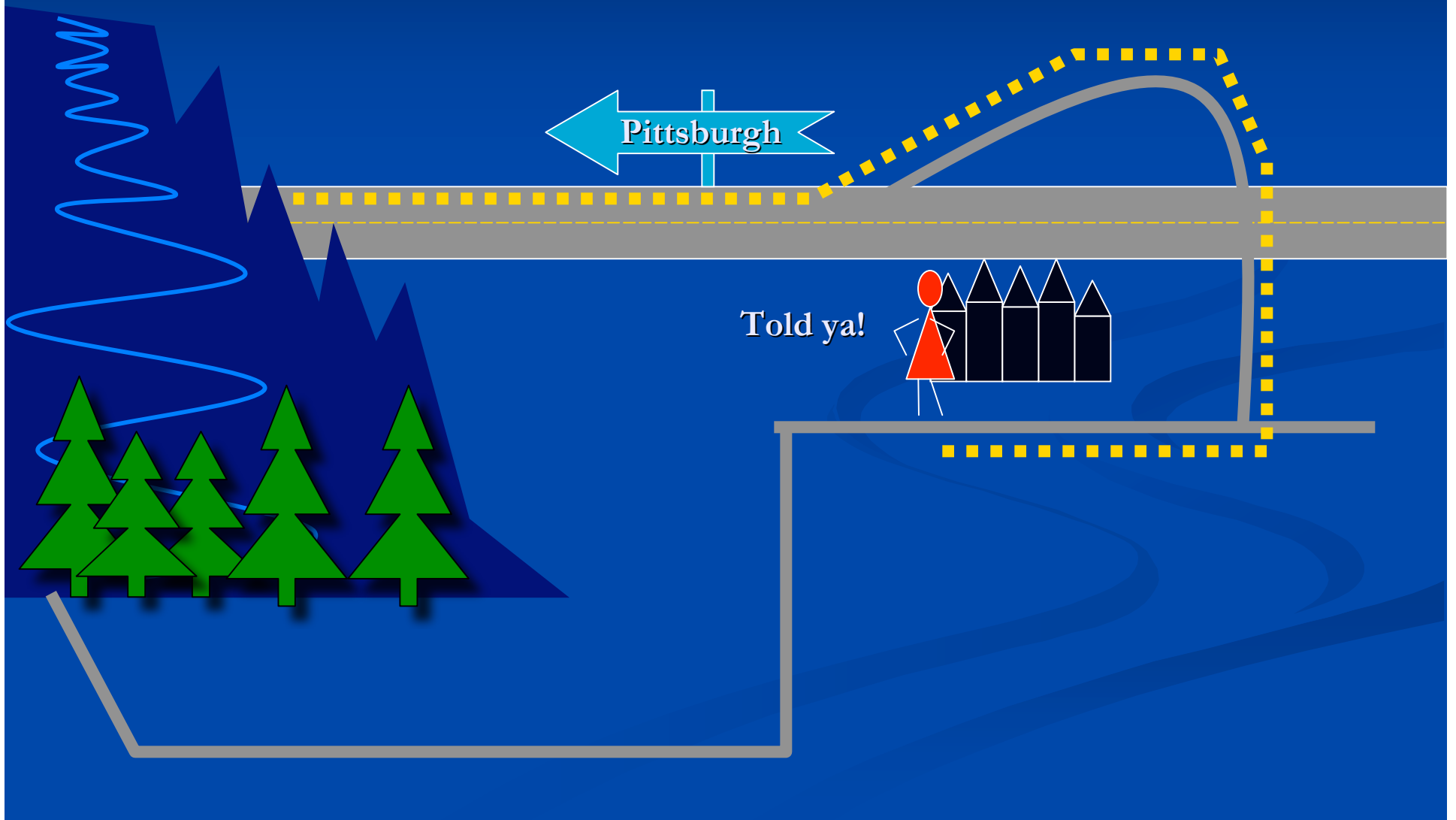
The U-Turn



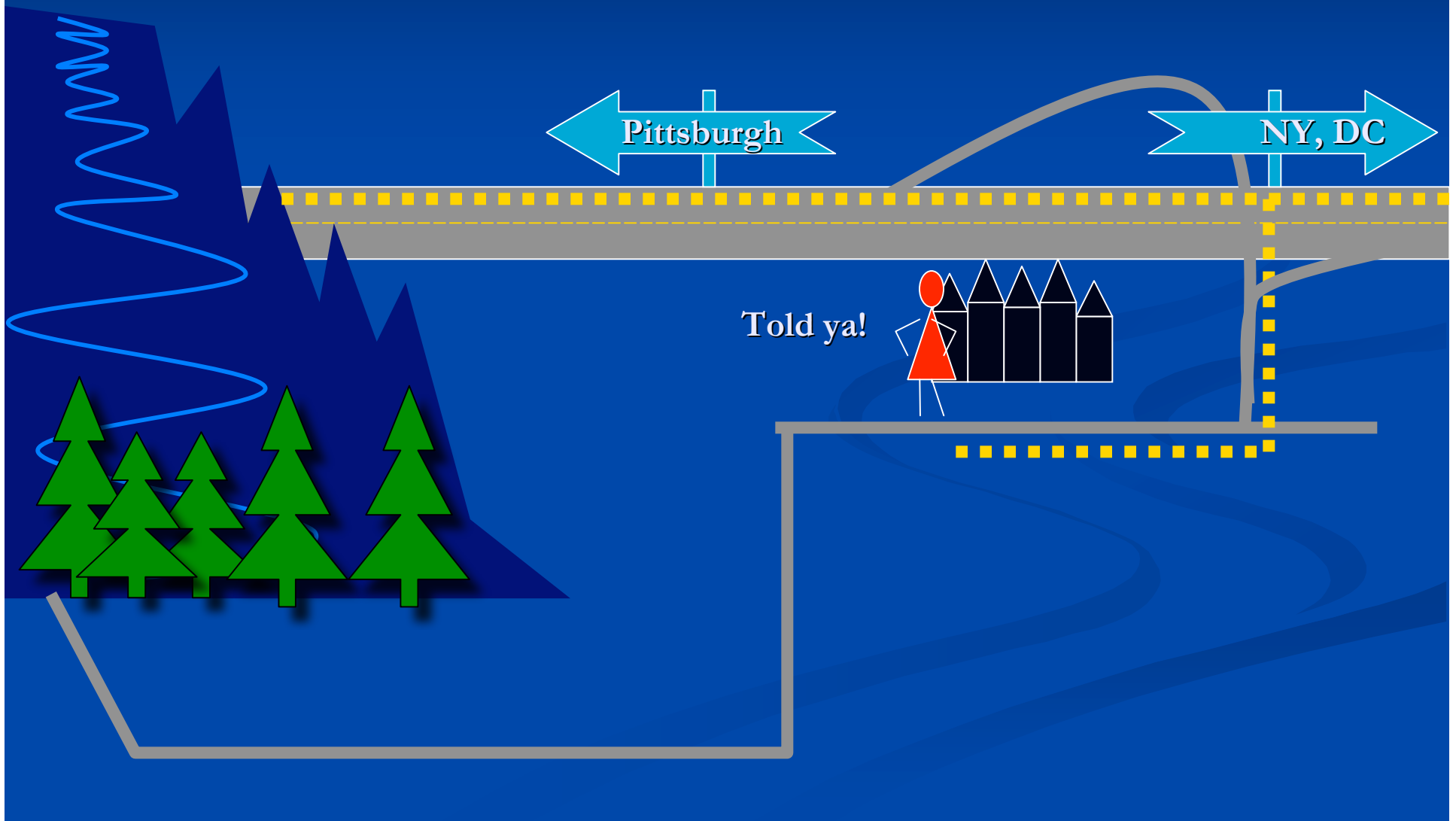
Your Route



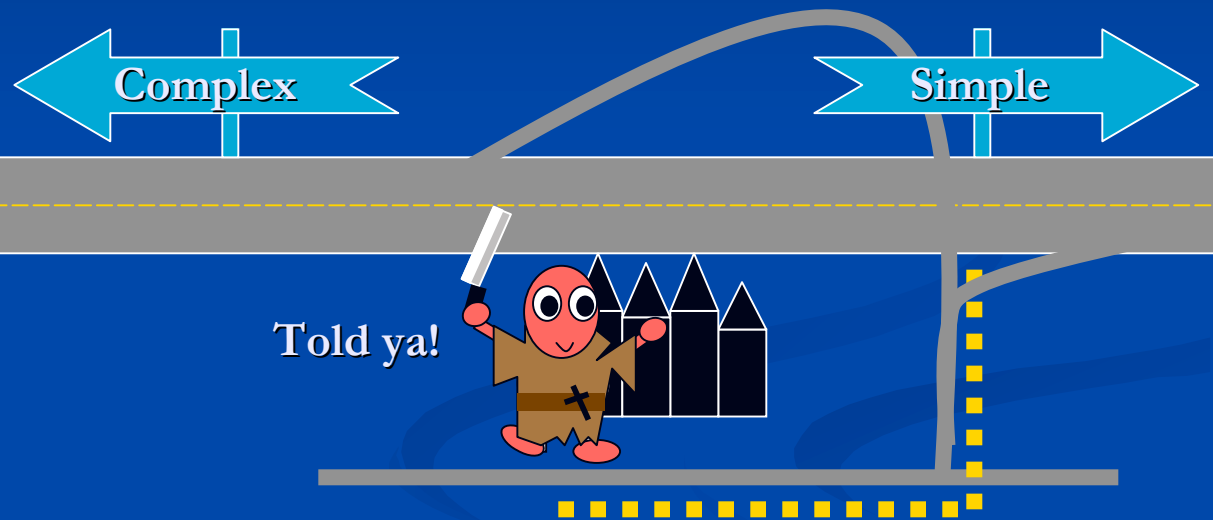
The Best Route



The Best Route Anywhere from There

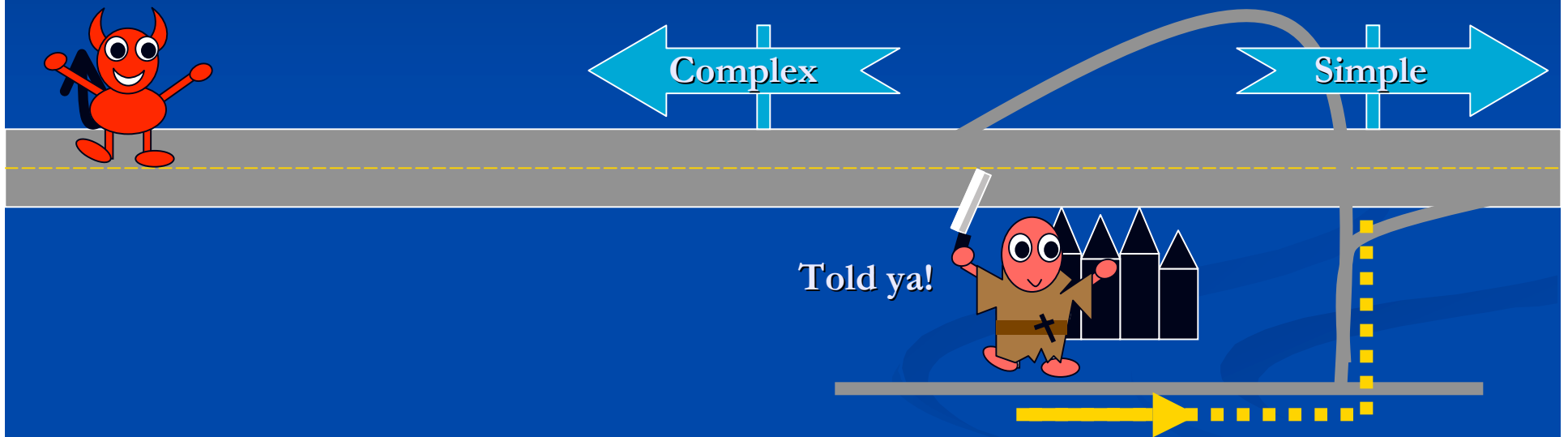


The Freeway to the Truth



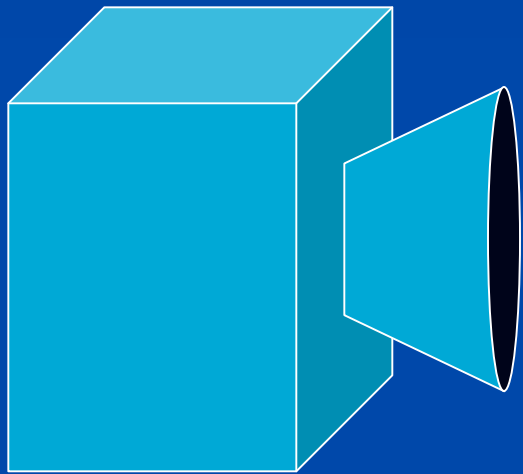
- Fixed advice for all destinations
- Disregarding it entails an extra course reversal...

The Freeway to the Truth

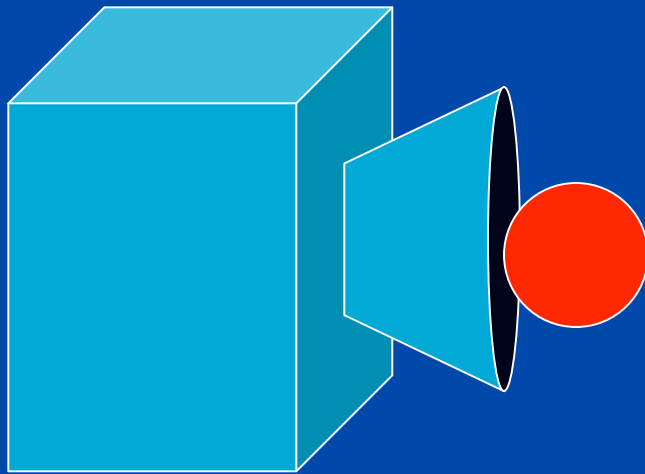


- ...even if the advice points **away from** the goal!

