Probability Logic and Logical Probability

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Many writers who entertain the use of subjective probability or credal probability for, among other things, the evaluation of expected values of options in decision problems insist that such credal probability judgments should be determined by the agent's "available evidence" or "available knowledge". Notable 20th century examples are H. Jeffreys (1921, 1951), J.M. Keynes (1921), R. Carnap (1950, 1962, and 1971) and H.E.Kyburg (1961, 1974, 2001). Typically these authors restricted evidence to the results of observation and experimentation. I am skeptical of the clarity of this restriction and would include in the available knowledge theoretical presuppositions and other so called-background information.

X's total evidence is X's state of full belief or standard for serious possibility. But X's state of full belief **K** cannot, in general, determine X's state **B** of credal probability judgment by itself. It needs to be supplemented by what I call a "confirmational commitment" (Levi, 1980 ch.4) which is a rule specifying for each potential state of full belief relevantly accessible to X what X's credal state should be when X is in that state of full belief.

The authors cited above tend to overlook or underemphasize this point because they advocate of a "logical" or "epistemological" interpretation of probability. According to those who favored this type of interpretation, inquirers ought rationally to undertake to fulfill the requirements of a standard confirmational commitment whose status as the standard is secured by the principles of probability logic (also known as inductive logic). Because of this presupposition, the standard confirmational commitment could be called the logical confirmational commitment.

Those authors who thought or hoped that probability logic could secure a logical confirmational commitment representable by a probability function are said to advocate a logical interpretation of the formal calculus probability. Sometimes the epithet includes authors, like Keynes and Kyburg who did not think that probability logic supports a numerically determinate logical confirmational commitment as the standard confirmational commitment. These authors continued to insist, however, that probability logic singles out a standard confirmational commitment even though that standard is not representable by a numerically determinate probability function.

Although I think that there is a good case for insisting that rational agents should be committed by their available evidence, available knowledge, available information or states of full belief and their confirmational commitments to a state of credal probability judgment as the views of these authors imply, I do not think that probability logic singles out a definite confirmational commitment that every rational agent is obliged as a rational agent to use in determining their credal states.

The authors I have just cited, as I understand them, did at least hope that a probability logic could be constructed that could guarantee that two agents X and Y sharing the same evidence or state of full belief K would be obliged to endorse the same state of credal probability judgment B over a given domain of shared concern. All rational agents would be obliged to endorse a standard confirmational commitment sanctioned by probability logic or criteria for rational probability judgment.

F.P.Ramsey (1924), B.De Finetti (1972, 1974) and L.J. Savage (1954) were right to call into question the presuppositionn that this hope can be achieved. In (Carnap, 1952), Carnap tacitly acknowledged similar doubts and began to change his project from a quest for a standard logical probability to identifying the set of probability functions conforming to the requirements of the logic of probability. Carnap continued to speak of such functions as logical probability measures and sanctioned an interpretation of probability as logical probability even though probability logic or principles of rational probability judgment fail to single out a unique probability function.

Clarity requires distinguishing between probability judgments *mandated* by probability logic and probability judgments *permitted* by probability logic. It appears that Carnap and the other necessarians (as Savage called them) gradually abandoned their necessarianism. Like the personalists they agreed that there are principles of rational probability judgment – i.e., probability logic. And according to the later views of Carnap, they agreed that a complete probability logic might not come close to singling out a unique logically permissible numerically determinate probability that qualifies therefore as a logically necessary one although Carnap continued to hope that his dream would one day be fulfilled. But insofar as Carnap's hope was a pipe dream, the quest for a logical probability that could serve as the standard confirmational commitment for all rational agents should have been abandoned They should have rested content with identifying the logically permissible probabilities. And talk of a logical interpretation of the calculus of probability should have ceased.

