Barriers to Inference

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Hume's Law

You can't get an 'ought' from an 'is'

It has often been said—in fact, I have said it quite emphatically myself—that it is impossible to deduce ethical conclusions from non-ethical premises. This now seems to me to be a mistake.[Prior, 1960]

Prior's Argument

P1	Tea-drinking is common in England
C	Tea-drinking is common in England or all New Zealanders ought to be shot

P1	Tea-drinking is common in England or all New Zealanders ought to be shot
P2	Tea-drinking is not common in England.
C	All New Zealanders ought to be shot.

More Arguments

$$D \wedge \neg D \vdash N$$

$$D \vdash N \vee \neg N$$

$$D \vdash \neg D \rightarrow N$$

Relevance?

1. $\ulcorner D \wedge N \urcorner$ relevantly entails N.
2. $\ulcorner D \wedge N \urcorner$ is not a descriptive statement.
3. $\ulcorner D \wedge N \urcorner$ is a normative statement.
4. $\ulcorner \neg(D \wedge N) \urcorner$ is a normative statement
5. $\ulcorner \neg D \urcorner$ relevantly entails $\ulcorner \neg(D \wedge N) \urcorner$
6. $\ulcorner \neg D \urcorner$ is descriptive.
7. Some descriptive statement relevantly entails a normative one.

Informal Arguments

1 Jones uttered the words "I hereby promise to pay you, Smith, five dollars."
2 Jones promised to pay Smith five dollars.
3 Jones placed himself under (undertook) an obligation to pay Smith five dollars.
4 Jones ought to pay Smith five dollars.

Jones wants the car to go faster.
If Jones puts pressure on the accelerator the car will go faster.

Jones has a reason to put pressure on the accelerator.

Gideon's Argument

Definition 1 (to flurg) To flurg is to do something that one ought not to do in front of children.

$$\frac{\text{Jones is in the presence of children.}}{\text{Jones ought not to flurg.}}$$

Other Inference Barrier Theses

Russell's Law: You can't derive a universal sentence from particular sentences.

> You can never arrive at a general proposition by inference from particular propositions alone. You will always have to have at least one general proposition in your premises. [Russell, 1918, P.101]

Hume's Second Law: You can't derive a claim about the future from claims about the past and present.

all inferences from experience suppose, as their foundation, that the future will resemble the past...If there be any suspicion that the course of nature may change, and that the past may be no rule for the future, all experience becomes useless, and can give rise to no inference or conclusion. It is impossible, therefore, that any arguments from experience can prove this resemblance of the past to the future; since all these arguments are founded on the supposition of that resemblance.[Hume, 1777, IV,II,32]

Kant's Law: You can't derive a claim about the way things have to be from claims about the way things are.

Collateral Damage

$$\frac{\text{Bird A is white.}}{\text{Bird A is white or all ravens are black.}}$$

$$\frac{\text{The sun has risen every day so far.}}{\text{The sun has risen every day so far or the future will resemble the past.}}$$

$$\frac{\text{Event A was followed by event B}}{\text{Event A was followed by event B or it is necessary that A causes B}}$$

Prior II

Bird A is white or all ravens are black.
Bird A is not white.

All ravens are black.

Conceptual Arguments

The only chair in the room is black.

All the chairs in the room are black.

Hesperus exists.

Necessarily, Hesperus is Hesperus.

Dracula is immortal.

At all future times it will be the case that Dracula is alive.

The Structure of the Flurg-style Examples

The form of the definition of flurg is:

$$\forall \mathsf{x}(\mathsf{Fl}(\mathsf{x}) \Leftrightarrow \mathsf{C} \longrightarrow \mathsf{O}\neg\mathsf{D}\mathsf{x})$$

where 'Fl' is the predicate 'flurg', 'D(x)' is 'x is done' and 'C' is 'there are children around'. The argument against Hume's Law then has the form:

$$\mathsf{C} \vdash \forall \mathsf{x}(\mathsf{Fl}(\mathsf{x}) \longrightarrow \mathsf{O}\neg\mathsf{D}\mathsf{x})$$

Eternal Blue and Determinate Earnestness

Definition 2 (eternal blue) Something is eternal blue, iff, providing colour is determined by chemical structure, it will be blue at all future times.

$$\frac{\text{Colour is determined by chemical structure.}}{\text{Eternal blue things will be blue at all future times.}}$$

Definition 3 (Determinately earnest) Someone is determinedly earnest iff, if character traits are inherited, he is necessarily earnest.

$$\frac{\text{Character traits are inherited}}{\text{Anyone who is determinedly earnest is necessarily earnest.}}$$

Projectable Pink

The generality case is more difficult. But we look at more general features of the flurg argument. The RHS of the definition consists of a condition, under which a statement of the conclusion kind will hold.