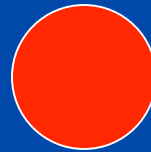
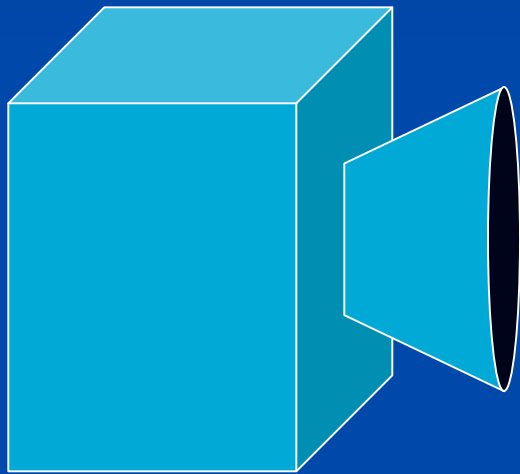
Counting Marbles



Counting Marbles

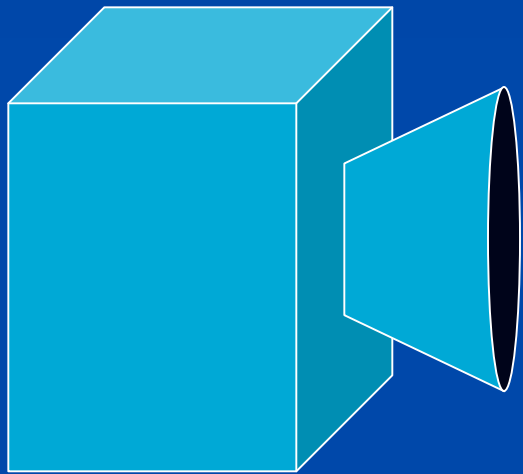


Counting Marbles



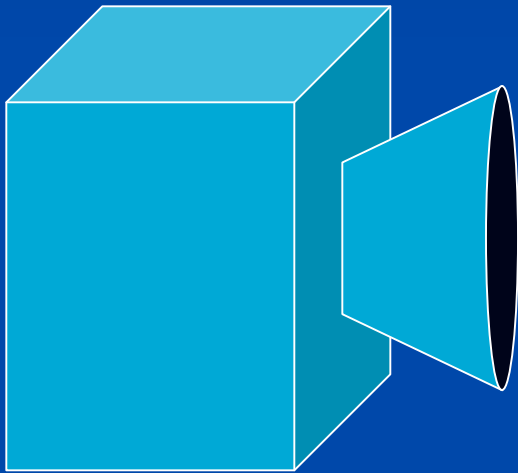
May come at any time...

Counting Marbles



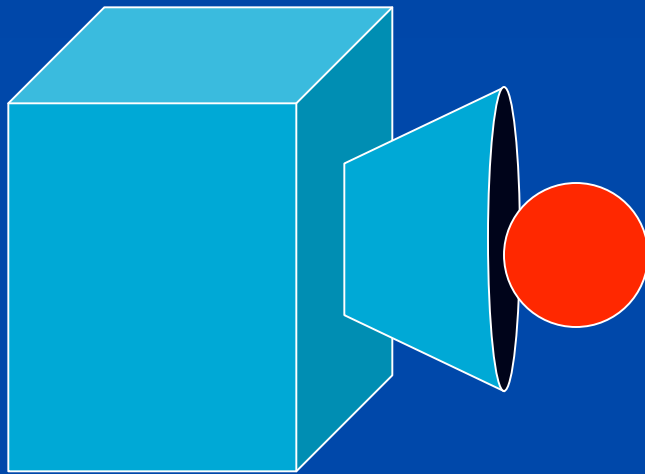
May come at any time...

Counting Marbles



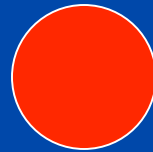
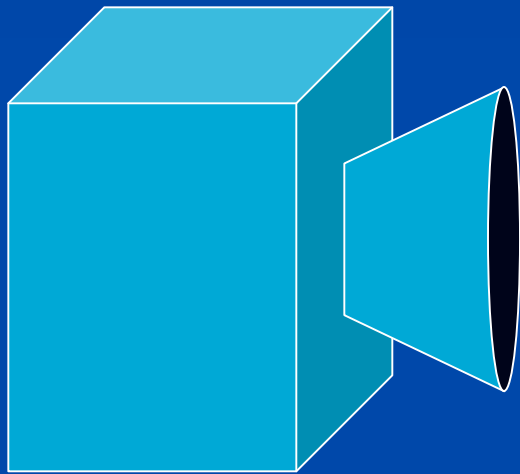
May come at any time...

Counting Marbles



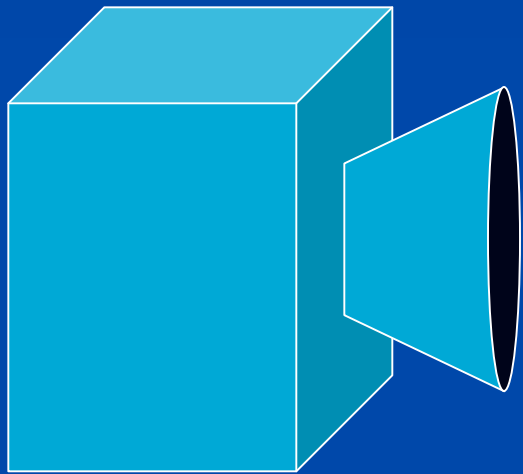
May come at any time...

Counting Marbles



May come at any time...

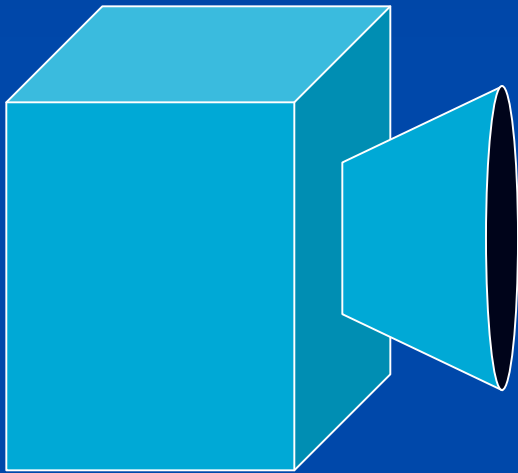
Counting Marbles



May come at any time...



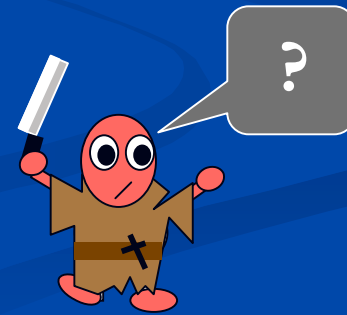
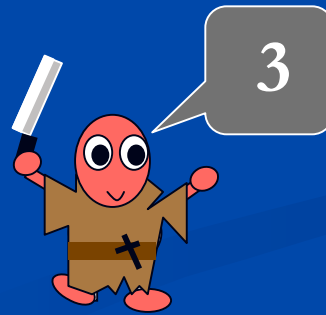
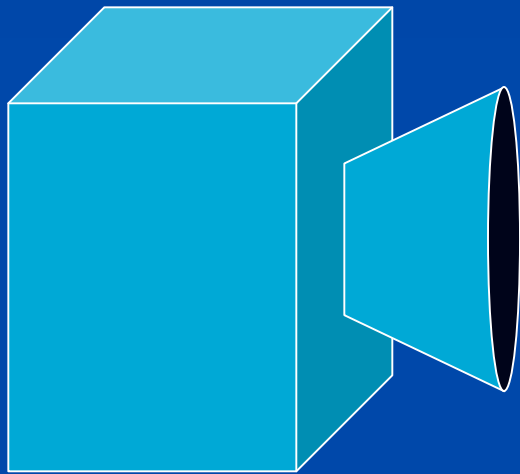
Counting Marbles



May come at any time...

Ockham's Razor

- If you answer, answer with the **current count**.

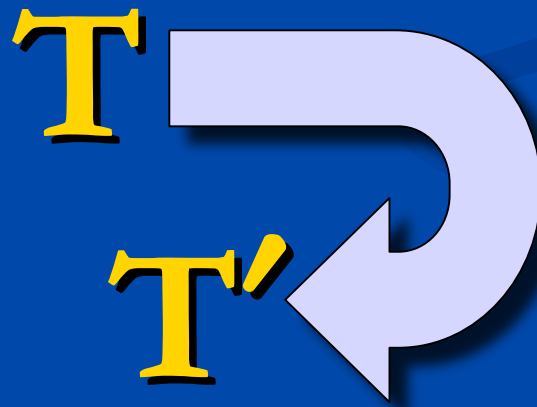


Analogy

- **Marbles** = detectable “effects”.
- **Late appearance** = difficulty of detection.
- **Count** = model (e.g., causal graph).
- **Appearance times** = free parameters.

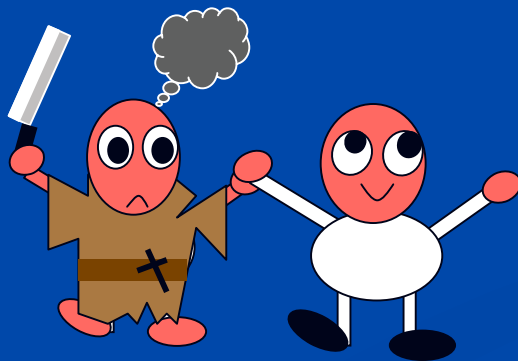
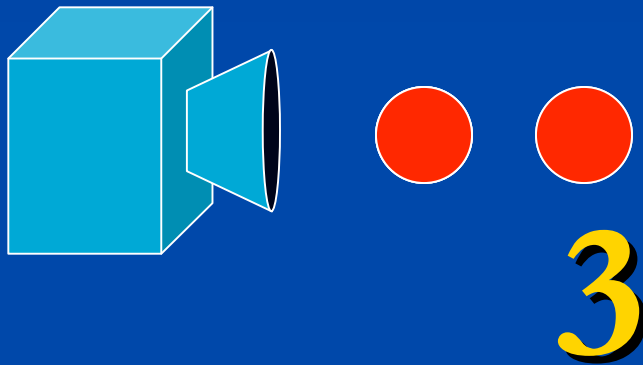
Analogy

- **U-turn** = model **revision** (with content loss)
- **Highway** = revision-efficient truth-finding method.



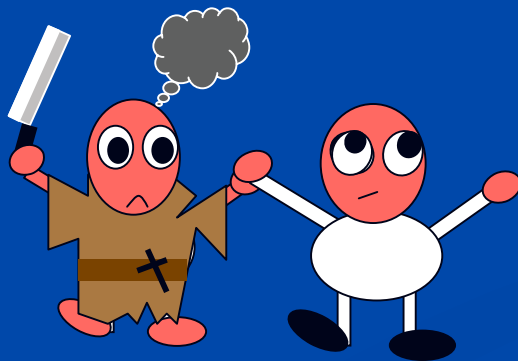
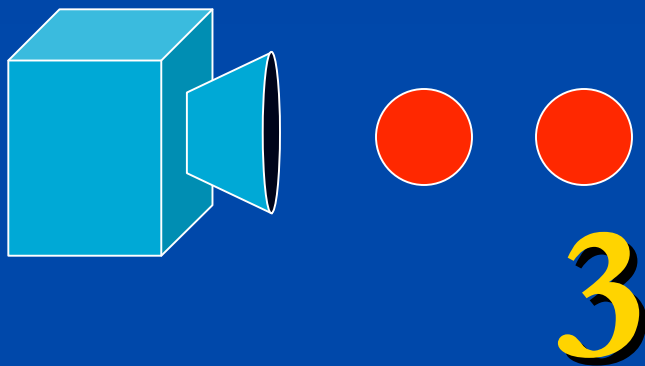
The U-turn Argument

- Suppose you converge to the truth but
- violate Ockham's razor along the way.



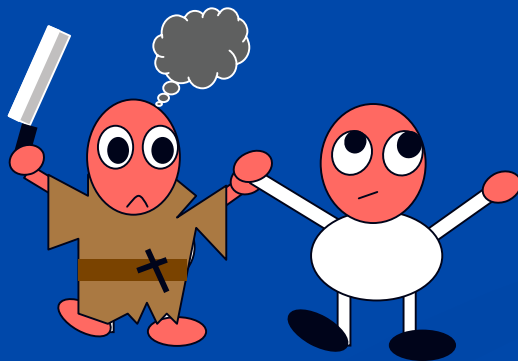
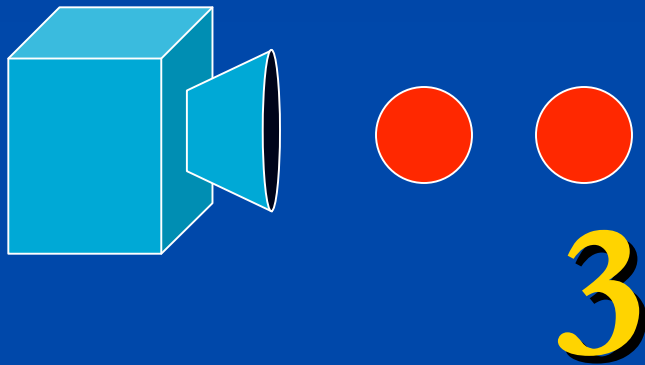
The U-turn Argument

- Where is that extra marble, anyway?