Clarity on this point should have suggested that logical probability could not be characterized by a single probability but, at best by the set of all logically permissible probabilities. As I understand them, Keynes and Kyburg thought that the principles of probability logic restrict the set of logically permissible probabilities sufficiently to allow the use of the logical probability as the guide in life. But others thought that the set of logically permissible credal probabilities could be used as the basis for mandating a rule for assigning credal states on the basis of the available evidence – i.e., the state of full belief. They took for granted (with the exception of Keynes and Kyburg) that ideally rational agents ought to make credal probability judgments that are numerically determinate. But they could no longer say that rational agents ought to derive these credal probability judgments from the state of full belief or evidence using a confirmational commitment mandated by probability logic. The personalists tended to acknowledge this point without apology. And they made no mention of logical probability at all.

Carnap's failure to make explicit the distinction between probability functions permitted by probability logic and logical probability which, if it is logical, should be mandated by probability logic seemed to have led him astray on matters more substantial than this mainly terminological matter. He conceded that there is no single standard confirmational commitment (or credence function) that all rational agents ought to adopt as a matter of probability or inductive logic. Unfortunately, he also seemed to think that when a mature rational agent adopted such a confirmational commitment or credence function, he was or should be saddled with it forever as if it were such a standard.

This is a serious flaw in his thinking that has misled subsequent authors. Confirmational commitments are sometimes revisable for good reason. In this respect, they are no different than states of full belief or credal states.

On the view I am proposing, a rational agent X is at a given time (or context) committed to a *state of full belief* K belonging to *a space of potential states of full belief* *K* that are relevant to X's inquiries at t. The members of *K* are partially ordered by a consequence relation so that they constitute a Boolean algebra closed under meets and joins of subsets of *K* even when these sets are not finite. If K_2 is a consequence of K_1, K_1 is stronger or carries more information than K_2. X is committed to fully believing and judging true every potential state of full belief that is either X's state of full belief or a consequence of X's state of full belief.

K need not be the set of potential states conceptually accessible to X but only that subalgebra of potential states that are relevant to X's inquiry or demands for information at the time.

When X is in state K $\in$ *K*, X is committed to judging all consequences of K in *K* to be true, to judging the complements of all consequences of K in *K* to be false and to suspending judgment with respect to all elements *h* of *K* such that neither that *h* nor its complement is a consequence of K. *h* is seriously possible according to K if and only if the complement of *h* is not a consequence of K.

I have just described some of the salient features of the logic of consistency for full belief. According to that logic, X's state of full belief that **K** specifies the set of potential states in *K* X is committed to judging true (false) and to judging seriously possible and impossible (that is, consistent or inconsistent with **K**.

X has other commitments to attitudes. In this discussion, attention shall be restricted to states of credal probability judgment and to confirmational commitments that together with states of full belief determine what the commitments to a credal state should be.[1]

Def. A real valued function Q(x/y) defined for every potential state x in *K* and all potential states y consistent with **K** is a *finitely additive and normalized conditional probability function defined for K* relative to **K** if and only if the following conditions are satisfied:

(1) For y consistent with **K**, $Q(x/y) \geq 0$
(2) If $\mathbf{K} \wedge x = \mathbf{K} \wedge x'$ and $\mathbf{K} \wedge y = \mathbf{K} \wedge y'$, $Q(x/y) = Q(x'/y')$
(3) If $x \wedge z$ is incompatible with $\mathbf{K} \wedge y$, $Q(x \vee z/y) = Q(x/y) + Q(z/y)$.
(4) If $\mathbf{K} \wedge y$ has x as a consequence, $Q(x/y) = 1$.
(5) $Q(x \wedge z/y) = Q(x/z \wedge y)Q(z/y)$.

Def.: *Credal State* B relative to state of full belief **K** is a set of conditional credal functions Q(x/y) where x is any element of *K* and y is any element of *K* consistent with **K**. Members of B are *permissible credal functions* according to the credal state B. The set of *potential credal states* is *B*.

Def.: C: $K \rightarrow B$ is a *confirmational commitment*.

If X endorses *C* at t, it represents X's commitment to regard the credal functions in B to be permissible to use in assessing expected utilities if **K** is X's state of full belief and *C*(**K**) = B.

Probability Logic imposes necessary conditions for the rational coherence or consistency of confirmational commitments. A *Complete Probability Logic* specifies necessary and jointly sufficient conditions for the coherence of confirmational commitments.

