Sentences, Propositions, and Beliefs

(Draft)

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May 22, 2006

Abstract: *We offer an account of beliefs as patterns of behaviour, including linguistic behaviour. Such an account of beliefs is implicit in Ramsey's treatment of probability, but is extended here to a much wider domain. Various puzzles then appear to dissolve.*

What is it that we know or believe? Is it sentences? Or is it propositions? In other words, in "Jack believes that X", what kind of object takes the place of the second argument? And is *believes* actually a two place relation between something and something else? The answers to these questions lie at the heart of issues like logical omniscience and Frege's problem.

Let us temporarily assume that *believes* is indeed a two place relation and we are not too concerned about the nature of the first argument (although in my view we should be at least in some contexts). Let us consider the nature of the second argument.

If it is indeed sentences which we believe then it seems that two people who speak two different languages could not believe the same thing. And that seems unreasonable. Surely, if they are both watching a sky full of dark clouds then very probably they both believe it is going to rain and the difference in language should not matter. Moreover, someone who believes that Jack is married to Jane also believes that Jack is married. But if these are just two *sentences* which are believed, then nothing seems to prevent someone believing that Jack is married to Jane but is not married. (We will ignore strange situations where we might wish to say this).

That seems to make a case that it is propositions which are the objects of our beliefs.[1] Then it is possible for people speaking different languages to believe the same proposition

[1] We put aside for the moment the question of what propositions are in the first place.

p which is expressed in their respective languages by two different sentences s and s'. When Jack says "It is raining," and Ravi says, "Baarish ho rahi hai," they are expressing belief in the same proposition that it is raining.

Moreover, if one subscribes to propositions as sets of possible worlds, then one can give a very nice mathematical account of beliefs this way. For there is then a set W_j of possible worlds which Jack is willing to allow,[2] and Jack believes a proposition P, regarded now as a set of possible worlds, *just in case* $W_j \subseteq P$. This is nice. We can set up logics of belief based on this inclusion relation and discover that they satisfy the logic KD45, or at least the logic KD4.

But our troubles are not over yet. For $0 = 0$ is the same proposition as Fermat's theorem, it is true in all possible worlds. Those of us who knew[3] $0 = 0$ also knew Fermat's theorem all along! This is essentially Plato's argument in Meno. If something could be proved to you then you knew it all along. Of course even Plato did not reckon with the possibility of Gödel's theorem according to which there are truths which cannot be proved.

Morever, the propositional account immediately leads to logical omniscience. If I know some propositions then I also know all their consequences. But in that case, as Dummett asks in [3], why bother with deduction at all? Deduction can never lead to new propositions which were not already implicit in what we knew. It can only lead to new *sentences*. So sentences must play *some* role.[4]

I shall argue that we believe *both* sentences and propositions, but of course the word "believe" will then have to have two different, but related, meanings.

Let us first consider propositions. It can be said, very roughly, that Nature speaks to us, and to inanimate devices, in propositions. First consider the latter. Suppose a thermostat is designed to turn on the heat when the outside temperature is under 50^oF. Now 50^oF is also 18^oC. If the thermostat is taken to France, will it have to be *told* that $50^oF = 18^oC$? Of course not. It will turn on the heat also when the temperature falls

[2] W_j represents Jack's strongest belief, so to say

[3] For convenience I shall use "know" as an abbreviation for "believe truly"

[4] It has been suggested that a proposition is not a *chunk* of all logically equivalent sentences but has a finer grain, but making that graininess precise is problematic. We shall not pursue this option here.

below 18^oC.

If a thermostat is said to believe a proposition *Cold* when it turns on the heat, and to believe *Warm* when it turns off the heat then this sort of belief will indeed obey the logic KD45. For instance, if it is cold when it snows, then the proposition *Snow* implies the proposition *Cold*, and indeed, when it snows, the thermostat will turn on the heat.

When listening to Nature, *we* also use propositions and not sentences. When the temperature is 0^oC we feel cold, and we also feel cold when it is 32^oF. However, we will utter different sentences depending on what our mother tongue is.