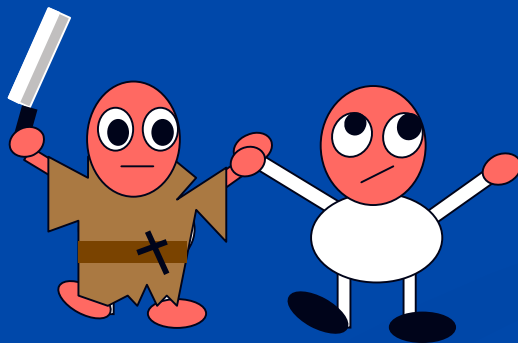
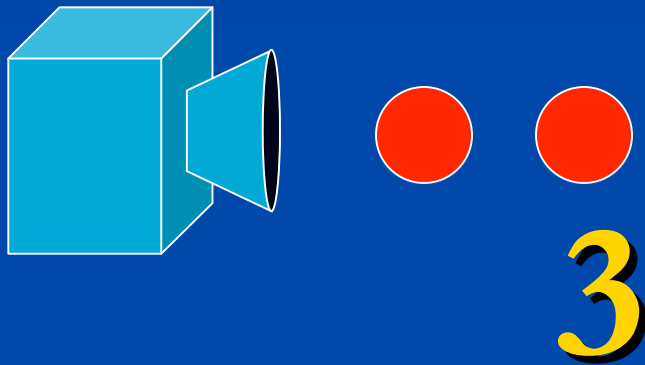
The U-turn Argument

- It's not coming, is it?



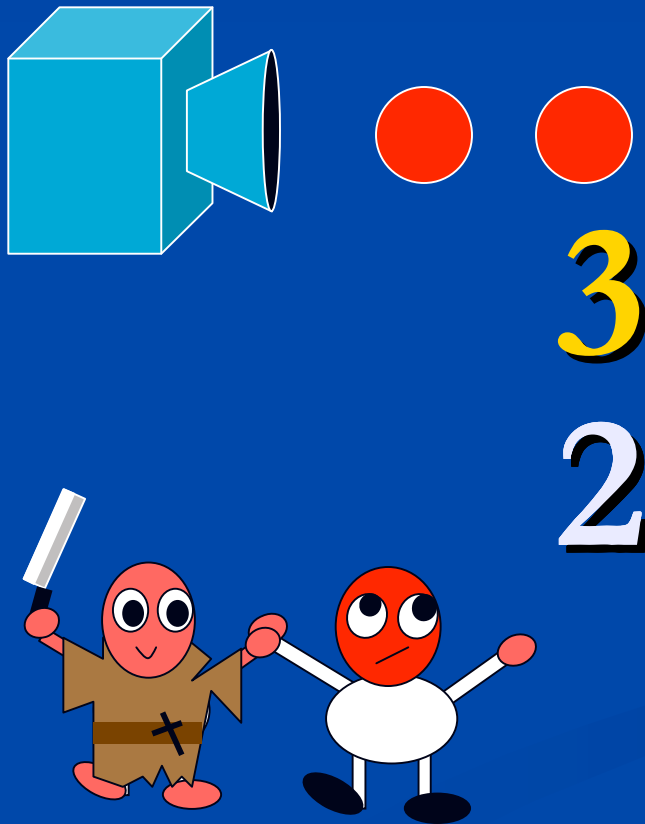
The U-turn Argument

- If you never say 2 you'll never converge to the truth....



The U-turn Argument

- That's it. You should have listened to Ockham.



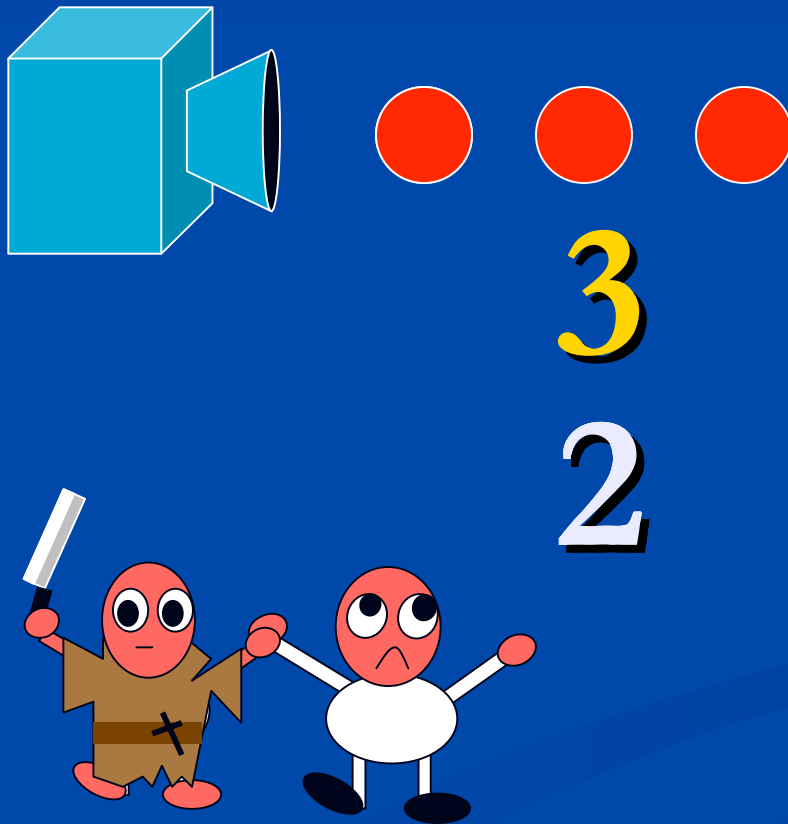
The U-turn Argument

- Oops! Well, no method is infallible!



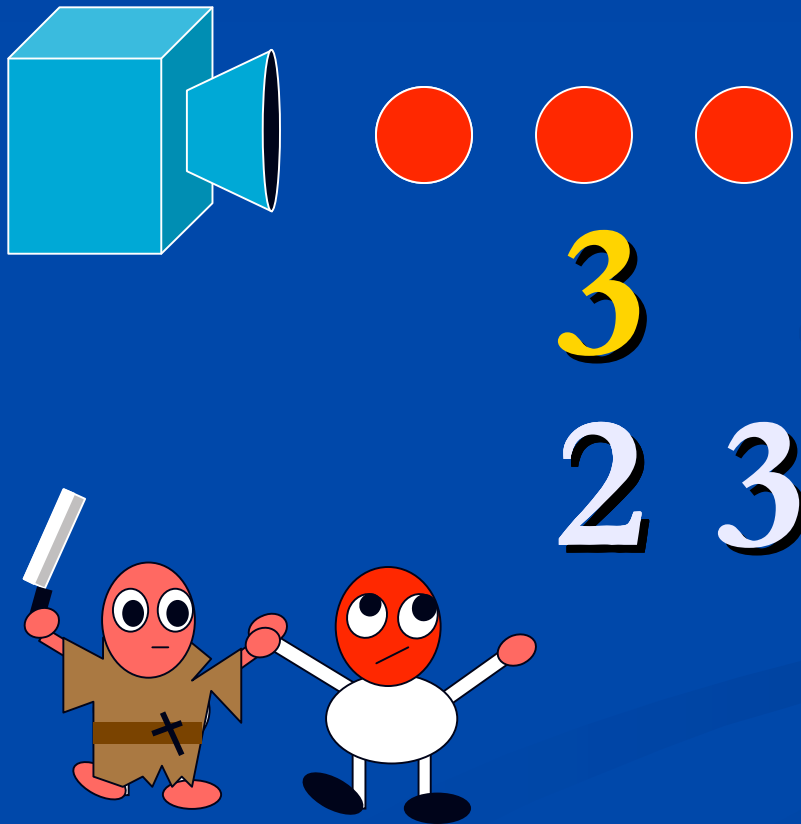
The U-turn Argument

- If you never say 3, you'll never converge to the truth....



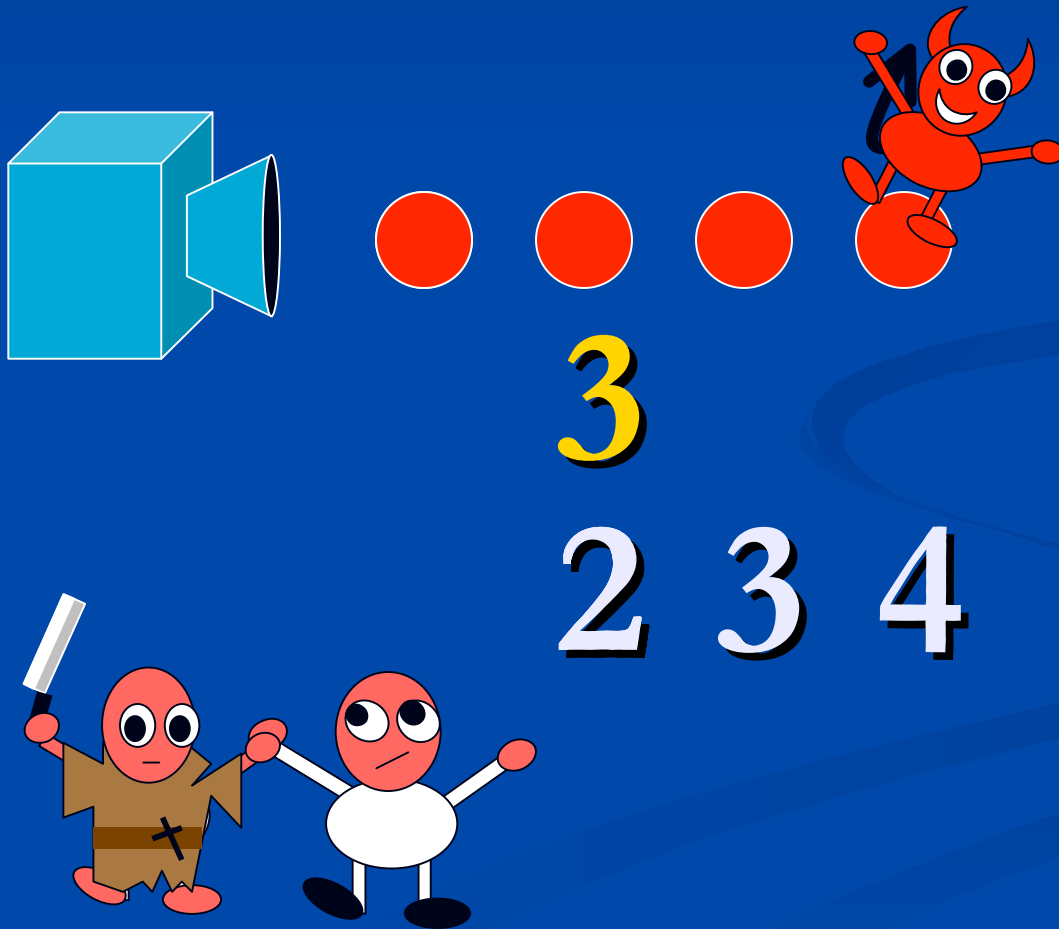
The U-turn Argument

- Embarrassing to be back at that old theory, eh?



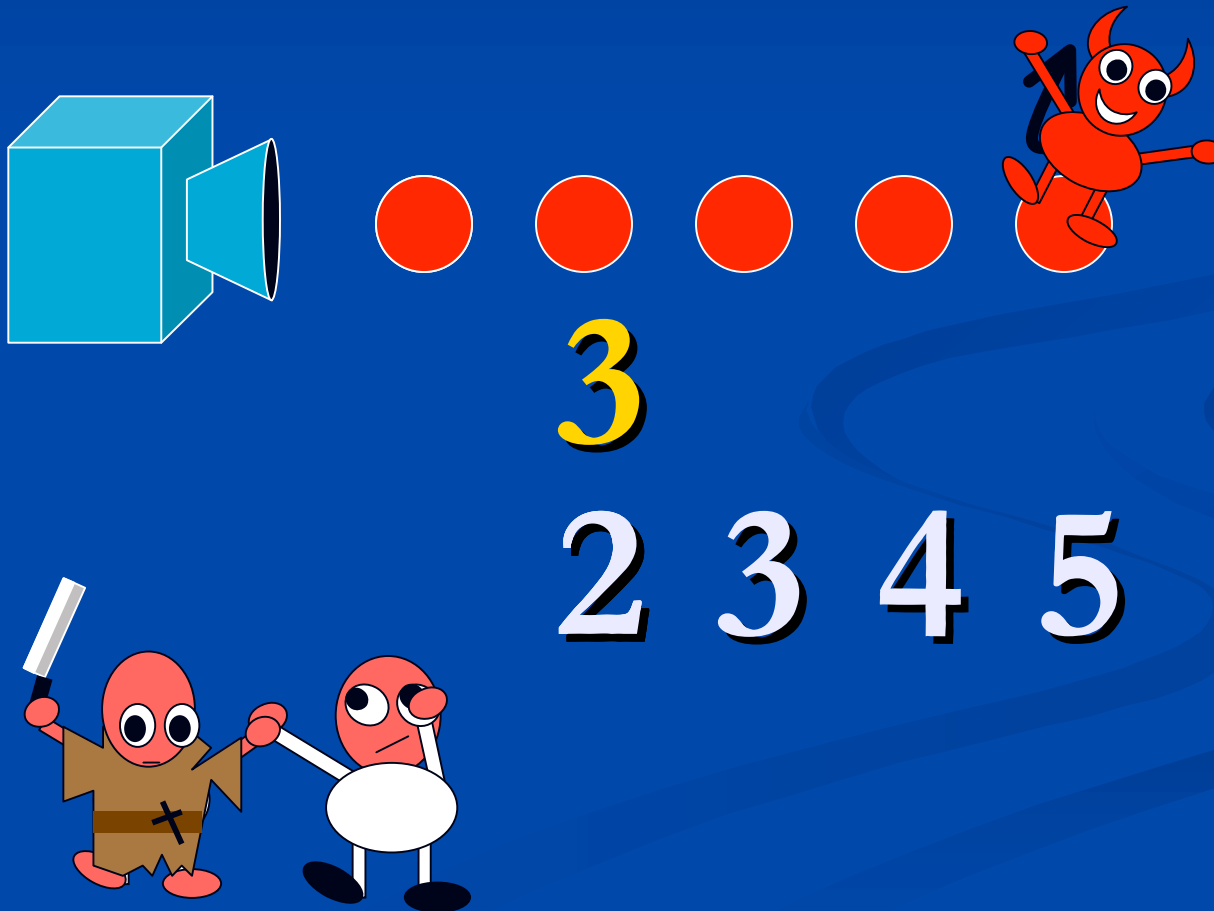
The U-turn Argument

- And so forth...