Definition 4 (Projectable Pink) Something is projectable pink iff, if it is quartz, everything that is quartz is pink.

$$\frac{\text{This is quartz.}}{\text{If it is projectable pink, all quartz is pink.}}$$

Universal Case

$$\neg\exists x \neg Fx \vdash \forall x Fx$$

- premise is not genuinely particular
- it's just a universal claim in disguise

An idea

- Particular claims do not constrain the whole world in the way that universal ones do.
- They are local where universal claims are global.
- If a particular claim is true, it is also true in extensions of that model.
- If a universal claim is true, it can be made false by some extension of that model.

Genuine Particularity and Universality

Let R be the relation of model-extension ($\supseteq$).

Definition 5 (Genuine Particularity) A sentence is genuinely particular iff for each $\mathfrak{M}, \mathfrak{M}' \in \mathcal{U}$, if $\mathfrak{M} \models A$ and $\mathfrak{M}' \supseteq \mathfrak{M}$ then $\mathfrak{M}' \models A$.

Definition 6 (Genuine Universality) A sentence is genuinely universal iff for each $\mathfrak{M} \in \mathcal{U}$ where $\mathfrak{M} \models A$, there is some $\mathfrak{M}' \supseteq \mathfrak{M}$ where $\mathfrak{M}' \not\models A$.

Which type of sentence am I?

Genuinely particular: Fa, $\neg Gb$, $Fa \wedge Gb$, $\exists x Fx$.

Genuinely universal: $(\forall x)Fx$, $(\forall x)(Fx \supset Gx)$, $\forall x Fx \wedge p$

Neither: $Fa \vee (\forall x)Gx$

Both: $Fa \wedge \neg Fa$

Russell's Law

Theorem 7 (Russell's Law) If Σ is a satisfiable set of sentences, each of which is genuinely particular, and A is genuinely universal, then $\Sigma \nvdash A$.

- Prior counterexamples: conclusion is not genuinely universal
- Inconsistency counterexample: Law doesn't cover those cases

Proof: Russell's law is an instance of the Barrier Construction Theorem

More General Definitions

Definition 8 (R-Fragility) A formula A is R-fragile if and only if $(\forall \mathfrak{M} \in \mathcal{M})(\text{if } \mathfrak{M} \models A \text{ then } (\exists \mathfrak{M}')(\mathfrak{M} R \mathfrak{M}' \text{ and } \mathfrak{M}' \not\models A))$

Definition 9 (R-Preservation) A formula A is R-preserved if and only if $(\forall \mathfrak{M}, \mathfrak{M}' \in \mathcal{M})$ $(\text{if } \mathfrak{M} \models A \text{ and } \mathfrak{M} R \mathfrak{M}' \text{ then } \mathfrak{M}' \models A)$

Barrier Construction Theorem

Theorem 10 (Barrier Construction Theorem) Given a class $\mathcal{M}$ of models, and a collection $\mathsf{X} \cup \{\mathsf{A}\}$ of formulas. If (a) X is satisfied by some model in $\mathcal{M}$; (b) A is R-fragile; and (c) each element of X is R-preserved, then $\mathsf{X} \not\vdash_{\mathcal{M}} \mathsf{A}$.

Necessity

We'll take propositional S5 as our example, since it is i) simple and ii) strong.

Models are sets of words on which a reflective, symmetric and transitive accessibility relation is defined, and they contain a privileged world g which is used for defining truth in the model.

Let R be the relation of modal model extension ($\sqsupseteq$).

Definition 11 (modal particularity) A sentence A is modally particular iff it is $\sqsupseteq$-preserved, that is, for each $\mathfrak{M}, \mathfrak{M}' \in \mathcal{V}$, if $\mathfrak{M} \models A$ and $\mathfrak{M}' \sqsupseteq \mathfrak{M}$ then $\mathfrak{M}' \models A$.

