Is Bayesian Coherentism Impossible?
On Bovens and Hartmann’s Impossibility Result

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1. background and B&H’s impossibility result
   - the intuition
   - bayesian coherentism
   - “ceteris paribus”
   - notation
   - the impossibility result

2. a case study - Shogenji’s work on coherence
   - Shogenji’s measure
   - ...and b&h’s ceteris paribus conditions
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3. a possibility result for bayesian coherentism
   - the "expectedness of reports" condition
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4. implications
Coherence??

- Traditional epistemology - coherence theories of truth and justification.
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- C. I. Lewis’s “congruence” - has to do with how well information “meshes or fits together.”
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Three witnesses give us reports ($S = \{R_1, R_2, R_3\}$)

$R_1 = “\text{The culprit had a French accent}”$

$R_2 = “\text{The culprit drove away in a Renault}”$

$R_3 = “\text{The culprit was wearing Coco Chanel shoes}”$
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Three witnesses give us reports (\(S = \{R_1, R_2, R_3\}\))

\(R_1 = \) “The culprit had a French accent”
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Three other witnesses give us reports (\(S' = \{R'_1, R'_2, R'_3\}\))

\(R'_1 = \) “The culprit was a woman”
\(R'_2 = \) “The culprit had a Danish accent”
\(R'_3 = \) “The culprit drove a Ford”
Three Fundamental Tenets of Bayesian Coherentism:

- **Separability** (BC$_1$): For all information sets $S, S' \in S$, if $S$ is no less coherent than $S'$ ($S \gtrless S'$), then our degree of confidence that the content of $S$ is true is no less than our degree of confidence that the content of $S'$ is true, ceteris paribus.
Three Fundamental Tenets of Bayesian Coherentism:

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- **Probabilism**: (BC$_2[i]$): The binary relation of "...being no less coherent than..." [i.e., $\succcurlyeq$] over $S$ is fully determined by the probabilistic features of the information sets contained in $S$. 

*Note: Following Bovens & Hartmann's (B&H) example, I will refer to the conjunction of (BC$_2[i]$) and (BC$_2[ii]$) as (BC$_2$).*
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- **Ordering**: (BC$_2[\text{ii}]$): The binary relation of "...being no less coherent than..." [i.e., $\succ$] is an ordering; i.e., the relation $\succ$ is transitive and complete.

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B&H's Taxonomy of Epistemically Relevant Factors:

1. *Expectedness of the information set* (i.e., of the conjunction of the information in the set):

   \[ P(R_1, R_2, \ldots, R_n) \]
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**Thus, "ceteris paribus" conditions in the testing for separability include (1) and (2).

**Note: B&H’s discussion of Bayesian Coherentism assumes the general prerequisite that the witnesses are independent: “R_i screens off REPR_i from all other fact variables R_j and from all other report variables REPR_j.”
Definition

A weight vector $< a_0, a_1, ..., a_n >$ for each $n$-membered information set specifies that set’s probabilistic information, where each element of the weight vector $a_i$ is the sum of the joint probabilities of all combinations of $i$ negative values and $n - i$ positive values of the variables $R_1, ..., R_n$. 

Example for information triple $\{R_1, R_2, R_3\}$, $a_2 = P(\neg R_1, \neg R_2, R_3) + P(\neg R_1, R_2, \neg R_3) + P(R_1, \neg R_2, \neg R_3)$.

The sum of the elements of the weight vector is one. $a_0$ represents the prior of the conjunction of the information.

Given the independence of witnesses, B&H rewrite Bayes’s Theorem:

$$P(R_1, ..., R_n | \text{REPR}_1, ..., \text{REPR}_n) = \frac{P(R_1, ..., R_n)}{a_0} = a_0 + a_1 \bar{r}_1 + ... + a_n \bar{r}_n.$$
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B&H's Impossibility Result

- The impossibility result seeks to show that "there cannot exist a measure of coherence that is probabilistic and induces a coherence ordering for information triples (BC₂) and that simultaneously makes it the case that the more coherent the information set, the more confident we are that the information is true, ceteris paribus (BC₁)."