[1] Ramsey (1926) contended that the principles of rational probability judgment constitute a "logic of consisteny" without a "logic of truth". Ramsey explored the possibilities for constructing a logic of truth for probability judgment that avoided regarding credal probability judgments as truth-value bearing but, as far as I can make out his remarks are conjectural and tentative.

Here are three principles of probability logic that are widely endorsed.

> ***Confirmational Consistency*:** If **K** is consistent, *C*(**K**) contains at least one permissible *Q*-function.
>
> ***Confirmational Coherence*:** If *Q* is a permissible credal function according to *C*(**K**) = B, *Q*(x/y) is a finitely additive and normalized probability function conditional probability function relative to **K**.
>
> ***Confirmational Consistency*:** **K** is consistent if and only if *C*(**K**) is nonempty.

In addition, the following requirement has until recently been widely endorsed"

> ***Confirmational Uniqueness*:** ***C***(**K**) is a singleton.

Confirmational uniqueness was taken for granted by Jeffreys, Ramsey, Carnap, von Wright, De Finetti, Savage and Jeffrey. It was rejected by Keynes, Kyburg, C.A.B. Smith, I.J.Good (maybe), Ellsberg, Levi, Seidenfeld, Walley.

I replace confirmational uniqueness by the following requirement:

> ***Confirmational Convexity*:** ***C***(***B***) is convex.

When I first presented my ideas in the 1970's, Duncan Luce objected (in oral communication) that the convexity condition rendered the characterization of probabilistic independence difficult. Many others have echoed this complaint. A relatively early discussion is Laddaga (1977). A recent rehearsal of the same matter is found in Halpern (2003, p.66 and exercise 4.12. I responded to this worry in Levi, 1980, ch.10 and 1997, ch.7 by suggesting that when credal probability judgment goes indeterminate, only discriminations between credal irrelevance (independence) and relevance (dependence) can be made. Walley (1991) called the credal irrelevance "epistemic independence. He, Cosman and others have explored other concepts of irrelevance that might have usefulness when credal states go indeterminate. (See for example, Cosman 2001 and Cosman and Walley, 2001.)

Seidenfeld, Schervish and Kadane have recently raised a novel crop of objections to convexity. Since the points I mean to elaborate here do not depend upon convexity, I reserve discussion of the SSK objections for another occasion. I say only that I continue to endorse confirmational convexity. But for the purposes of the following discussion, we can insist only that confirmational uniqueness be rejected.

Confirmational consistency and coherence ought to be noncontroversial but advocates of interval valued probability might want to dissent. However, an interval valued unconditional probability function might be said to determine the set of unconditional probability functions enveloped by the interval valued function and conversely a set of unconditional probability functions determines the upper and lower probability function that envelops them.[2]

A credal state is representable by a set of *conditional* probability functions. Here a conditional credal probability function is required to satisfy the multiplication theorem. This makes sense when conditional probabilities determine conditional probabilities for called off bets. Here is an example:

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[2] Comparative and interval valued probabilities are representable by the sets of numerically determinate probabilities but the converse does not hold in general. In this respect, representing credal states by sets of numerically determinate conditional probabilities allows for describing more potential credal states than the alternatives. If someone thinks that the extra subtlety is of no value in deliberation and inquiry, they should explain why rather than begin with comparative or interval-valued probabilities.

Example 1:

	AE	~AE	~E
G	S – P	-P	k
R	0	0	k

In example 1 the decision maker X is offered a bet G on the truth of A that is called off if E should be false. I suppose that if AE is true, X receives S-P utiles. X loses P utiles if ~AE E is true. If E is false, X receives k utilities. If X chooses R X receives nothing in the first two states and receives the same utiles under the third eventuality.

Although the falsity of E is a serious possibility according to X's state of full belief **K**, that serious possibility may be ignored because X equiprefers G and R in case E is false so that that serious possibility is not a *relevant* possibility in the context of the decision problem on offer.