Perhaps a thermostat cannot be said to have beliefs, and perhaps we ourselves cannot be said to have a belief when we merely experience some sensation. But if we were willing to be flexible here, then we would have the proper logic.

But suppose now that we do not have a thermostat, but only a thermometer, and that some person is told, "Turn on the heat when the temperature falls below 50^oF." She might not turn on the heat if she only has a centigrade thermometer which shows 18^oC. She would need to be *told* that $50^oF = 18^oC$. Her lack of logical omniscience would have exhibited itself.

Similar issues will arise with our friend Lois Lane. Suppose she has invited Clark Kent for dinner but he has not responded. If she then sees Superman standing on her balcony, she is unlikely to ask him, "Are you coming for dinner, then?" She has a sentential belief that Clark Kent has been invited to dinner, and she does not have the sentential belief that Superman has been invited to dinner, although (as a common account goes) the two sentences amount to the same proposition. Thus her belief in this context cannot be quite propositional.

Nonetheless, when we believe a sentence, we also *tend to* believe other equivalent sentences, or sentences implied by the sentences we do believe. And this is because the processes of deduction and information gathering which allow us to deduce some sentences from others have *pragmatic value*. If Lois Lane *has* invited Clark Kent to dinner, and believes the sentence "Superman is standing on my balcony," it is in her interest to also come to believe the sentence "Clark Kent is standing on my balcony," for then she can find out about the dinner. We carry out deductions and gather information because it is useful. But it certainly does not come for free in the shape of logical omniscience.

1 Some technical details

We assume given a space $\mathcal{B}$ for some agent whose beliefs we are considering. The elements of $\mathcal{B}$ are the belief states of that agent, and these are not assumed to be sentences in Mentalese although for some restricted purposes they could be. There are three important update operations on $\mathcal{B}$ coming about as a result of (i) events observed, (ii) sentences heard, and (iii) deductions made. Elements of $\mathcal{B}$ are also used to make *choices*. Thus in certain states of belief an agent may make the choice to take his umbrella and we could then say that the agent believes it is raining. Many human agents are also likely to make the choice to *say*, "I think it is raining and so I am taking my umbrella" but clearly only if the agent is English speaking. Thus two agents speaking different languages, both of whom are taking their umbrellas, but making different noises, have the same belief in one sense but a different one in another. Both of these will matter. Later on we shall look into the connection.

Deduction is an update which does not require an input from the outside. But it *can* result in a change in $\mathcal{B}$. Suppose for instance that Jane thinks it is clear, and is about to leave the building without an umbrella. She might say, "Wait, didn't I just see Jack coming in with a wet umbrella? It must be raining." The sight of Jack with a wet umbrella might not have caused her to believe that it was raining, perhaps she was busy with a phone call. But the memory of that wet umbrella may later cause a deduction to take place of the fact that it is raining.

Thus our three update operations are:

$\mathcal{B} \times \mathcal{E} \rightarrow_e \mathcal{B}$

A belief gets revised by witnessing an event.

$\mathcal{B} \times \mathcal{L} \rightarrow_s \mathcal{B}$

A belief gets revised through hearing a sentence. And hearing two logically equivalent sentences s, s' need not result in the same change occurring, although they may, in case the agent knows they are equivalent.

$\mathcal{B} \rightarrow_d \mathcal{B}$

A deduction causes a change in the belief state (which we may sometimes represent as an *addition*).

Here E is the set of events which an agent may witness and L is some language which an agent understands. The last map $\rightarrow_d$ for deduction is non-deterministic as an agent in the same state of belief may make one deduction or another.

Finally, we also have a space $\mathcal{S}$ of *choice sets* where an agent makes a particular choice among various alternatives. This gives us the map

$\mathcal{B} \times \mathcal{S} \rightarrow_{ch} \mathcal{B} \times C$

An agent with a certain belief makes a choice among various alternatives.

While $\mathcal{S}$ is the family of choice sets, C is the set of possible choices. Thus {*take umbrella, don't take umbrella*} is a choice set and an element of $\mathcal{S}$ but *take umbrella* is a choice and an element of C.

