Inductive Influence

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Abstract

Objective Bayesianism has been criticised for not allowing learning from experience: it is claimed that an agent must give degree of belief $\frac{1}{2}$ to the next raven being black, however many other black ravens have been observed. I argue that this objection can be overcome by appealing to *objective Bayesian nets*, a recently-developed formalism for representing objective Bayesian degrees of belief. Under this account, previous observations exert an *inductive influence* on the next observation. I show how this approach can be used to capture the Johnson-Carnap continuum of inductive methods, as well as the Nix-Paris continuum, and speculate as to how inductive influence might be measured.

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§1
Introduction

To what extent should I believe it will rain tomorrow? Objective Bayesianism is a theory which puts forward precise answers to questions like this.[1] In common with other Bayesians, objective Bayesians argue that an agent's degrees of belief should be probabilities. But objective Bayesians go further by isolating a single probability function as a candidate for an agent's degrees of belief.[2] This probability function is objectively determined by the extent of the agent's background knowledge.

Background knowledge isolates the most appropriate probability function in two ways. First, the agent's degrees of belief should make the commitments that are warranted by her background knowledge: those probability functions that do not satisfy constraints imposed by background knowledge should be eliminated from consideration. Second, the agent should not believe things to a greater extent than is warranted by background knowledge: the agent should select a probability function, from all those remaining, that embodies the most middling degrees of belief, those furthest from the extremes of 0 and 1.[3] Distance from the extremes is measured by entropy $H = -\sum_{\omega \in \Omega} p(\omega) \log p(\omega)$; hence the Maximum Entropy Principle: an agent should adopt as her belief function, from all the probability functions that satisfy constraints imposed by background knowledge, that which has maximum entropy.

Objective Bayesianism faces a number of challenges,[4] not least the charge that learning from experience becomes impossible on the objective Bayesian account (§2). In §3 I shall argue that this charge is a mistake, attributable to a misapplication at the first stage of the entropy maximisation process: the constraints imposed by background knowledge have not been correctly assessed. In order to elucidate these constraints I introduce the machinery of *objective Bayesian nets* in §4. Such nets offer a way of representing maximum entropy probability functions that renders probabilistic dependence and independence relationships perspicuous. They are useful here, I claim, because when learning from experience past observations exert an *inductive influence*—a type of dependence relationship—on future observations (§5).

When objective Bayesian nets are applied to the problem of learning from experience, the resulting formalism yields the Johnson-Carnap continuum of inductive methods as a natural special case (§6). In §7 we see that the Nix-Paris continuum of inductive methods emerges as another special case—though arguably a less central special case. The question now arises as to which point in the Johnson-Carnap continuum yields the most appropriate inductive method from the objective Bayesian perspective. In §8 I tentatively suggest that the classification efficiency of the agent's language might provide the answer to this question. Finally, §9 addresses possible extensions to the method presented here.

[1] (Rosenkrantz, 1977; Jaynes, 2003)

[2] I will only be considering finite probability spaces in this paper. The extension of objective Bayesianism to the infinite case is steeped in controversy and arguably proceeds at the expense of uniqueness of the most appropriate probability function—see Williamson (2006b, §19).

[3] (Williamson, 2006a)

[4] (Williamson, 2006b, Part III)