**Note:** B&H's "expectedness of the information" ceteris paribus condition is explicitly being enforced in this specification of the weight vectors.
B&H’s Impossibility Result

- The impossibility result seeks to show that "there cannot exist a measure of coherence that is probabilistic and induces a coherence ordering for information triples (BC$_2$) and that simultaneously makes it the case that the more coherent the information set, the more confident we are that the information is true, ceteris paribus (BC$_1$)."

- Take two information triples $S = \{R_1, R_2, R_3\}$ and $S' = \{R'_1, R'_2, R'_3\}$ with the respective weight vectors:

$$< a_0, a_1, a_2, a_3 > = < .05, .30, .10, .55 >$$

$$< a'_0, a'_1, a'_2, a'_3 > = < .05, .20, .70, .05 >$$

** Note: B&H’s "expectedness of the information" ceteris paribus condition is explicitly being enforced in this specification of the weight vectors.
Graphically:
Consequences

- If our chosen coherence measure gives the result that $\text{coh}(S') \geq \text{coh}(S)$: For $r \in (.8, 1)$, $(BC_1)$ is violated; i.e., $P^*(R_1', R_2', R_3') < P^*(R_1, R_2, R_3)$ even though $\text{coh}(S') \geq \text{coh}(S)$ and the ceteris paribus conditions are enforced.

- If our chosen coherence measure gives the result that $\text{coh}(S') < \text{coh}(S)$: For $r \in (0, .8]$, $(BC_2)$ is violated; i.e., $P^*(R_1', R_2', R_3') \geq P^*(R_1, R_2, R_3)$ even though $\text{coh}(S') < \text{coh}(S)$ and the ceteris paribus conditions are enforced.

Upshot: $(BC_1)$ and $(BC_2)$ are inconsistent for triples (and B&H note that the impossibility result applies to information sets of size $n \geq 3$). Thus, if coherence can be formalized using a complete probabilistic ordering, then coherence cannot be separable from reliability.

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- If our chosen coherence measure gives the result that \( \text{coh}(S') \geq \text{coh}(S) \): For \( r \in (0.8, 1) \), \((BC_1)\) is violated; i.e., \( P^*(R'_1, R'_2, R'_3) < P^*(R_1, R_2, R_3) \) even though \( \text{coh}(S') \geq \text{coh}(S) \) and the ceteris paribus conditions are enforced.

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A Case Study - Shogenji’s Work on Coherence

Shogenji’s coherence measure:

\[ C(R_1, \ldots, R_n) = \text{def} \frac{P(R_1, \ldots, R_n)}{P(R_1) \times \ldots \times P(R_n)} \]

1. “total individual strength” as a measure of the number and specificity of pieces of information in the set:

\[ P(R_1) \times \ldots \times P(R_n) \]

Total individual strength is negatively correlated with our level of confidence in an information set. The more information a set contains, and the more specific that information, the less our confidence.

2. **Thus, for Shogenji, “ceteris paribus” conditions in the testing for separability must include (1).**

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2. coherence

**Thus, for Shogenji, "ceteris paribus" conditions in the testing for separability must include (1).
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What if one supports Shogenji’s measure and accepts B&H’s taxonomy?

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\frac{P(R_1, \ldots, R_n)}{P(R_1) \times \cdots \times P(R_n)} = \frac{P(R'_1, \ldots, R'_n)}{P(R'_1) \times \cdots \times P(R'_n)}
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According to B&H’s taxonomy, we need to include the expectedness of the information \(P(R_1, \ldots, R_n)\) in our ceteris paribus conditions; thus, the numerators of Shogenji’s measure are equal under ceteris paribus.
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- As Shogenji argues, denominators are also held equal when enforcing the ceteris paribus conditions.
- Accepting both B&H’s taxonomy and Shogenji’s measure (along with Shogenji’s "total individual strength" condition), we can only compare information sets of equal coherence ceteris paribus!
This isn’t - in and of itself - a point against B&H’s taxonomy or ceteris paribus conditions.