Among the choices that agents make are choices to assent to or dissent from sentences. But there is no *logical* reason why an agent who assents to "It is raining" must take an umbrella or a raincoat. It is just the more pragmatic choice to take the umbrella when one says it is raining, because otherwise either one gets wet, or one is suspected of insincerity. We could say that the agent believes the sentence "It is raining", and dis-believes the proposition that it is raining. But we tend to be uncomfortable with such an account and prefer to say that the agent is either lying or confused.

Of course an agent may prefer to get wet and in that case, saying "It is raining," and not taking an umbrella are perfectly compatible choices. This shows that an agent's preferences need to be taken into account when correlating the actions which an agent takes and what an agent believes. But we usually do not want to get wet and to make such choices, and usually we do not say what we do not believe. It does not work for us.

Thus our theory of an agent presupposes such a belief set $\mathcal{B}$, and appropriate functions u, u', d, ch. We can understand an agent (with some caveats) if what we *see* as the effects of these maps conforms to some theory of what an agent wants and what the agent thinks. And we succeed pretty well. *Contra* Wittgenstein, we not only have a theory of what a lion wants, and what it means when it growls, we even have theories for bees and bats. Naturally these theories do not have the map $\rightarrow_s$ except with creatures like dogs or cats and some parrots (who not only "parrot" our words but understand them to some extent [6]).

2 Sentences and Propositions

Speaking *conventionally*, an agent may have beliefs as to how the world is, and beliefs as to what is the best thing to do. Still speaking conventionally, we imagine agents as making the best choices, given their view of the world and their preferences. So strictly speaking our map $\rightarrow_{ch}$ should be a map from $\mathcal{B} \times \mathcal{S} \times \mathcal{P}$, where $\mathcal{P}$ is the space of preference orders. Note that elements of $\mathcal{S}$ also contain choice sets which consist of choices of what to *say*. There is no *logical* connection between choices which an agent makes when faced with various elements of $\mathcal{S}$, but there is a *pragmatic connection*. An agent who prefers to be thought of as truthful, and who is known not to like getting wet, will either take an umbrella and *say* "It is raining," or *not* take an umbrella and say "I think it is clear."

Of course an agent may have different beliefs at different times (again speaking conventionally) and also have different preferences at different times. But an element of $\mathcal{B}$ will induce a map from $\mathcal{S} \times \mathcal{P}$. We could legitimately call such a map (or the hypothetical element which induces it) a *belief state*. And typically, a belief state will be revealed by its values on $\mathcal{S}_L \times \mathcal{P}$. Here $\mathcal{S}_L$ consists of those choice sets where the agent is asked to choose among some purely linguistic acts. A multiple choice question (or indeed *any question*) on an examination demands such a linguistic choice.

Thus for an agent for whom we have already somehow established that she does not want to get wet, we will expect that the choice whether to take an umbrella or not will correlate with a choice between saying “It is raining,” and saying “It is clear.” The belief that it is raining is itself revealed by taking the umbrella, but may also (usually) be revealed by saying, “It is raining.” If we define a proposition as a map from $\mathcal{S} \times \mathcal{P}$ into C, then it is clear in what sense a sentence corresponds to a proposition. An agent who assents to some sentence s also tends to act in a way as he would if he believed that the actual world was an element of the proposition (w-proposition) p expressed by s.

But we also have other notions of *proposition.* A proposition is a set of possible worlds. Or a proposition is defined by a sentence s and is the set of all sentences s' logically equivalent to s. How do these notions come to correspond?

If we take a naive theory of rational choice, then given an agent's beliefs and the agent's preference, there is often also a most rational choice for the agent to make. Thus we can say that given the agent's preference (presumed known to us), and the agent's actual choice (observed by us), the agent believes that the state of the world is such that this actual choice is the best one. Suppose the agent chooses c from the set X of presented choices. Let P be the set of all worlds in which given the agent's actual preference, c *would* be the best choice from X. We can now say that the agent believes P.

Given this semantics we can readily deduce that when Lois Lane sees Superman standing on her balcony and *fails* to ask him if he is coming to dinner, she does *not* believe the proposition that he has been asked to dinner, and she also does not believe the *sentence* “Clark Kent is standing on my balcony.” She does not, and cannot deduce that sentence from the sentence “Superman is standing on my balcony,” which she does believe.