The book theorems may be extended to show that fair betting rates for called off bets cohere with betting rates for unconditional bets if and only if such fair betting rates are controlled by conditional probabilities satisfying the multiplication theorem. Of course, this is on the assumption, which is often counterfactual, that such fair betting rates exist. They need not exist, however, because of indeterminacy in probability judgments. Nonetheless, if credal states are intended to represent X's judgments as to how to take risks, it seems reasonable to interpret the several permissible conditional probability functions in an indeterminate credal state as specifying permissible fair betting rates. Each such permissible conditional probability is evaluated on the counterfactual supposition that fair betting rates exist. Given this supposition, it may be argued that a permissible conditional credal probability function should satisfy the multiplication theorem in order to avoid the threat of incoherence.

There is another sense in which conditional probability could understood. X might consider what X's credal state should be were X's current state **K**' where **K'** is the expansion of **K** by adding E consistent with **K** to **K** – i.e., **K**' = **K**∧E. The conditional probability of A given E is understood as the probability of A on that supposition.[3]

According to the approach taken here, the set of permissible conditional probabilities on the supposition that E is given by the confirmational commitment *C*(**K**∧E) that specifies the set of permissible credal probabilities according to the credal state to which X is committed on the supposition that X's state of full belief is **K**∧E.

The supposition that E is the supposition that ~E is not a serious possibility. So we can envisage a decision problem like that in example 1 except that ~E is by supposition not a serious possibility. Call this example 2.[4]

One of the characteristic implications of the Bayesian view is the insistence that the called off bet interpretation notion of conditional probability and the suppositional interpretation of conditional probability ought to be equivalent as a matter of probability logic. That is to say, rational X is obliged to choose the same way in example 1 and example 2. (Levi, 1980, ch.10.4.)

The explicitly non Bayesian authors Dempster, Fisher and Kyburg dissent from this prescription. I count myself a dissenter from strict Bayesian doctrine. Yet, I agree

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[3] For a more elaborate discussion of conditionals and suppositional reasoning, see Levi, 1996.

[4] Example 2 is obtained from example 1 by expanding the state of full belief **K** for example 1 by adding E (for the sake of the argument).

with the recommendation that the called off bet interpretation and the suppositional interpretation of conditional probability should be equated. My dissent from strict Bayesian doctrine derives from my rejection of confirmational uniqueness.

Fisher, Kyburg and Dempster reject both confirmational uniqueness and confirmational conditionalization. So their opposition to Bayesian doctrine is more profound than my own.

The equation of the called off bet and suppositional interpretations of conditional probability can be equated with the endorsement of another familiar principle.

Definition of conditionalization: C(K∧y) is the conditionalization of C(**K**) if and only if for every credal probability Q^y permissible according to C(**K**∧y) there is a permissible Q according to C(**K**) such that Q(x/z∧y) = Q^y(x/z) and for every permissible Q there is a Q^y satisfying the same condition.

According to the Bayesian view, a confirmational commitment should satisfy the following principle *as a matter of probability logic*.

Confirmational Conditionalization**:** If **K** is consistent and y is consistent with **K**, C(**K**∧y) is the conditionalization of C(**K**).

It should be clear that if confirmational conditionalization is enforced as a minimal condition of probabilistic rationality or probability logic, the called off bet and suppositional interpretations of conditional probability merge into one.

If confirmational conditionalization is enforced, then any confirmational commitment can be determined by specifying the credal state relative to the weakest potential state of full belief $\mathbf{K}_T$ where X is in the state of ignorance. By confirmational conditionalization, $C(\mathbf{K}_T \wedge y) = C(\mathbf{K})$ is the conditionalization of $\mathbf{K}_T$ and permissible conditional probability functions that satisfy the multiplication theorem also qualify as suppositional conditional probabilities.

One does not have to be a Bayesian to endorse confirmational conditionalization. Classical strict Bayesian doctrine also requires rational agents to conform to confirmational uniqueness. I, for one, reject confirmational uniqueness and, for this reason, reject Bayesianism. One might say, I am quasi Bayesian or a qualified Bayesian.