Definition 12 (modal generality) A sentence A is modally general iff is $\sqsupseteq$-fragile, that is, for each $\mathfrak{M} \in \mathcal{V}$ where $\mathfrak{M} \models A$, there is some $\mathfrak{M}' \in \mathcal{V}$, such that $\mathfrak{M}' \sqsupseteq \mathfrak{M}$ and $\mathfrak{M}' \not\models A$.

Kant's Law

Theorem 13 (Kant's Law) If Σ is a satisfiable set of sentences, each of which is modally particular, and A is modally general, then $\Sigma \not\models A$.

Proof: Kant's law is an instance of the Barrier Construction Theorem

Which sentences are modally general?

The class of modally particular sentences includes sentences from three groups:

i) sentences with no modal operators, such as p, and $\mathsf{p} \vee \mathsf{q}$,

ii) sentences such as $\Diamond\mathsf{p}$ and $\neg\Box\mathsf{p}$ whose truth can be secured by a single world, or a structure of worlds, regardless of additions to that structure. For S5, $\Box\Diamond\mathsf{p}$ falls into this category.

iii) the logical truths of modal logic, such as $\Box p \vee \neg \Box p$.

Hume's 2nd Law

We let the R relation between models be the symmetric relation of history sharing ($\curlyvee$) where two models stand in this relation if they are the same with respect to the present moment and all earlier moments.

Definition 14 (genuinely historical) A sentence A is genuinely historic iff it is $\curlyvee$-preserved, that is for each $\mathfrak{M}, \mathfrak{M}' \in \mathcal{T}$, if $\mathfrak{M} \models A$ and $\mathfrak{M}' \curlyvee \mathfrak{M}$ then $\mathfrak{M}' \models A$.

Definition 15 (genuinely future-constraining) A sentence A is genuinely future-constraining iff it is $\curlyvee$-fragile, that is, for each $\mathfrak{M} \in \mathcal{T}$ where $\mathfrak{M} \models A$, there is some $\mathfrak{M}' \in \mathcal{T}$, such that $\mathfrak{M}' \curlyvee \mathfrak{M}$ and $\mathfrak{M}' \not\models A$.

Theorem 16 (Hume's Second Law) *If Σ is a satisfiable set of sentences, each of which is genuinely historic, and A is genuinely future-constraining, then $\Sigma \not\models A$.*

Proof: Hume's 2nd law is an instance of the Barrier Construction Theorem

Which sentences are historic/future-constraining?

Suppose that the $>$ relation is transitive, irreflexive and anti-symmetric. Then:

i) p, Pp, Hp, GPp are genuinely historic,

ii) Fp and Gp are genuinely future-constraining and

iii) $\mathsf{p} \vee \mathsf{Fp}$ and PFp are neither genuinely historic nor genuinely future-constraining.

An Ambiguity in Hume's Law

Does Hume's Law say that one cannot get 'ought'-sentences from 'is'-sentences? Or does it say that one cannot get normative sentences from descriptive sentences?

Reading Hume suggests the former.

In every system of morality, which I have hitherto met with, I have always remark'd, that the author proceeds for some time in the ordinary way of reasoning, and establishes the being of a God, or makes observations concerning human affairs; when of a sudden I am surpriz'd to find, that instead of the usual copulations of propositions, is, and is not, I meet with no proposition that is not connected with an ought, or an ought not. This change is imperceptible; but is, however, of the last consequence. For as this ought, or ought not, expresses some new relation or affirmation, 'tis necessary that it shou'd be observ'd and explain'd; and at the same time that a reason should be given, for what seems altogether inconceivable, how this new relation can be a deduction from others, which are entirely different from it.” [Hume, 1978, III, i, 3]

But the literature suggests the latter.

We think both are true and so will formulate both.

Normative Extension

Definition 17 (normative extension - $\Supset$) $\mathfrak{M}' \Supset \mathfrak{M}$ just in case you can obtain $\mathfrak{M}'$ from $\mathfrak{M}$ by adding new worlds and extending the $\mathcal{S}$ relation relation.