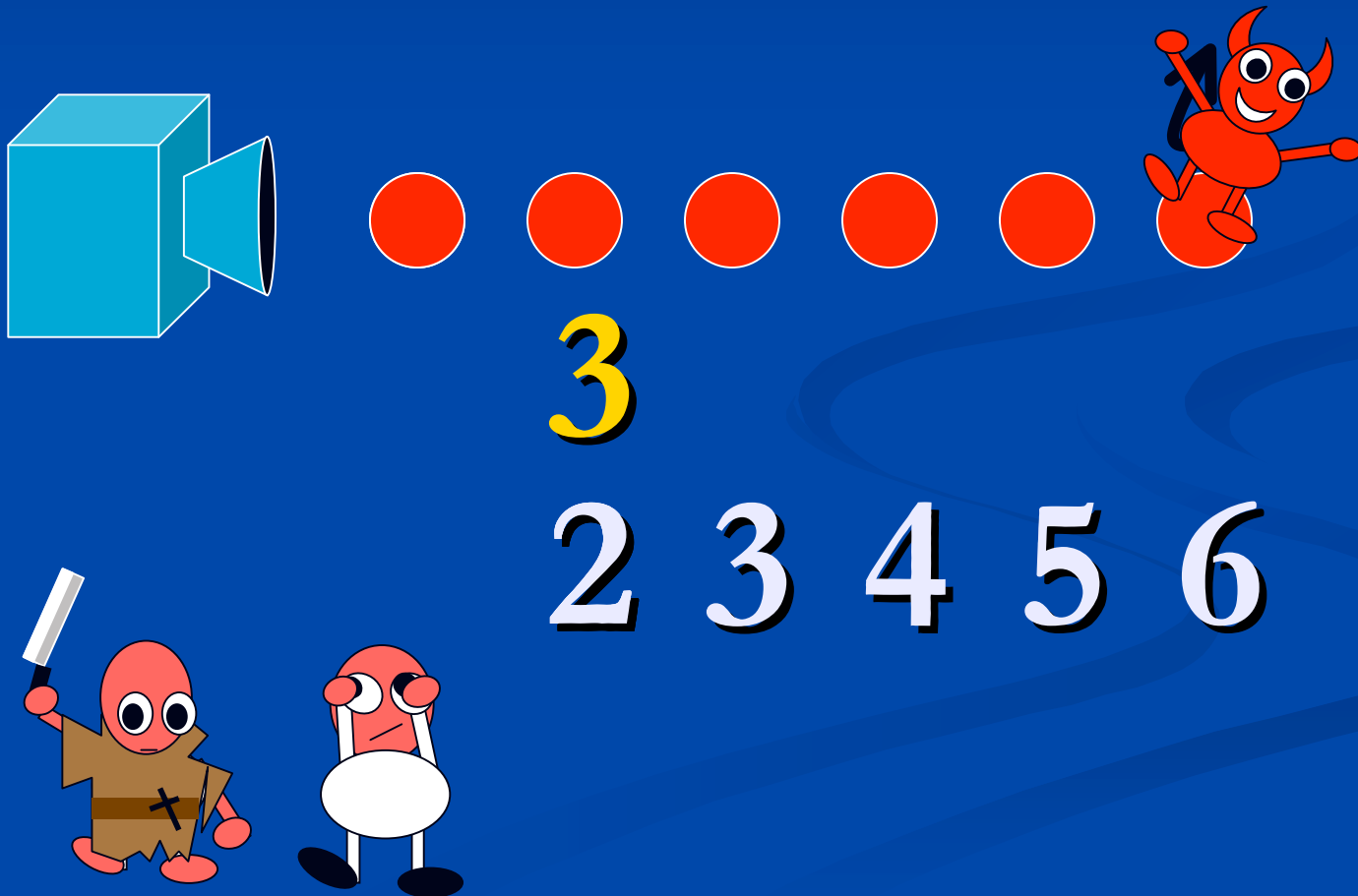
The U-turn Argument

- And so forth...



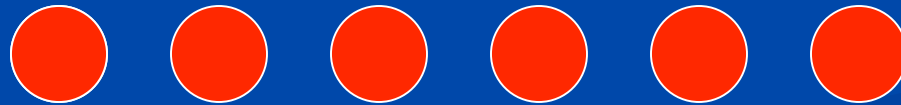
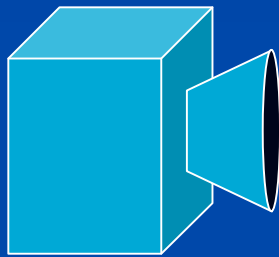
The U-turn Argument

- And so forth...



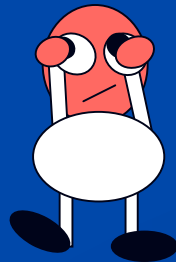
The U-turn Argument

- And so forth...



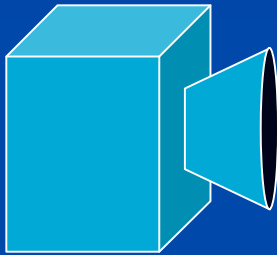
3

2 3 4 5 6 7



The Score

■ You:



Subproblem



3

2

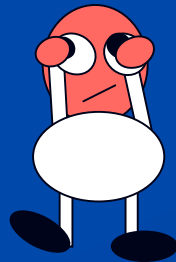
3

4

5

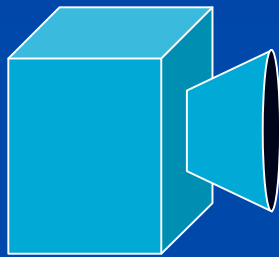
6

7



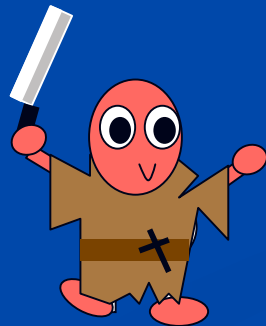
The Score

■ Ockham:



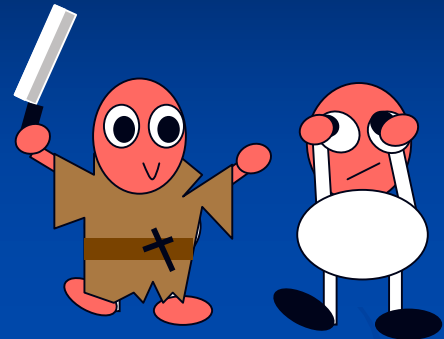
Subproblem

2 3 4 5 6 7



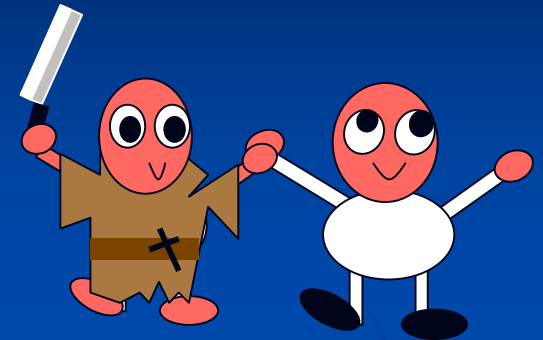
Ockham is Necessary

- If you **converge to the truth**,
and
- you **violate Ockham's razor**
then
- some convergent method **beats your worst-case revision bound** in each answer in the subproblem entered at the time of the violation.



Ockham is Sufficient

- If you **converge to the truth**,
and
- you **never violate Ockham's razor**
then
- You **achieve the worst-case revision bound** of
each convergent solution in each answer in
each subproblem.



Efficiency

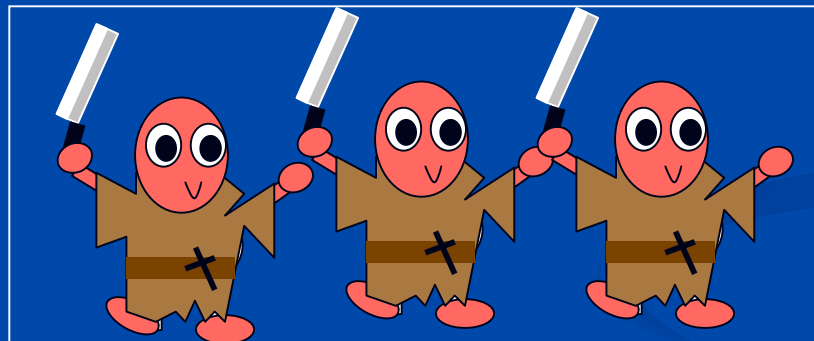
- **Efficiency** = achievement of the best worst-case revision bound in each answer in each subproblem.

Ockham Efficiency Theorem

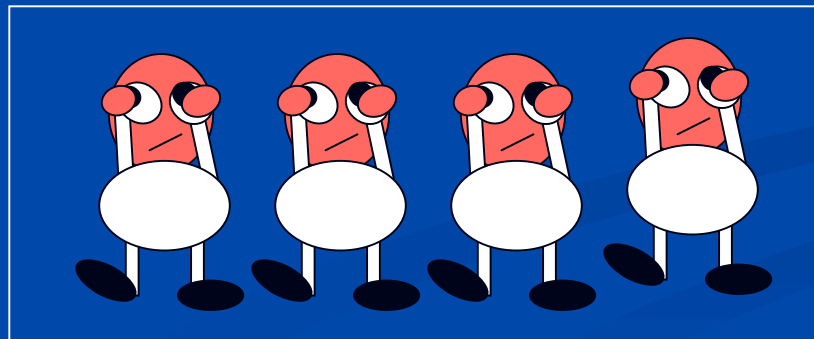
- Among the convergent methods...

Ockham = Efficient!

Efficient



Inefficient



“Mixed” Strategies

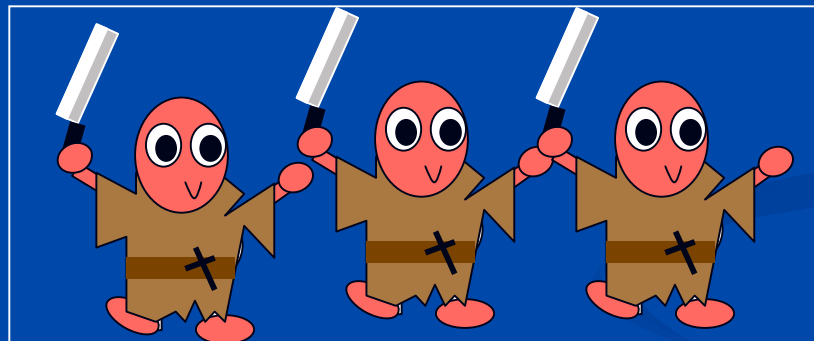
- **mixed strategy** = chance of output depends only on actual experience.
- **convergence in probability** = chance of producing true answer approaches 1 in the limit.
- **efficiency** = achievement of best worst-case **expected** revision bound in each answer in each subproblem.

Ockham Efficiency Theorem

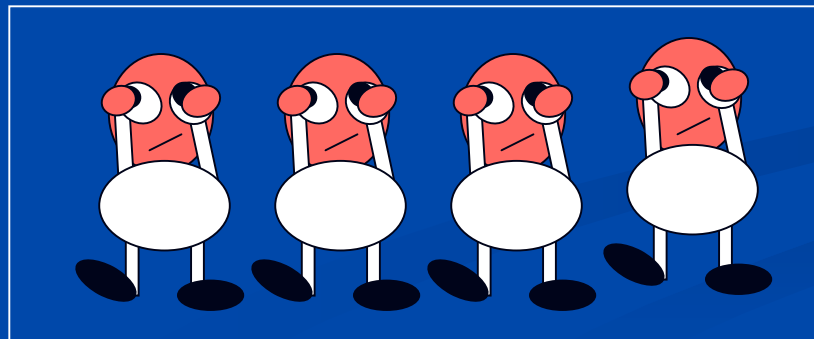
- Among the **mixed** methods that converge in probability...

Ockham = Efficient!