Other principles have been advocated as requirements of probability logic. The two basic types of requirement have been:

(i) *Principles of Direct Inference* that constrain the making of credal probability judgment on the basis of statements of chance.

(ii) *Principles of Insufficient reason and cognate requirements like E.T.Jayes, Maxent principle.*

Controversy concerning these principles suggests how difficult it is to provide a characterization of a complete inductive logic. For the sake of the argument let us suppose that we have a complete inductive logic *IL*.

Given any consistent state of full belief **K**, let *CIL*(**K**) be the union of the sets *C*(**K**) for all coherent confirmational commitments *C* according to the principles of probability logic.

Construct the function *CIL* from *K* to *B* whose value for **K** is *CIL*(**K**). If the requirements of inductive logic allow this function to be a confirmational commitment, it must be the weakest confirmational commitment. Any credal probability that is permissible relative to **K** according to some logically coherent confirmational commitment is permissible according to *CIL*. If *CIL* is a confirmational commitment, probability logic should recognize it as the logical confirmational commitment. *CIL* is under these circumstances the logical probability and it is, as far as I can see, the only candidate for being the logical probability according to probability logic.

Suppose that confirmational conditionalization is enforced by probability logic as Bayesian doctrine requires. The set $CIL(\mathbf{K_T})$ uniquely determines $CIL(\mathbf{K})$ via confirmational conditionalization for every consistent potential state of full belief. If *CIL* qualifies as a confirmational commitment according to probability logic, it is the weakest confirmational commitment in the sense that it rules out no logically permissible *Q* function relative to any given state of full belief.

Strict Bayesians who endorse confirmational conditionalization but who also endorse confirmational uniqueness cannot recognize $CIL(\mathbf{K_T})$ as a value of a confirmational commitment unless that value is a singleton. If they acknowledge (as Carnap did acknowledge in his later writings) that probability logic does not restrict the set of logically permissible probabilities relative to $\mathbf{K_T}$ to a unique probability, $CIL(\mathbf{K_T})$ should be disbarred as a confirmational commitment and cannot serve as a logical probability.

In his later writings, Carnap's credibility functions correspond roughly to my confirmational commitments. There are, however, some important differences.

(1) Carnap's credibility functions are strictly Bayesian and, in my terminology, must satisfy confirmational uniqueness.

(2) Carnap's credibility functions are incorrigible whereas confirmational commitments are subject to modification when good reasons arise.

(3) Carnap's credibility functions represent permanent dispositions whereas confirmational commitments are undertakings of commitments on the part of agents to follow certain procedures for adjusting credal states with changes in evidence. An agent may have such a commitment without having a disposition to fulfill the commitment. Having the commitment obliges the inquirer to behave according to the requirements of the commitment insofar as the inquirer is able and, in cases where he or she is not able to accept therapy, use prosthetic devices or undergo retraining costs and opportunities permitting so as to improve his or her capacity to fulfill the commitment. A confirmational commitment is an attitudinal commitment in the sense I have explained elsewhere just as a full belief and a value is. (Levi, 1980, ch.1.5, 1991, ch.2, 1997, ch.1.) In spite of our lack of computational capacity, emotional stability and good health, all of which contribute to our lack of logical omniscience, we should not modify the demands of rationality so as to be able to meet the demands but should seek ways and means to improve our performance.

Glossing over these differences, I contend that no strictly Bayesian credibility (or no strictly Bayesian confirmational commitment) can qualify as a logical probability *in the sense that a logical probability is mandated for use by probability or inductive logic*. Carnap agreed with this. But he seemed to think that because a credibility is permitted by inductive logic and can be represented by some mathematical function, it can qualify as logical. Perhaps it can in some sense; but the significance of such terminological practice is obscure to me. Calling a logically permissible credence "logical" does not make it true. It does not make it possibly true. As Ramsey, De Finetti and Savage rightly saw, no such probability carries a truth value. And calling it "logical" does not mandate its use or support the idea that conditional probability characterizes partial entailment. Nor does calling a logically permissible credence "logical" justify its use. At best, calling a probability "logical" acknowledges the probability to be coherent or consistent according to the principles of probability logic or rationality.