Definition 18 (normative particularity) A sentence A is normatively particular iff it is $\Supset$-preserved, that is, for each $\mathfrak{M}, \mathfrak{M}' \in \mathcal{N}$, if $\mathfrak{M} \models A$ and $\mathfrak{M}' \Supset \mathfrak{M}$ then $\mathfrak{M}' \models A$.

Definition 19 (normative generality) A sentence A is normatively general iff is $\Supset$-fragile, that is, for each $\mathfrak{M} \in \mathcal{N}$ where $\mathfrak{M} \models A$, there is some $\mathfrak{M}' \in \mathcal{N}$ such that $M' \Supset \mathfrak{M}$ and $\mathfrak{M}' \not\models A$.

Which sentences are normatively particular/general?

Three kinds of sentence turn out to be normatively particular:

i) no deontic operators e.g., p and $\mathsf{p} \supset \mathsf{q}$;

ii) sentences containing deontic operators whose truth can be secured by some structure of worlds at least one of which is in the relation of moral satisfaction to the actual world (regardless of what else we add to that structure), e.g., Pp, PPp and $\neg\mathsf{Op}$

iii) the deontic tautologies.

The normatively general sentences include $\mathsf{O}p$ and $\neg\mathsf{P}p$.

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Normative Translation

Definition 20 (normative translation - $\between$) $\mathfrak{M}' \between \mathfrak{M}$ just in case you can obtain $\mathfrak{M}'$ from $\mathfrak{M}$ simply by changing the pairs of worlds related by the S relation.

Definition 21 (descriptiveness) A sentence A is descriptive iff it is $\between$-preserved, that is, for each $\mathfrak{M}, \mathfrak{M}' \in \mathcal{N}$, if $\mathfrak{M} \models A$ and $\mathfrak{M}' \between \mathfrak{M}$ then $\mathfrak{M}' \models A$

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Definition 22 (normativity (sufficient condition)) A sentence A is normative if it is $\between$-fragile, that is, for each $\mathfrak{M} \in \mathcal{N}$ where $\mathfrak{M} \models \mathsf{A}$, there is some $\mathfrak{M}' \in \mathcal{N}$ such that $\mathsf{M}' \between \mathfrak{M}$ and $\mathfrak{M}' \not\models \mathsf{A}$.

Which sentences are descriptive/normative?

The descriptive sentences are, intuitively, those that make no appeal to the deontic structure of the model (at least, not the parts that can change. Deontic tautologies will be descriptive.) They include p, $\mathsf{p} \vee \mathsf{q}$ and $\mathsf{Op} \vee \neg\mathsf{Op}$.

The normative sentences are, intuitively, the ones which make demands on the arrangement of the relation of moral satisfaction, e.g., Pp.

N.B. important normatively general sentences such as $\mathbf{Op}$ are not $\between$-fragile. Consider a model in which every world is one where $\mathbf{p}$ is true. No rearranging of the S relation can make it the case that $\mathbf{Op}$ is false, and so $\mathbf{Op}$ not $\between$-fragile (though it is not $\between$-preserved either).

Normativity and Hume's Laws

Definition 23 (Normativity) A sentence A is normative iff it is either $\between$-fragile or $\Supset$-fragile, that is, either i) for each $\mathfrak{M} \in \mathcal{N}$ where $\mathfrak{M} \models \mathsf{A}$, there is some $\mathfrak{M}' \in \mathcal{N}$ such that $\mathfrak{M} \between \mathfrak{M}$ and $\mathfrak{M}' \not\models \mathsf{A}$ or ii) for each $\mathfrak{M} \in \mathcal{N}$ where $\mathfrak{M} \models \mathsf{A}$, there is some $\mathfrak{M}' \in \mathcal{N}$ such that $\mathfrak{M}' \Supset \mathfrak{M}$ and $\mathfrak{M}' \not\models \mathsf{A}$

Theorem 24 (Hume's Law (Ought-formulation)) If Σ is a satisfiable set of sentences, each of which is normatively particular, and A is normatively general, then $\Sigma \not\models A$.

Theorem 25 (Hume's Law (Normativity-formulation If Σ is a satisfiable set of sentences, each of which is descriptive, and A is normative, then $\Sigma \not\models A$.

Proof: The ought-formulation is follows from the Barrier Construction Theorem, and the normativity formulation follows from the Barrier Construction Theorem and lemma 26.

Lemma 26 All descriptive sentences are normatively particular.

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