Efficient



Inefficient



Dominance and “Support”

- Every convergent method is weakly dominated in revisions by a clone who says “?” until stage n .



Convergence

Must leap eventually.



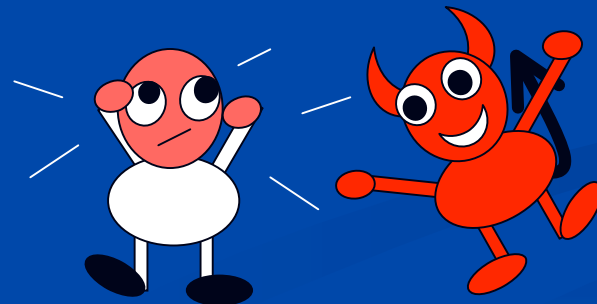
Efficiency

Only leap to simplest.



Dominance

Could always wait longer.



Can't wait forever!

III. Ockham on Steroids



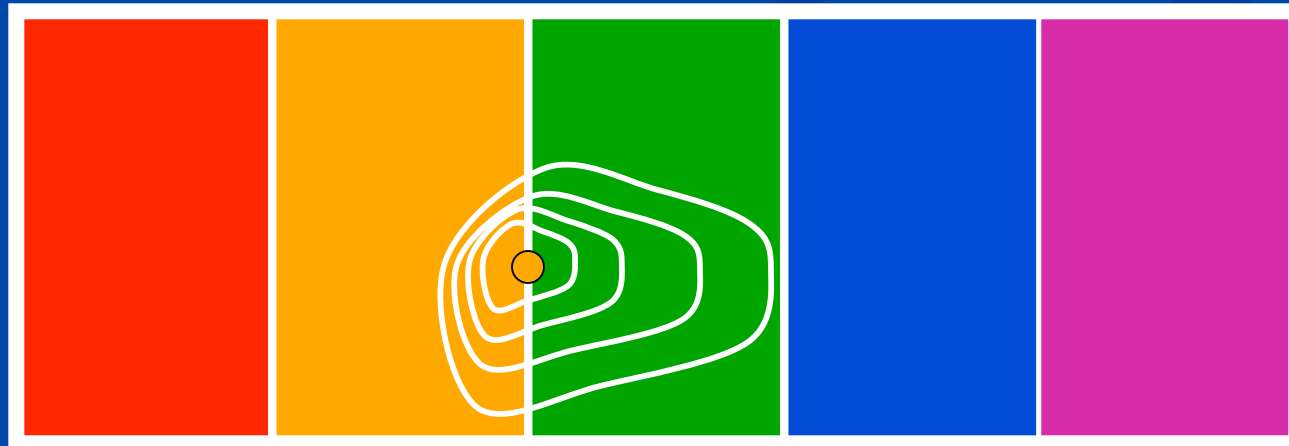
Ockham Wish List

- General definition of Ockham's razor.
- Compare revisions even when not bounded within answers.
- Prove theorem for arbitrary empirical problems.

Empirical Problems

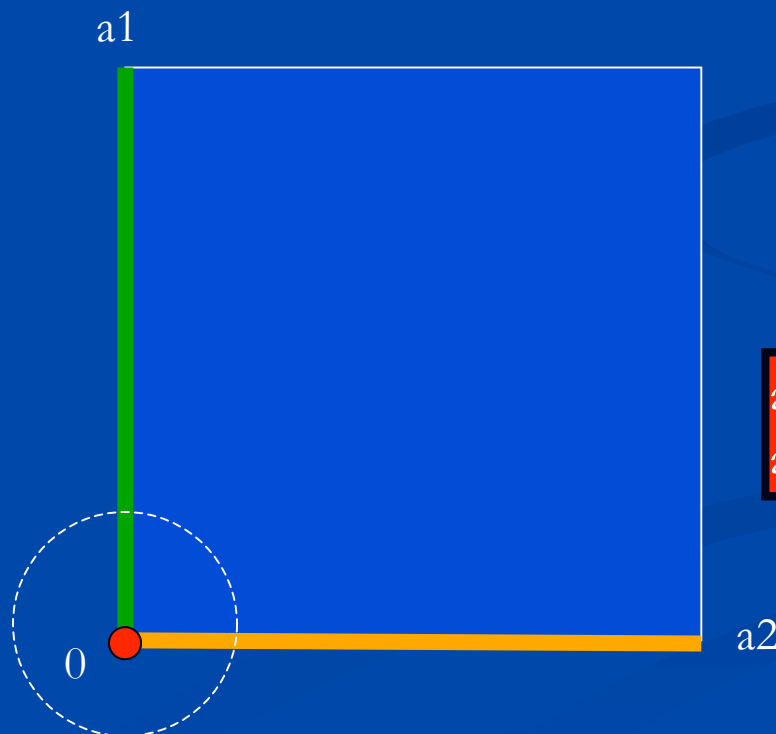
- **Problem** = partition of a topological space.
- **Potential answers** = partition cells.
- **Evidence** = open (verifiable) propositions.

Example: Symmetry







Example: Parameter Freeing

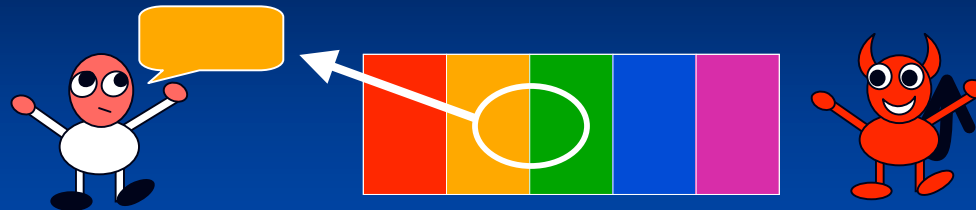
- Euclidean topology.
- Say which parameters are zero.
- Evidence = open neighborhood.



• Curve fitting

			
$a_1 = 0$ $a_2 = 0$	$a_1 > 0$ $a_2 = 0$	$a_1 = 0$ $a_2 > 0$	$a_1 > 0$ $a_2 > 0$

The Players

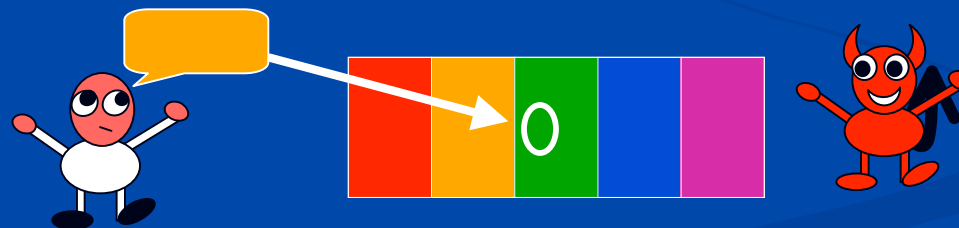


■ Scientist:

- Produces an answer in response to current evidence.

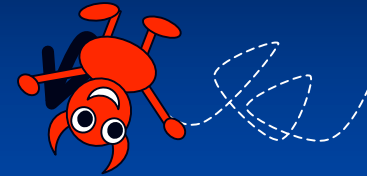
■ Demon:

- Chooses evidence in response to scientist's choices

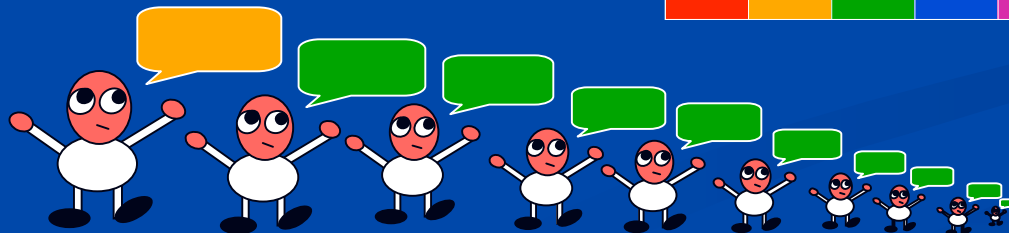


Winning

■ Scientist wins...

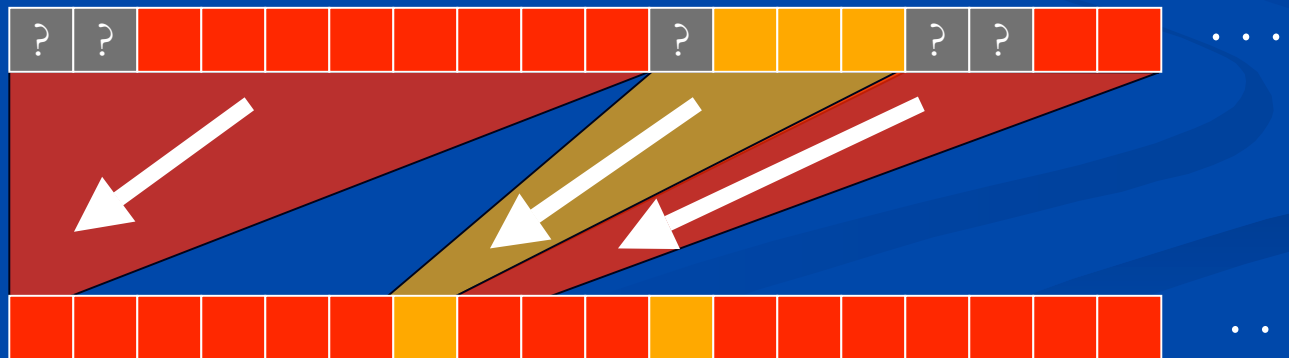


- **by default** if demon doesn't present an infinite nested sequence of basic open sets whose intersection is a singleton.
- **else by merit** if scientist eventually always produces the true answer for world selected by demon's choices.



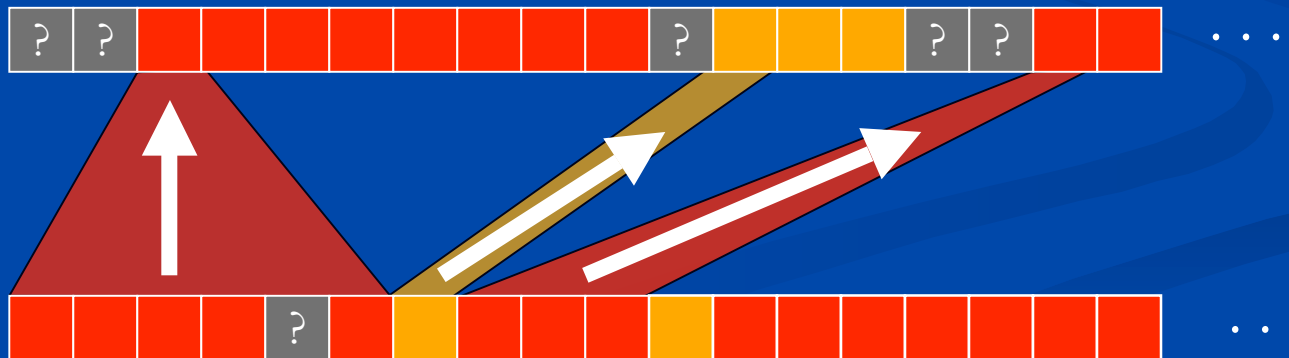
Comparing Revisions

- One answer sequence **maps into** another iff
 - there is an order and answer-preserving map from the first to the second (? is wild).
- Then the revisions of first are **as good as** those of the second.



Comparing Revisions

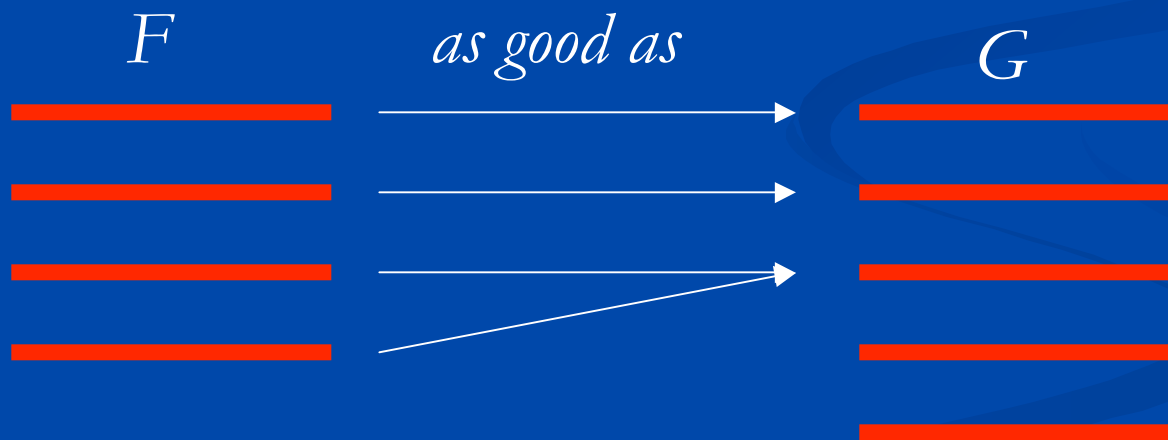
- The revisions of the first are **strictly better** if, in addition, the latter doesn't map back into the former.