In my judgment, the personalist strict Bayesians were far clearer headed on this matter. As long as credal probability judgments conform to the requirements of probability logic, reason cannot complain. There is no need for logical probability.

Whether a logical confirmational commitment is defined when confirmational conditionalization is abandoned as for example in the approach taken by Fisher and Kyburg or in the rather different approach adopted by A.P.Dempster has not, to my knowledge been explored. Given confirmational conditionalization, confirmational commitments are representable by $C(\mathbf{K_T})$ – i.e., by a set of probability functions. The confirmational commitments associated with Fisher, with Kyburg and with Dempster cannot be represented by se ts of probability functions in this manner.

Thus, by disallowing confirmational uniqueness while insisting on confirmational conditionalization, we obtain a perfectly intelligible conception of logical probability that avoids the obscurities of the notion of logical probability as found in the Cambridge tradition as well as in Carnap and his followers. In particular, it calls into question the idea of a logical probability (determinate or indeterminate) that probability logic mandates as the standard confirmational commitment.

Confirmational commitments are subject to critical review. One way to appreciate this point is to consider the treatment of conditionalization in the literature.

Confirmational Conditionalization should not be confused with *Temporal Credal Conditionalization* (*TCC*)-a requirement most authors call simply "conditionalization".

> ***Temporal Credal Conditionalization*** **(TCC)**: If inquirer X shifts from **K** at t to **K**∧y at t', $B_{t'}$ should be the conditionalization of B_t.
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> ***Inverse Temporal Credal Conditionalization*** **(ITCC)**: If X shifts from **K**∧y at t to **K** at t', B_t should be the conditionalization of $B_{t'}$.

TCC amd ITCC are *diachronic* constraints on confirmational commitmemnts. Confirmational conditionalization is a *synchronic* constraint. I deny that rationality imposes constraints on changes in credal probability judgment.

Confirmational conditionalization implies TCC and ITCC provided it is supplemented with the assumption that the confirmational commitment *C* remains the same from t to t'. I deny that confirmational commitments are incorrigible in this sense. If *C* is modified in that time interval, neither TCC nor ITCC are mandated even though the confirmational commitments at t and t' both obey confirmational conditionalization.

TCC is easily recognized as "updating" by conditionalization via Bayes theorem – a principle widely used by Bayesian statisticians. Those who endorse it as a normative requirement may be interpreted within the framework adopted here as adopting confirmational commitments satisfying confirmational conditionalization that should be held fixed. Harold Jeffreys (1939) and Rudolf Carnap (1937) illustrate this view well.

The chief difficulty with the personalist or subjectivist view as construed by many probabilists is that Temporal Credal Conditionalization is enforced as if it were a requirement of minimal rationality along with Confirmational Uniqueness even though personalists deny that there are principles of rationality that mandate one confirmational commitment rather than another. Such personalism ends up maintaining that X and Y are rationally entitled to start off with different numerically determinate confirmational commitments that both should recognize as rational. Yet both of them are required on pain of irrationality to stick with their different confirmational commitments throughout their inquiries.

Matters are still worse than this. If TCC is the sole diachronic principle endorsed in this way, once X fully believes that *h*, X cannot give it up. Like Bush, Cheney and Rumsfeld we are to admire as rational the resolution to stay the course come hell or high water.

Some authors retreat from TCC by abandoning confirmational conditionalization. R.A. Fisher, H.E. Kyburg and A.P. Dempster have followed this route.

Others have proposed allowing modifications of confirmational commitments without abandoning confirmational conditionalization. Such proposals have come in two forms.

Richard Jeffrey suggested replacing TCC with another temporal principle – Jeffrey updating. Jeffrey updating calls for changing confirmational commitments in response to sensory inputs in a manner that is beyond the critical control of the inquiring or deliberating agent. Jeffrey, of course, did not exploit the notion of a confirmational commitment or a credibility. Although Jeffrey's idea can be formulated so that the constantly changing confirmational commitments obey confirmational conditionalization, changes in credal state always take place without changes in state of full belief. Consequently, reference to changes in confirmational commitment deprives confirmational commitments of useful function.