But what about *our belief* that the two sentences express the same proposition? But this is simple, for surely *we* can deduce “Clark Kent is standing on Lois Lane's balcony,” from ”Superman is standing on Lois Lane's baclony,” and if our preference is to help her, we might tell her that Clark Kent is Superman. Fron our point of view, the two sentences belong to the same proposition, either is deducible from the other. For her they do not.

2.1 More on propositions

We now have several definitions for propositions. They are respectively,

1. A proposition is a set of possible worlds.

2. A proposition is an equivalence class of sentences under some sort of deductive process. (Sometimes people do not take an entire equivalence class but only sentences 'near' some given sentence. But it seems *ad hoc* and we shall not go into it.)

3. A proposition is a map from $\mathcal{S} \times \mathcal{P}$ to C. (Here $\mathcal{S}$ is a subset of the power set of C).

These seem different, and surely the last one will seem unrecognizable to most people who have thought about them. There is of course a pre-cursor to that notion in Ramsey's work [5] where he derives subjective probabilities (a form of belief) from choices made by individuals who are offered certain bets.

What we shall now do is to correlate the three notions in those situations where no throny issues arise, and show that they are equivalent. Suppose that we have a purely propositional language based on (say) three propositional constants p, q, r. Now a sentence is of course a formula in the resulting language. E.g. $p \rightarrow \neg q$ is a sentence. A possible world is merely a truth assignment so that there are eight possible 'worlds' and therefore 256 sets of possible worlds.[5] It is well known that there is a formula in disjunctive normal form for each set of possible worlds so that the correspondence between sentences and propositions is perfect. If two sentences are truth functionally equivalent then they yield the same set of possible worlds, i.e., the same proposition. Finally, imagine an individual who believes that a certain set of X worlds is possible. Since there are only finitely many such worlds his belief can be expressed in a single formula. But how

[5]Of these, 255 are nonempty and correspond to possible states of belief. If an agent j believes nothing non-trivial about p, q, r then W_j will be all 16 truth assignments, and if an agent is sure that p, q, r are all true, then W_j will be the set conataining the unique truth assignment making $p \wedge q \wedge r$ true.

does this yield a choice function?

Given a preference order P and a set X of truth assignments, consider two actions a and b such that if an element of X is true, then action a yields a better state than action b and if some element of $W - X$ is true, then action b is better than action a. If, given a choice between a, b the agent chooses a then we can say that the agent believes X. Thus if a rat in a maze, and seeking cheese, is given a choice between going left and going right, and goes right, we may say that the rat believes that the cheese is on the right.

Thus we have shown that *in this simple setup* a proposition as a choice function uniquely determines an agent's propositional beliefs as a set of possible worlds. We now get three definitions:

Definition 2.1 *A* w-proposition *is a set of possible worlds.*

Definition 2.2 *An* s-proposition *is a set of sentences, closed under the equivalence relation of being logically equivalent (another equivalence relation might well be used).*

Given a w-proposition p, we *could* say that Jill believes p if there is a sentence s such that p is the set of worlds in which s is true and Jill assents to s. One difficulty is that s and s' may be logically equivalent and therefore define the same set of possible worlds, and yet Jill assents to s and not to s'. If Jill is logically omniscient, and there is a deduction from s to s' and *vice versa* (as there will be in first order logic) then it is only a logical failure on Jill's part that she assents to s and not to s'. This is bad enough, but worse can happen. s might be of the form $P(\ell)$ and s' of the form $P(\ell')$ where ℓ, ℓ' are names of the same city. If Jill does not know this fact, then we cannot blame her logical abilities for her failure to see the equivalence. So *does* Jill believe p or not?

Definition 2.3 *A* ch-proposition *is a map from choice situations and preference orders into choices.*

Given a ch-proposition f we could say that Jill believes f just in case she acts according to f (or would if she had certain preferences and was presented with certain choices).