Comparing Methods

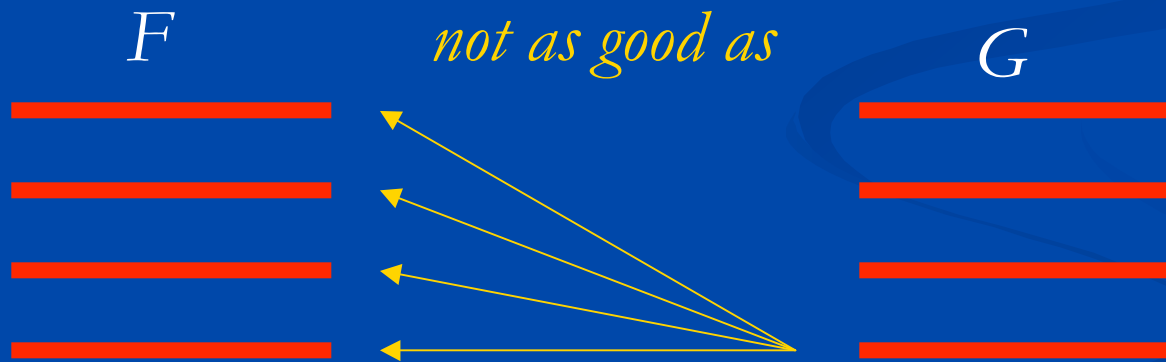
- F is as good as G iff

each output sequence of F is as good as some output sequence of G .



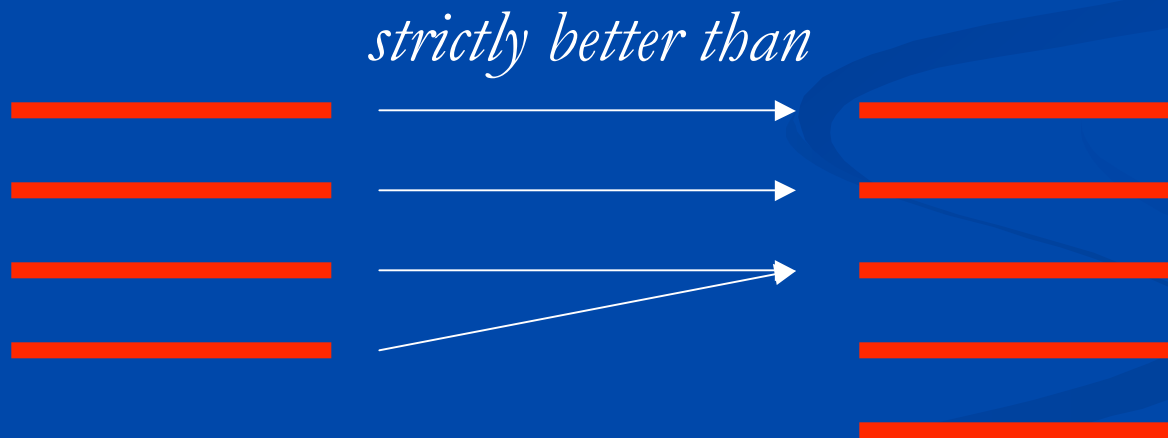
Comparing Methods

- F is **better than** G iff
 F is as good as G and
 G is **not** as good as F



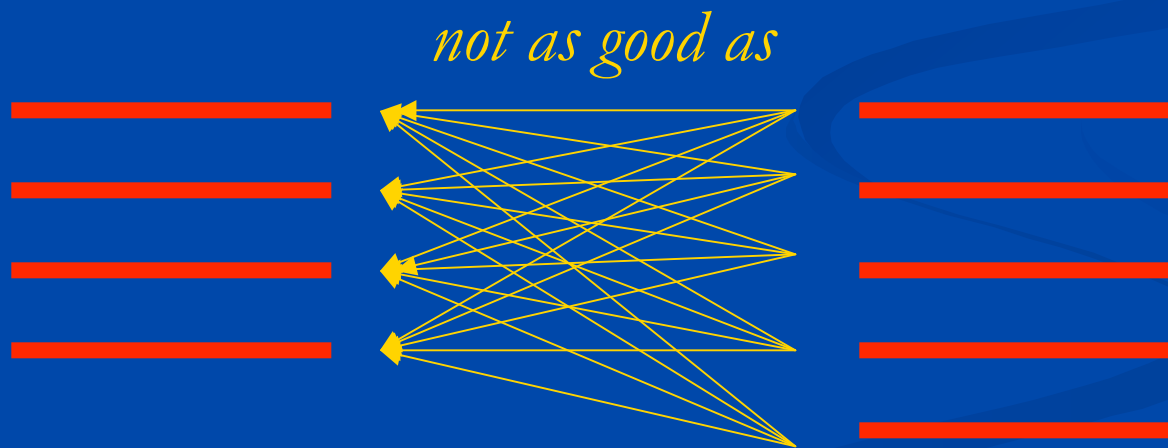
Comparing Methods

- F is **strongly better than** G iff each output sequence of F is strictly better than an output sequence of G but ...



Comparing Methods

- ... no output sequence of G is as good as any of F .



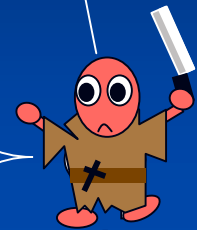
Terminology

- **Efficient solution:** as good as any solution in any subproblem.

What Simplicity Isn't

- Syntactic length.
- Data-compression (MDL).
- Computational ease.
- Social “entrenchment” (Goodman).
- Number of free parameters (BIC, AIC).
- Euclidean dimensionality

Only by
accident!!



Symptoms...



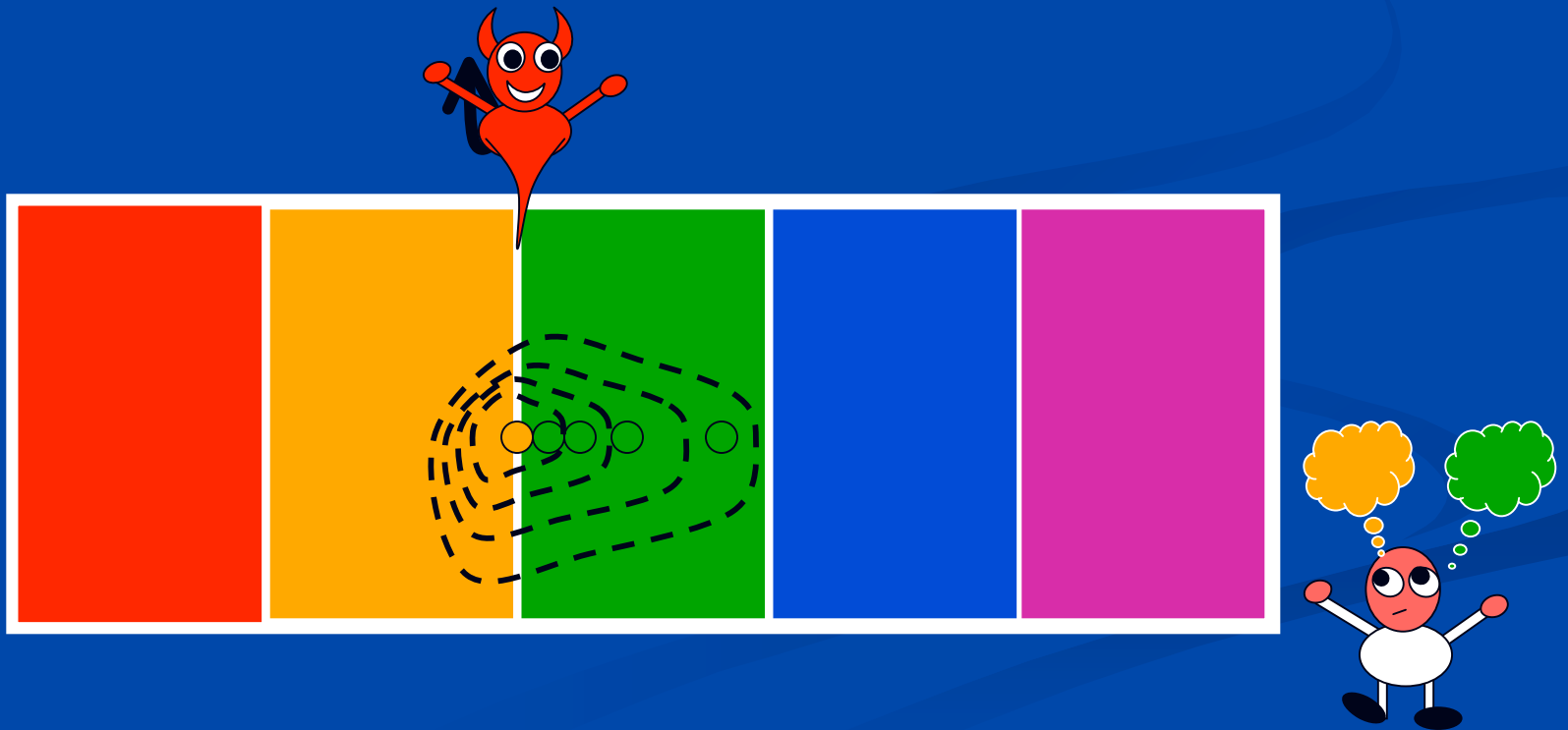
What Simplicity Is

- Simpler theories are compatible with deeper problems of induction.



Problem of Induction

- No true information entails the true answer.
- Happens in **answer boundaries**.

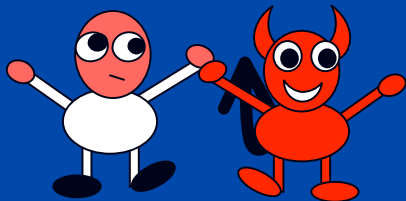


Demonic Paths

A **demonic path** from w is a sequence of alternating answers that a demon can force an arbitrary convergent method through starting from w .



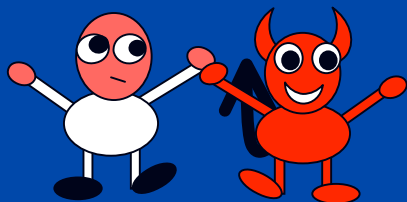
0...1...2...3...4



Simplicity **Defined**

The *A*-sequences are the demonic sequences beginning with answer *A*.

A is **as simple** as *B* iff each *B*-sequence is as good as some *A*-sequence.



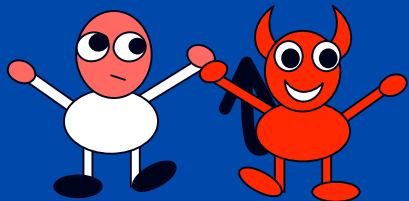
$$\begin{array}{lcl} 3 & < & 2, 3 \\ 3, 4 & < & 2, 3, 4 \\ 3, 4, 5 & < & 2, 3, 4, 5 \end{array}$$



So 2 is simpler than 3!

Ockham Answer

- An answer as simple as any other answer.
- = number of observed particles.



$$\begin{array}{lcl} n & < & 2, \dots, n \\ n, n+1 & < & 2, \dots, n, n+1 \\ n, n+1, n+2 & < & 2, \dots, n, n+1, n+2 \end{array}$$



So 2 is Ockham!

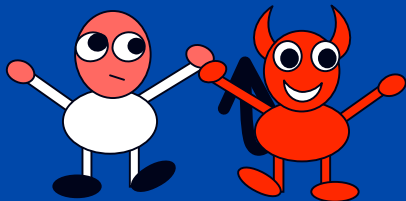
Ockham Lemma

A is Ockham iff

for all demonic p , $(A^*p) \leq$ some demonic sequence.



3



I can force you
through 2
but not
through 3,2.

So 3 isn't
Ockham

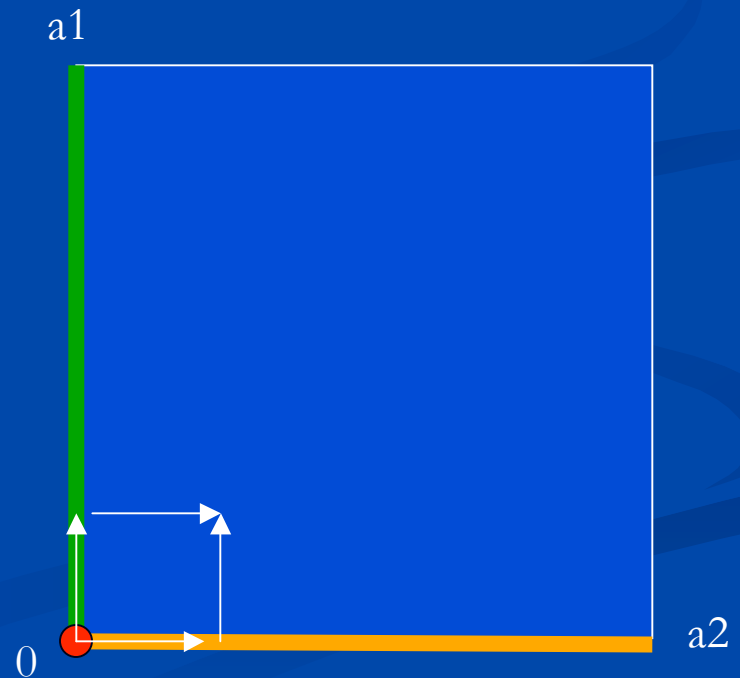


Ockham Answer

E.g.: Only simplest curve compatible with data is Ockham.

