Why Epistemology Can't be Operationalized

(L)	Condition C is luminous if and only if in every case in which C obtains, one is in a position to know that C obtains.	

(O0)	C is luminous.	Assumption
($O1_i$)	If at t_i one is in a position to know that C obtains, then at t_{i+1} C obtains.	Assumption
($O2_i$)	If at t_i C obtains, then at t_i one is in a position to know that C obtains.	O0, LP
($O3_i$)	At t_i C obtains.	Assumption
($O4_i$)	At t_i one is in a position to know that C obtains.	$O2_i$, $O3_i$, MP
($O3_{i+1}$)	At t_{i+1} C obtains.	$O1_i$, $O4_i$, MP
($O3_0$)	At t_0 C obtains.	Assumption
($O3_n$)	At t_n C obtains.	$O3_i \rightarrow O3_{i+1}$, $i=0, \ldots, n-1$
(O5)	At t_n C does not obtain.	Assumption

(LP1)	Condition C is *luminous in probability 1* if and only if in every case in which C obtains, the probability that C obtains is 1.	

(0)	C is luminous in probability 1.	Assumption
(1_i)	If at t_i the probability that C obtains is 1, then at t_{i+1} C obtains.	Assumption
(2_i)	If at t_i C obtains, then at t_i the probability that C obtains is 1.	0, LP
(3_i)	At t_i C obtains.	Assumption
(4_i)	At t_i the probability that C obtains is 1.	2_i, 3_i, MP
(3_{i+1})	At t_{i+1} C obtains.	1_i, 4_i, MP
(3_0)	At t_0 C obtains.	Assumption
(3_n)	At t_n C obtains.	$3_i \rightarrow 3_{i+1}$, $i=0, \ldots, n-1$
(5)	At t_n C does not obtain.	Assumption

(0)	C is luminous in probability 1.	Assumption
(1)	If C obtains at no time in a nonempty interval (*t, t***), then at no time in the interval (t-ε, t**+ε) is the probability that C obtains 1.	Assumption
(2)	If at no time in the interval (t-ε, t**+ε) is the probability that C obtains 1, then C obtains at no time in the interval (t-ε, t**+ε).	0, LP
(6)	If C obtains at no time in a nonempty interval (*t, t***), then C obtains at no time in the interval (t-ε, t**+ε).	1, 2
(7_0)	C obtains at no time in the interval (t-δ, t).	Assumption
(7_i)	C obtains at no time in the interval (t-δ-iε, t+iε).	(6), (7_0)
(7)	C obtains at no time in the interval [t_0, t_n].	(7_i)
(3_0)	At t_0 C obtains.	Assumption

(LPx) Condition C is *luminous in probability x* if and only if in every case in which C obtains, the probability that C obtains is x.

C……………………….t*_____________C)[¬C---¬C
Prob 0.5 Prob ε Prob 0.5 - ε

(LP>x) Condition C is *luminous in probability* > *x* if and only if in every case in which C obtains, the probability that C obtains is more than x.

In every case in which C obtains, the probability that C obtains is more than 0.5.

Expectation (finite case) $E(X) = \sum_{x\varepsilon I} xP(X = x)$

Example. Time is discrete. At each time t: P(T = t-1) = P(T = t) = P(T = t+1) = 1/3.
Thus always E(T) = T.

There are countably many atomic variables X, Y, Z, … (informally, denoting real numbers; they correspond to random variables).

For each rational number *c* there is an atomic constant [*c*] (informally, denoting *c*).

If T and U are terms then T+U is a term (+ is informally read as 'plus').

If A is a formula then ¬A is a formula (¬ is informally read as 'it is not the case that').

If A and B are formulas then A & B is a formula (& is informally read as 'and').

If A is a formula then V(A) is a term (V is informally read as 'the truth-value of').

If T and U are terms then T≤U is a formula (≤ is informally read as 'is less than or equal to').