Thus Jill believes *that it is raining* just in case she takes an umbrella on those occasions when we know she does not want to get wet. Her belief is revealed by how she acts, and she does not need to say anything.[6]

This case does not create the problem which the other cases presented. If Jill has seen Jack come in with a wet umbrella, but goes out without one herself, then she does *not* believe that it is raining, even though this fact is deducible from Jacks' umbrella and Jill's other beliefs.

In the case we looked at, there is a 1-1 correspondence between propositions-w and propositions-s. The correspondence could break down in more complex situations, like those involving rigid designators for instance. But in the simple setup it works. Finally, a ch-proposition determines a s-proposition (or w-proposition) but is not determined by it.

The last notion of proposition, ch-proposition is much richer. For instance, as Ramsey showed, one can also find an agent's subjective probabilities by watching the choices which the agent makes, but these cannot be found from merely knowing which sentences the agent (fully) believes.

Moreover, the notion of ch-proposition allows us to understand agents whose beliefs seem apparently inconsistent.

Suppose that a husband asks his wife on a Friday if their bank is open on Saturdays and she says yes. He then says that they have to make a payment by Saturday evening, and that a payment on Monday will incur a penalty. At this point she makes a decision to call the bank, a decision that she had not made earlier. Does she believe that the bank is open on Saturdays? Perhaps she assigns to *bank is open on Saturdays* a probability greater than .5, but not high enough to accept the risk of a penalty. Her action can clearly be understood as a choice function, but not as a belief in a proposition as a set of possible worlds. Of course, her choice can also be understood probabilistically - perhaps

[6]Certainly many readers will rebel at the thought that taking one's umbrella when one does not want to get wet is the *same* as *the belief that it is raining.* The reader has our sympathy – we feel the same way ourselves!

she assigns to *bank is open on Saturdays* a probability greater than .5 (allowing her to say it is open) but a probability less than .9 (making her reluctant to take a risk). But even probability will not accomodate our next example.

Travis and brill-bream: In this example [18], Jones has heard that certain fish called brill are delicious. She has also seen bream in a display somewhere and taken them to be brill. Today she has decided to have fish for dinner and decides on brill, or so she thinks. She now goes to a grocer and asks him for a brill. Travis wants now to know if Jones wants brill, or bream, or fish that are brill but look like bream, or none of these.

But let us see what might happen if Jones does go shopping for a brill. Here are two possible dialogues:

Dialogue 1:

Grocer: Here you are, madam.
Jones: No, I don't like the look of that, do you have any tofu?

Dialogue 2:

Grocer: Here you are, madam.
Jones: But surely that isn't brill. Aren't brill yellow?
Grocer: No, ma'am, it is bream which are yellow. But they taste something awful and I do not stock them. I know many people do not like the blue color of brill, but they are delicious.
Jones: All right, I will trust you. Please tell me what I owe you.

Neither of these dialogues is strange. Do they answer Travis' questions? We might say that in the first case Jones wanted fish that actually looked like bream, and in the second that she really wanted brill (given that they were tasty). But in either case her behaviour can be understood in itself as behaviour. There *is* a proposition in *our sense* which she can be said to believe, but it is not expressible in normal English.

2.2 From Sentences to Propositions

Suppose that an agent believes a sentence s in some language, say English in the sense of asserting s or assenting to it when asserted by someone else. Then we expect the agent's other choices to correspond to this purely linguistic behaviour. Thus there is a correlation between an agent's linguistic behaviour and the same agent's other behaviour. But this correlation is not logically required. It happens because the agent belongs to a community and co-ordination of actions takes place via linguistic behaviour.

3 Concluding Remarks

We suggested that there is a way of formalizing the notion of *proposition* in terms of an agent's behaviour which includes our tradiional notion of a proposition but goes beyond it. Moreover, this wider notion of proposition does not lead to logical omniscience for sentences.

The resistance to thinking of beliefs *as* behaviour, rather than behaviour as *arising from beliefs* may be a remnant of Cartesianism. But not being an expert we will not venture further into this territory. Also, while we may assign preference orderings to animals, it is not clear how we can speak of the preferences of a thermostat. Clearly this is a figure of speech, and the preferences of the thermostat are merely the preferences of the person who designed it, and of the person who bought it.

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