Demonic sequence: ●

Non-demonic sequences:



General Efficiency Theorem

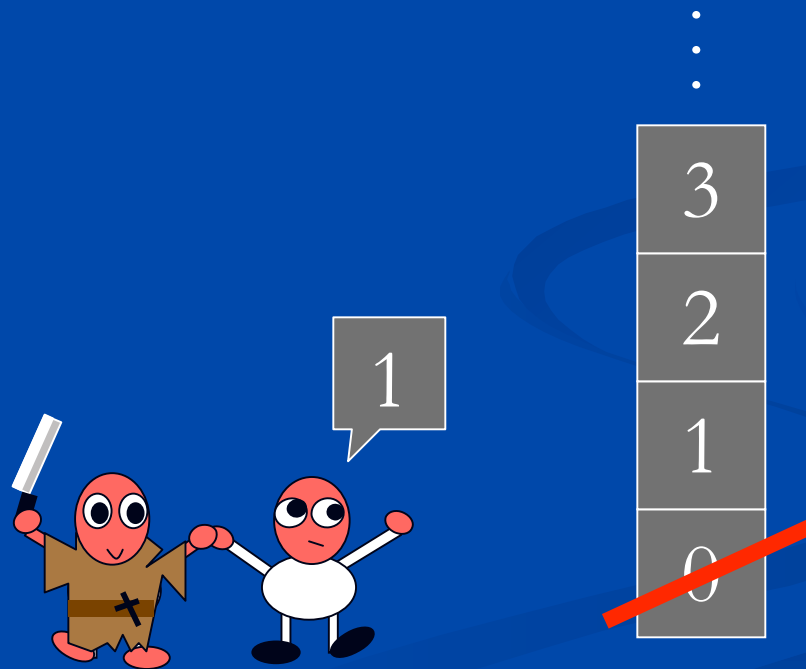
If the topology is metrizable and separable and the question is countable then:

Ockham = Efficient.

Proof: uses Martin's Borel Determinacy theorem.

Stacked Problems

There is an Ockham answer at every stage.



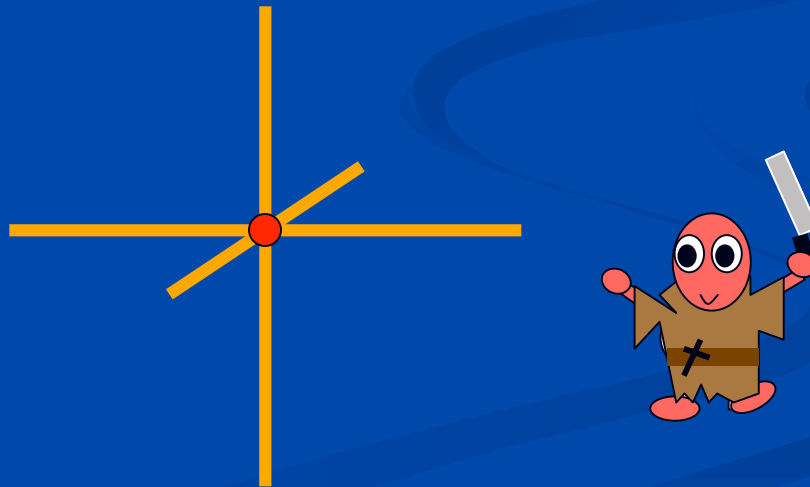
Non-Ockham \rightarrow Strongly Worse

If the problem is a **stacked** countable partition over a restricted Polish space:

Each Ockham solution is **strongly better than each non-Ockham solution in the subproblem entered at the time of the violation.**

Simplicity \neq Low Dimension

- Suppose God says the true parameter value is rational.



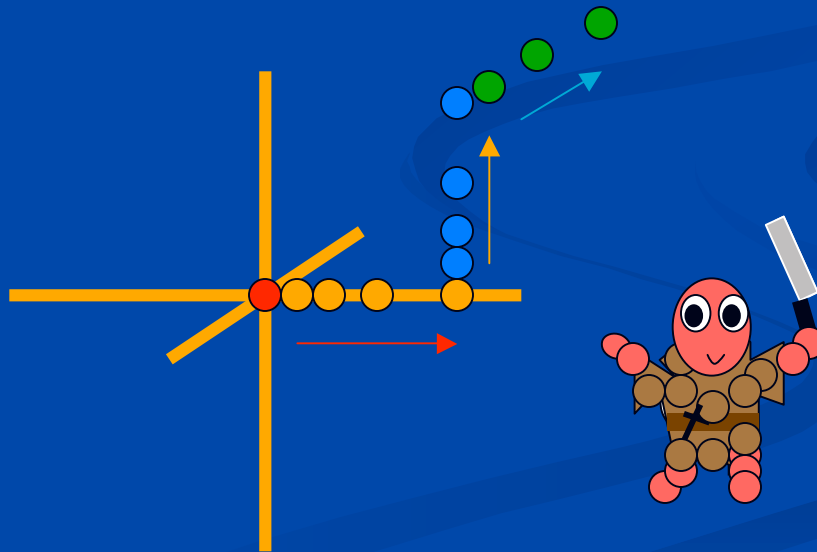
Simplicity \neq Low Dimension

- Topological dimension and integration theory dissolve.
- Does Ockham?

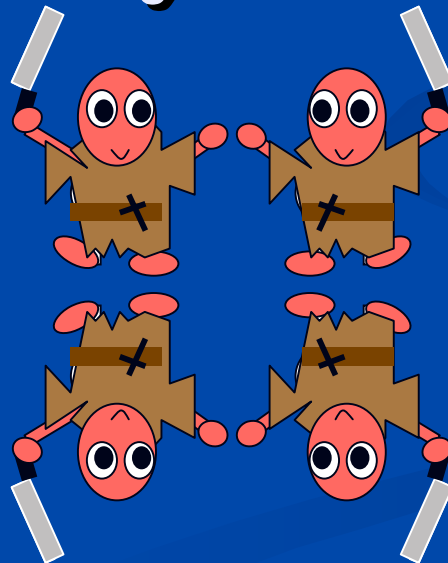


Simplicity \neq Low Dimension

- The proposed account survives in the preserved limit point structure.

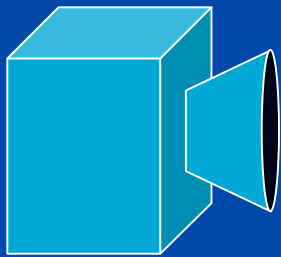


IV. Ockham and Symmetry

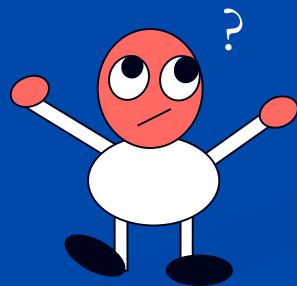


Respect for Symmetry

- If several **simplest** alternatives are available, don't break the symmetry.



- Count the marbles of each color.
- You **hear** the first marble but **don't see it**.
- Why **red** rather than **green**?



Respect for Symmetry

- Before the noise, $(0, 0)$ is Ockham.
- After the noise, **no answer is Ockham:**

Demonic

$(1, 0)$

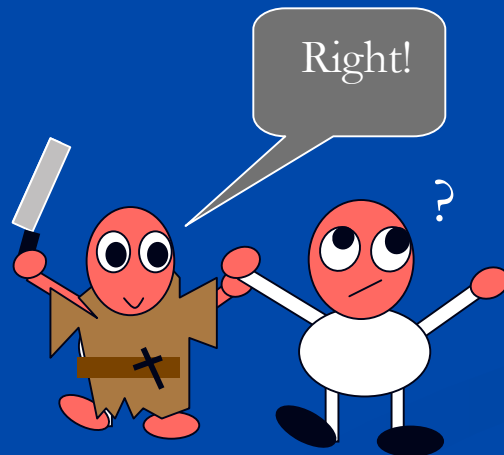
$(0, 1)$

Non-demonic

$(0, 0)$

$(1, 0)$ $(0, 1)$

$(0, 1)$ $(1, 0)$



Goodman's Riddle

- Count **oneicles**--- a oneicle is a particle at any stage but one, when it is a non-particle.
- Oneicle translation is auto-homeomorphism that **does not preserve the problem**.
- **Unique** Ockham answer is current oneicle count.
- **Contradicts** unique Ockham answer in particle counting.

Supersymmetry

- Say **when** each particle appears.
- **Refines** counting problem.
- **Every** auto-homeomorphism preserves problem.
- **No** answer is Ockham.
- **No** solution is Ockham.
- **No** method is efficient.

Dual Supersymmetry

- Say only whether particle count is **even or odd**.
- **Coarsens** counting problem.
- **Particle/Oneicle** auto-homeomorphism preserves problem.
- **Every** answer is Ockham.
- **Every** solution is Ockham.
- **Every** solution is efficient.

Broken Symmetry

- Count the even or just report odd.
- Coarsens counting problem.
- Refines the even/odd problem.
- Unique Ockham answer at each stage.
- Exactly Ockham solutions are efficient.

Simplicity Under Refinement

Supersymmetry
No answer is Ockham



Broken symmetry
Unique Ockham answer



Dual supersymmetry
Both answers are Ockham



Time of particle appearance

Particle counting

Oneicle counting

Twoicle counting

Particle counting
or odd particles

Oneicle counting
or odd oneicles

Twoicle counting
or odd twoicles

Even/odd

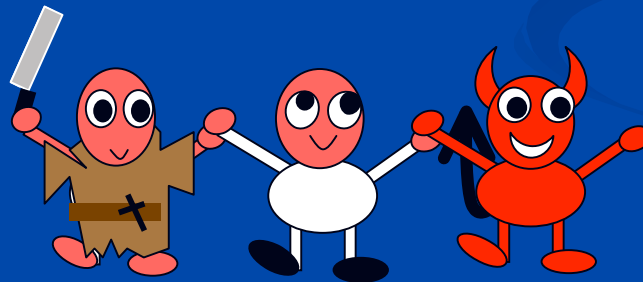
Proposed Theory is Right

- **Objective** efficiency is grounded in problems.
- Symmetries in the preceding problems would **wash out** stronger simplicity distinctions.
- Hence, such distinctions would amount to mere **conventions** (like coordinate axes) that couldn't have anything to do with objective efficiency.

Furthermore...

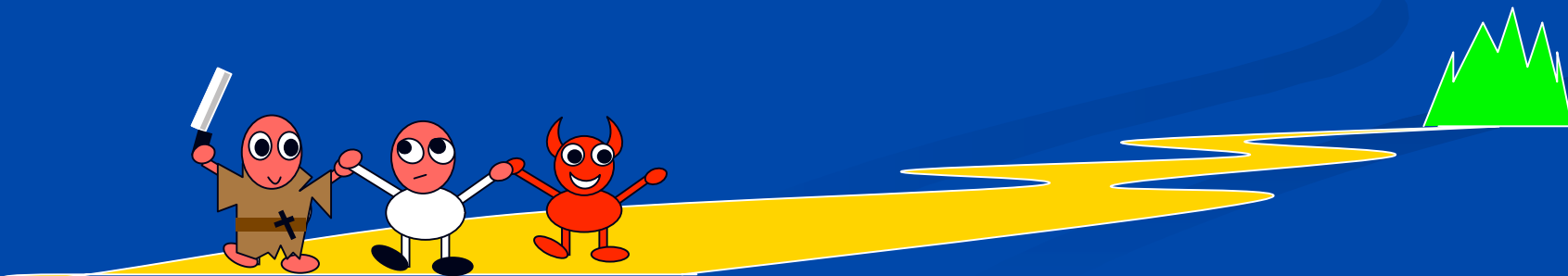
- If Ockham's razor is forced to choose in the supersymmetrical problems then either:
 - following Ockham's razor **increases revisions** in some counting problems
 - Or
 - Ockham's razor leads to **contradictions** as a problem is coarsened or refined.

V. Conclusion



What Ockham's Razor Is

- “Only output Ockham answers”
- Ockham answer = a topological invariant of the empirical problem addressed.



What it Isn't

- preference for
 - brevity,
 - computational ease,
 - entrenchment,
 - past success,
 - Kolmogorov complexity,
 - dimensionality, etc....



How it Works

- Ockham's razor is necessary for **mininizing revisions** prior to convergence to the truth.



How it Doesn't

- No possible method could:
 - Point at the truth;
 - Indicate the truth;
 - Bound the probability of error;
 - Bound the number of future revisions.



Spooky Ockham

- Science *without support or safety nets.*



Spooky Ockham

- Science *without support or safety nets.*



Spooky Ockham

- Science *without support or safety nets.*

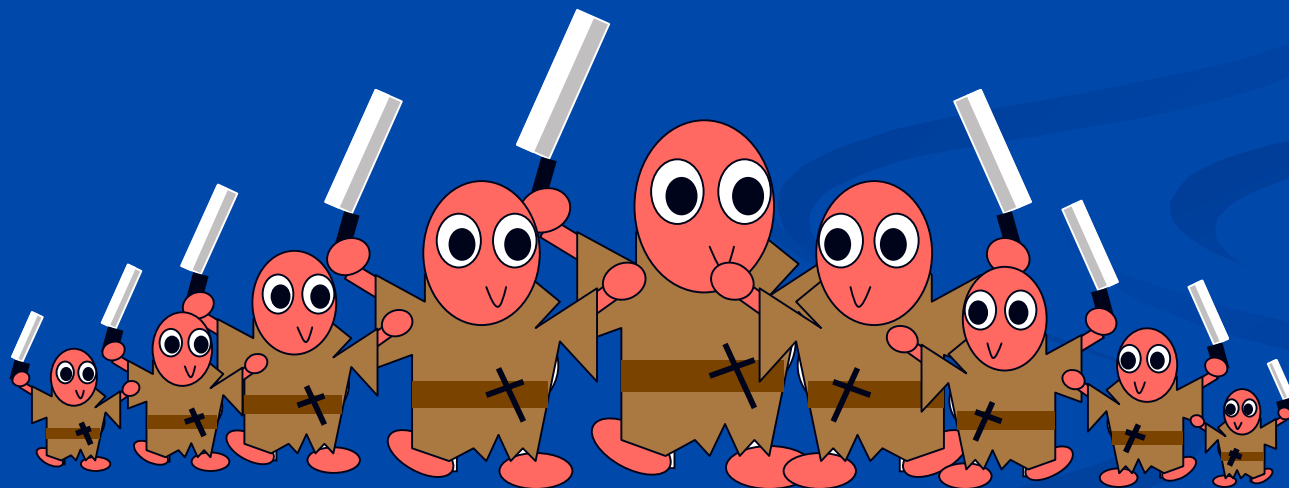


Spooky Ockham

- Science *without support or safety nets.*



VI. Stochastic Ockham



“Mixed” Strategies

- **mixed strategy** = chance of output depends only on actual experience.



$$P_e(M = H \text{ at } n) = P_{e|n}(M = H \text{ at } n).$$

Stochastic Case

- **Ockham** =

at each stage, you produce a non-Ockham answer with **prob** = 0.