If T is a term then E(T) is a term (E is informally read as 'the expectation of').

'T=U' abbreviates T≤U & U≤T.
'T<U' abbreviates T≤U & ¬U≤T
'P(A)' abbreviates E(V(A)) (P is informally read as 'the probability of').

A *model* is a triple <W, Prob, F>, where W is a nonempty set, Prob is a function from members of W to probability distributions over W and F is a function from atomic terms to their intensions (functions from members of W to real numbers).

val(*w*, A): truth-value at *w* of A
den(*w*, A): denotation at *w* of T, den(*w*, T)

If T is an atomic variable, den(*w*, T) = F(T)(*w*).

If *c* is a rational number, den(*w*, [*c*]) = *c*.

If T and U are terms, den(*w*, T+U) = den(*w*, T) + den(*w*, U).

If A is a formula, val(*w*, ¬A) = 1-val(*w*, A).

If A and B are formulas, val(*w*, A & B) = min{val(*w*, A), val(*w*, B)}.

If A is a formula, den(*w*, V(A)) = val(*w*, A).

If T and U are terms, val(*w*, T≤U) = 1 if den(*w*, T) ≤ den(*w*, U) and 0 otherwise.

If T is a term, den(*w*, E(T)) is the expectation with respect to the probability distribution Prob(*w*) of the random variable whose value at each world *x*εW is den(*x*, T).

For example if W is finite, then den(*w*, E(T)) = $\sum_{x\varepsilon W}$ den(*x*, T)Prob(*w*)({*x*})

Count <W, Prob, F> as a model only if den(*w*, E(T)) is well-defined for every *w*εW and term T.

Example. $W = \{u, v, w\}$.
$\text{Prob}(u)(\{u\}) = \text{Prob}(u)(\{v\}) = ½$ $\quad$ $\text{Prob}(u)(\{w\}) = 0$
$\text{Prob}(v)(\{u\}) = \text{Prob}(v)(\{v\}) = \text{Prob}(v)(\{w\}) = 1/3$
$\text{Prob}(w)(\{u\}) = 0$ $\quad$ $\text{Prob}(w)(\{v\}) = \text{Prob}(w)(\{w\}) = ½$
$\text{den}(u, X) = F(X)(u) = 8$; $\text{den}(v, X) = F(X)(v) = 4$; $\text{den}(w, X) = F(X)(w) = 0$

$\text{den}(u, E(X)) = 6$, $\text{den}(v, E(X)) = 4$, $\text{den}(w, E(X)) = 2$
$\text{den}(u, E(E(X))) = 5$, $\text{den}(v, E(E(X))) = 4$, $\text{den}(w, E(E(X))) = 3$

$E(E(X)) = E(X)$?

$\text{Var}(X) = E((X-E(X))^2$

Example. Where $E(T) = T$.
Wide-scope calculation: $\text{Var}(T) = ((T-1-T)^2 + (T-T)^2 + (T+1-T)^2)/3 = 2/3$.
Narrow-scope calculation: Since certainly $E(T) = T$, 'Var'(T) = 0.

SORITES $\quad$ For any w, x εW, there are $w_0, \ldots, w_n$ εW such that $w = w_0$, $x = w_n$ and for $0 \leq i, j \leq n$, if $|i-j| \leq 1$ then $\text{Prob}(w_i)(\{w_j\}) > 0$.

Proposition. Let the set of worlds be finite and SORITES hold. Then for any random variable T, if $E(T) = T$ at every world then T has the same value at every world.

UNCMAX $\quad$ For some wεW: for all xεW, $\text{den}(x, T) \leq \text{den}(w, T)$, but $\text{Prob}(w)(\{x: \text{den}(x, T) < \text{den}(w, T)\}) > 0$.

Proposition. Let the set of worlds be finite and SORITES hold. Then for any random variable T, the sequence T, E(T), E(E(T)), … converges to the same limit whichever world one is at.

TW 18.v.2006