In effect, Jeffrey's approach trivializes the demand that a rational agent's judgments of credal probability have to answer to X's evidence or state of full belief. The reasonable agent acquires a skill enabling him or her to respond to sensory input without being able to scrutinize explicitly what these responses are.

The alternative approach to retaining confirmational conditionalization avoids trivializing the function of confirmational commitments in this way. Sometimes confirmational commitments are retained when new information is added to the evidence and sometimes confirmational commitments are modified. According to this view, it becomes important that we explore conditions under which one might deliberately change confirmational commitments with good reason either by weakening them or strengthening them. The logical confirmational commitment represents the upper bound on potential rationality preserving weakenings of confirmational commitments. The lower bound would, of course, be the inconsistent confirmational commitment where $C(\mathbf{K_T}) = \varnothing$. Within those boundaries adjustments in confirmational commitments may be made when warranted. The question is: when are they warranted?

To focus the question in this manner is to allow for violations of TCC on occasions where confirmational commitments are modified while insisting that all coherent confirmational commitments obey confirmational conditionalization.

Legitimate changes then may come in two forms:

Confirmational Weakening: Sometimes X who endorses C_X may wish to engage in inquiry with Y who endorses C_Y. Not only should they embark on their joint inquiry from a state of full belief that represents their shared agreements as represented by the meet $\mathbf{K_X} \wedge \mathbf{K_Y} = \mathbf{K_{XY}}$ of their respective states of full belief but a shared confirmational commitment that recognizes as permissible all probability functions permissible according to $C_X(\mathbf{K_{XY}})$ and according to $C_Y(\mathbf{K_{XY}})$ and, if confirmational convexity holds, the convex hull of these.

Confirmational Strengthening: Sometimes when inquirer's find their credal states indeterminate, it may be possible to reach sensible decisions without strengthening their confirmational commitments. But sometimes such strengthenin may seem desirable. One kind of reasoning that seems legitimate in some cases arises in the design of experiments where the data to be obtained are to be used in efforts at inductive expansion. Such efforts will be useless if the confirmational commitment is too weak. On the other hand, strengthening the confirmational commitment should not prejudice the results of experimentation in advance of obtaining the data.

Suppose X is in state of full belief **K**. X has identified a roster $U_{\mathbf{K}}$ of conjectures exclusive and exhaustive given **K** and each consistent with **K** to be the strongest relevant potential answers to a question under investigation. The value of the information to be gained by rejecting an element h of U_K is represented by $M(h)$ where M is formally a probability distribution over U_K. This probability characterizes the increment in informational value of elements of U_K when they are added to **K** and is not used for calculating expectations. Q is an expectation determining credal probability distribution over U_K. If Q were uniquely permissible according to X's credal state, X should reject h

if and only if $Q(h) < qM(h)$. q is an index of caution and ranges from 0 to 1. The rationale for this rule is given in Levi (1967b) and shall not be repeated here.

X, so we suppose, recognizes all probability distributions over U_K to be permissible. Accordingly, X must fail to reject any element of U_K unless it is rejected according to all permissible distributions. And this means that no element of U_K is rejected even when X is maximally bold. Moreover, if X runs experiments and collects data, as long as no element of U_K is eliminated deductively, no matter what the outcome of experiment is, X will be in the same situation as before.

In order to obtain a useful experiment, the set of permissible distributions should be reduced. But one should not eliminate any distribution that is unbiased given U_K, *M*.and q. That is to say, X should recognize as permissible just those distributions that avoid rejection of any element of U_K when combined with *M* and q.