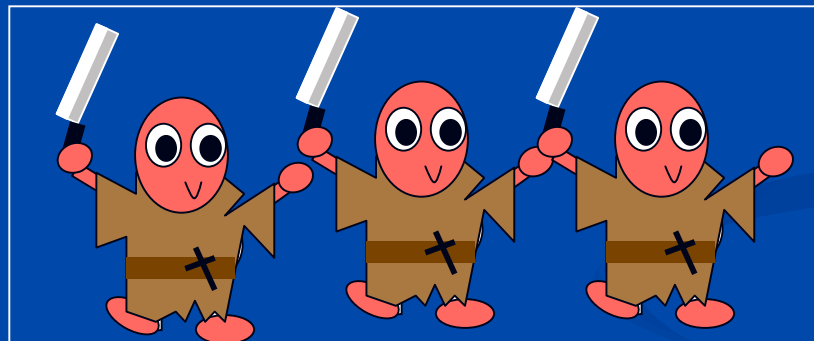
- **Efficiency** =

achievement of the best worst-case **expected** revision bound in each answer in each subproblem over all methods that converge to the truth **in probability**.

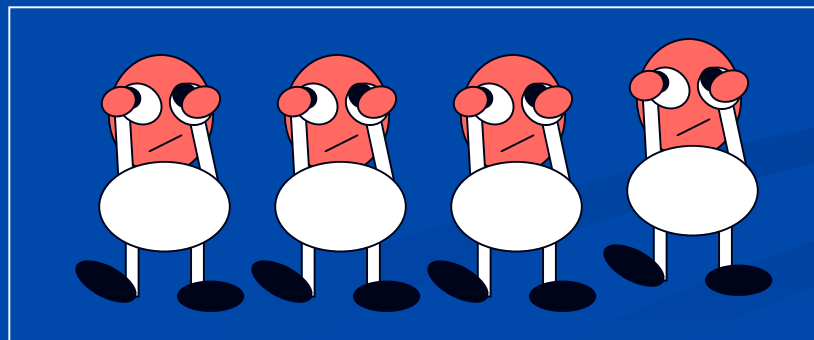
Stochastic Efficiency Theorem

- Among the stochastic methods that converge in probability, **Ockham = Efficient!**

Efficient

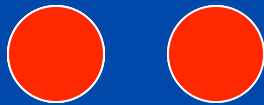
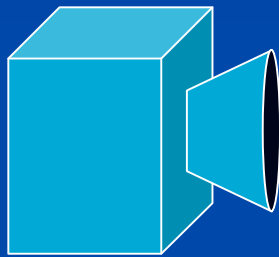


Inefficient



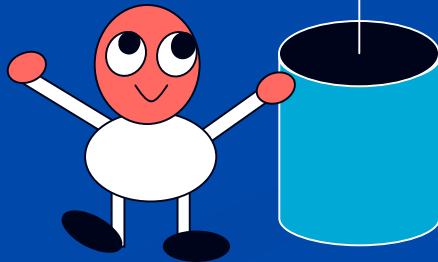
Stochastic Methods

- Your chance of producing an answer is a **function of observations made so far.**



2

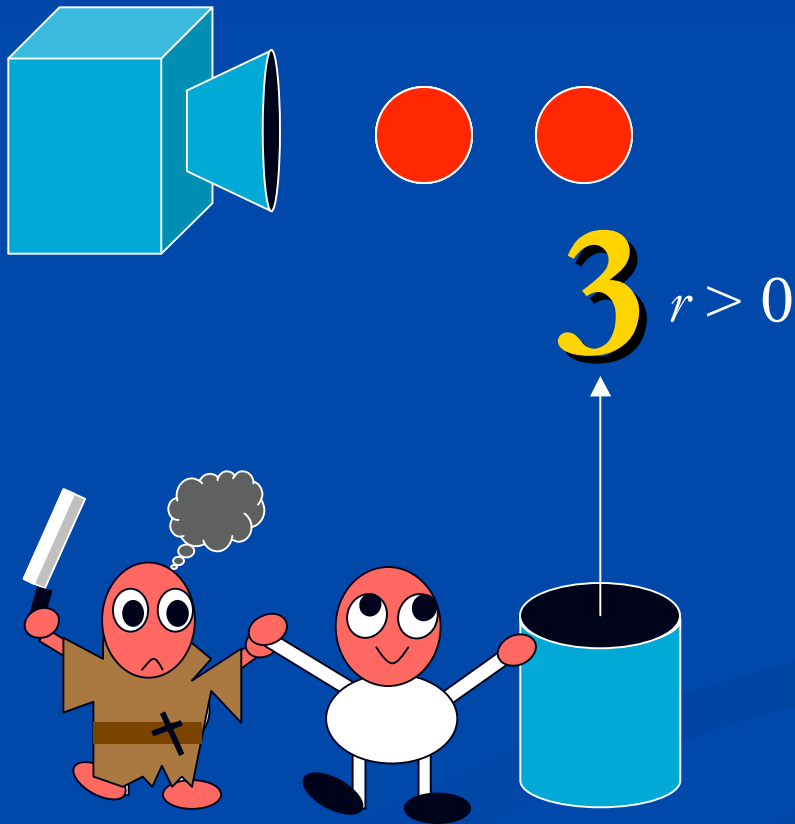
p



Urn selected **in light of observations.**

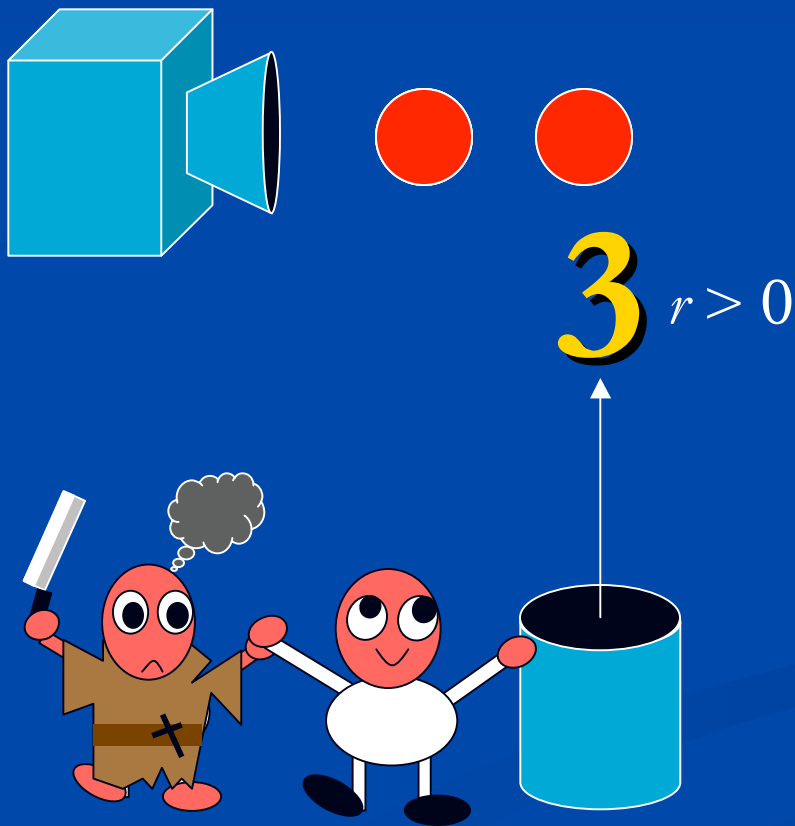
Stochastic U-turn Argument

- Suppose you **converge in probability** to the truth but produce a non-Ockham answer with prob > 0 .



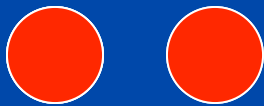
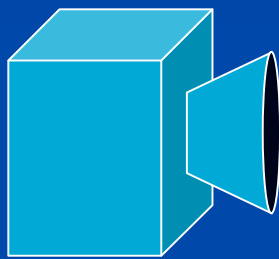
Stochastic U-turn Argument

- Choose small $\varepsilon > 0$. Consider answer 4.



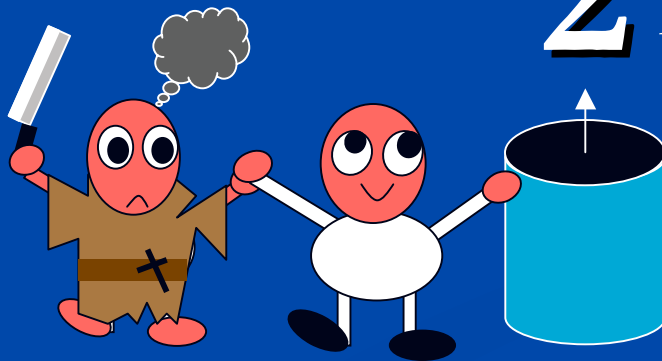
Stochastic U-turn Argument

- By convergence in probability to the truth:



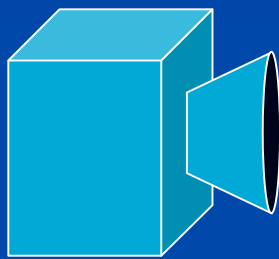
3 $r > 0$

2 $p > 1 - \epsilon/3$



Stochastic U-turn Argument

■ Etc.

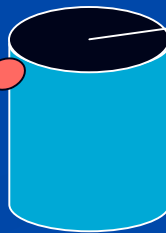
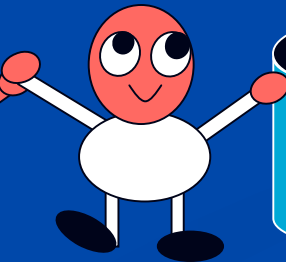
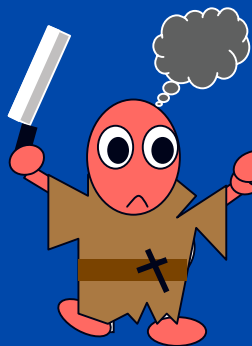


3 $r > 0$

2 $p > 1 - \epsilon/3$

3 $p > 1 - \epsilon/3$

4 $p > 1 - \epsilon/3$

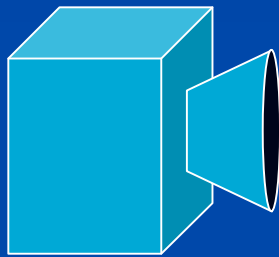


Stochastic U-turn Argument

■ Since ε can be chosen arbitrarily small,

■ sup prob of ≥ 3 revisions $\geq r$.

■ sup prob of ≥ 2 revisions $=1$

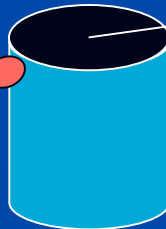
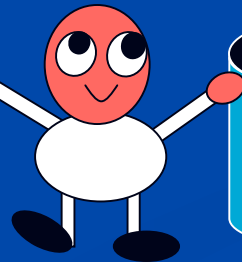
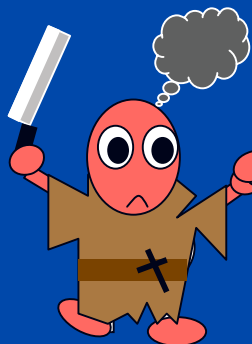


3 $r > 0$

2 $p > 1 - \varepsilon/3$

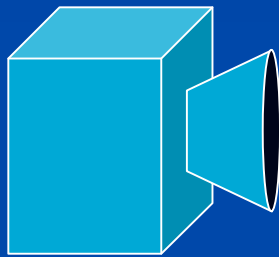
3 $p > 1 - \varepsilon/3$

4 $p > 1 - \varepsilon/3$



Stochastic U-turn Argument

- So sup Exp revisions is $\geq 2 + 3r$.
- But for Ockham = 2.



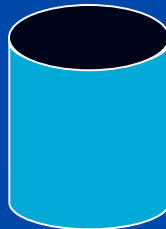
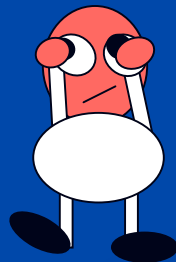
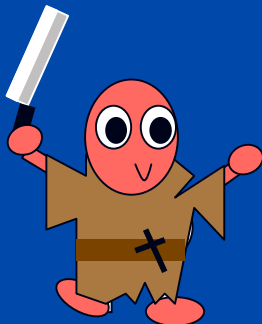
3 _{$r > 0$}

2 _{$p > 1-\epsilon/3$}

3 _{$p > 1-\epsilon/3$}

4 _{$p > 1-\epsilon/3$}

Subproblem



THE END