When U_K is finite, the permissible probabilities will be all probability distributions *Q* such that $Q(x) = qMx)$ for all elements x of U_K but one. The probability of the exception is the remaining probability required to yield total probability 1. Take the convex hull of these distributions as the credal state. This credal state is the same as the so called "epsilon contaminated class" of probabilities characterized by $(1-\varepsilon)M(x) + \varepsilon Q(x)$ where *Q* is allowed to be any distribution over U_K (Berger and Berliner, 1986, 462.)[5]

[5] Berger and Berliner cite several uses of ε-contaminated classes of distributions going back to the 1960's most but not all of which embrace frequentist interpretations rather than credal interpretations of probability. In Levi, 1980, ch.13.4ff, I proposed adoption of a family of prior distributions derived along the lines suggested in the text. Only much later did

Thus, the inquirers X and Y would be entitled to adopt as the set of "permissible prior credal probabilities" for use in subsequent investigation the class of those distributions none of which would recommend rejecting an element of U_K. Some may complain that the choice of priors to count as permissible is dependent on the informational value determining *M*-function. I respond that making such bias explicit permits us to keep bias under critical control. Thus, if the *M*-function is uniform over U_K, when q = 1, the prior distribution is the uniform distribution; but rather than considering this distribution to be recommended by some principle of insufficient reason, it is recognized as the product of the inquirer's demands for information.

A more serious difficulty is that the epsilon contaminated family recognizes for each element of U_K at least one permissible distribution according to which the prior credal probability of x is very high so that it becomes difficult to reject x at the given level of boldness for any data points unless likelihoods are very decisive. If this is a genuine objection, it can be remedied in several ways by imposing a maximum less than 1 on the value of *Q* in the formula for epsilon contamination.

Further issues need to be addressed if the *M* – function is allowed to go indeterminate as it should be. I shall not address them here.

Teddy Seidenfeld point out to me that the class of unbiased priors as I called them was an ε-contaminated class as understood in the robust Bayesian literature. The difference between my approach and that in the statistical literature is that the distribution *M* I picked out for distinction in defining the class was equated with an informational value determining probability characterizing the demands for information appropriate to some problem of inductive expansion. I avoided consideration of "most plausible". The notion of a plausible distribution seems to me to be obscure.

My main purpose in this discussion, however, is not to recapitulate the fragments of an account of revision of confirmational commitments that I and others have constructed. The fragments of proposals I have offered are intended to suggest the kind of project that I think is worth pursuing rather than an entrenched ideology. I wish to advocate a turning away from efforts to explicate a concept of logical probability and a focusing of attention on revision of confirmational commitments. There is no sensible interpretation of mathematical probability as logical probability. To be sure, there is a conception of logical probability as the weakest coherent confirmational commitment. But *CIL* cannot plausibly be recommended as the standard confirmational commitment. One cannot exploit *CIL* to develop a useful account of the notion of partial entailment that Ramsey found so mysterious in Keynes. And *CIL* cannot capture such ideas as increasing confirmation of hypotheses by increasing the number and variety of positive instances.

Inquiry involves changing one's point of view and doing so legitimately. A point of view includes both states of full belief and confirmational commitments as well as the credal states determined by them. Rather than seeking to elaborate a logical interpretation of probability, attention should be focused on developing an account of when and how confirmational commitments should be revised. Of course, such an account is a supplement to an account of conditions under which expansion and contraction of states of full belief are justified and, more generally, of conditions of rational choice.

Afterthought

Advocates of logical probability, especially strict Bayesians, have often though of confirmational commitments not only as standards secured by probability logic but as measures of evidential support. In this discussion, I have restricted attention to the role of confirmational commitments as determining credal states used in deriving expected utility. Using confirmation functions or probabilities derived from them to represent evidential support could and has meant other things.

For me, the useful notions of evidential support are assessments of hypotheses on the basis of evidence that can be used in inductive expansion of the evidence **K**. Without going into detail, there are measures of evidential support in the maximizing sense that recommends an expansion from those available that maximizes evidential support. There are also measures of evidential support in the satisficing sense that recommend adding information support for which has attained a sufficiently high level.

I contend that as long as inductive expansion is sometimes warranted, neither of these measures can be probability whether logical or extralogical and these two measures cannot be the same. One of them has the properties of expected epistemic utility and the other the properties of a Shackle measure of degree of belief, Baconian probability, Spohnian degree of belief. If this is right, there is no prospect for confirmation theory in the tradition of Carnap to be resuscitated. (Levi, 1997, ch.8., 2002.)

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