Only intended to show that acceptance of Shogenji’s measure calls for a rejection of B&H’s $P(R_1, \ldots, R_n)$ ceteris paribus condition.
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Two consequent questions:

1. Which of these accounts, if any, is enforcing the correct set of conditions?
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Two consequent questions:

1. Which of these accounts, if any, is enforcing the correct set of conditions?
2. Can a different set of ceteris paribus conditions (e.g., Shogenji’s) allow one to sidestep the impossibility result?

**I am only interested in this talk in answering the second question**
An Additional Condition and a New Taxonomy

*Intuitively,* the expectedness of reports $P(REPR_1, ..., REPR_n)$ is distinct from coherence; coherence - being a virtue of information sets *purely on the information level* - should not be affected by considerations having to do with the reporting of that information (including reliability of the sources and expectedness of the reports).
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**Epistemically relevant factors:**

1. **Total individual strength:** $P(R_1) \times ... \times P(R_n)$
2. **Expectedness of the reports:** $P(REPR_1, ..., REPR_n)$
3. **Reliability of the information sources:**
   
   $$1 - \frac{P(REPR_i|\neg R_i)}{P(REPR_i|R_i)}$$

4. **Coherence of the information**
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   \]
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"ceteris paribus": factors (1), (2), and (3) need to be equal between sets.
A Possibility Result for Bayesian Coherentism

Theorem

*If one enforces this set of ceteris paribus conditions and Shogenji’s measure, there can be no counter-examples to $(BC_1)$ and $(BC_2)$.***
A Possibility Result for Bayesian Coherentism

Theorem

*If one enforces this set of ceteris paribus conditions and Shogenji’s measure, there can be no counter-examples to (BC₁) and (BC₂).*

Consider the simple form of Bayes’s Theorem:

\[
P^*(R_1, \ldots, R_n) = \frac{P(R_1, \ldots, R_n) \times P(REPR_1, \ldots, REPR_n|R_1, \ldots, R_n)}{P(REPR_1, \ldots, REPR_n)}
\]
A Possibility Result for Bayesian Coherentism

Theorem

If one enforces this set of ceteris paribus conditions and Shogenji’s measure, there can be no counter-examples to (BC1) and (BC2).

Consider the simple form of Bayes’s Theorem:

\[ P^*(R_1, \ldots, R_n) = \frac{P(R_1, \ldots, R_n) \times P(REPR_1, \ldots, REPR_n|R_1, \ldots, R_n)}{P(REPR_1, \ldots, REPR_n)} \]

Given that it is a general prerequisite of B&H’s discussion of Bayesian Coherentism that information sources are independent - in the sense that “R_i screens off REPR_i from all other fact variables R_j and from all other report variables REPR_j”: 
a possibility result for bayesian coherentism

an end around the impossibility result

\[ P^*(R_1, \ldots, R_n) = \frac{P(R_1, \ldots, R_n) \times P(REPR_1|R_1) \times \ldots \times P(REPR_n|R_n)}{P(REPR_1, \ldots, REPR_n)} \]
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By ceteris paribus condition (3), we need only compare:

\[ \frac{P(R_1, \ldots, R_n)}{P(REPR_1, \ldots, REPR_n)} \]
\[
P^*(R_1, \ldots, R_n) = \frac{P(R_1, \ldots, R_n) \times P(REPR_1|R_1) \times \ldots \times P(REPR_n|R_n)}{P(REPR_1, \ldots, REPR_n)}
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**Result 1**

Ceteris paribus, relative values of posterior probability will be directly proportional to those of the expectedness of the information.
Utilizing Shogenji’s measure, and by ceteris paribus condition (1):

Result 2

Ceteris paribus, \( C(S) = \frac{P(R_1, \ldots, R_n)}{P(R_1) \times \ldots \times P(R_n)} > \frac{P(R'_1, \ldots, R'_n)}{P(R'_1) \times \ldots \times P(R'_n)} = C(S') \) iff

\( P(R_1, \ldots, R_n) > P(R'_1, \ldots, R'_n) \) - and more generally, ceteris paribus, relative values of coherence between sets will be directly proportional to those of the expectedness of the information.
Utilizing Shogenji’s measure, and by ceteris paribus condition (1):

Result 2

Ceteris paribus, \( C(S) = \frac{P(R_1, \ldots, R_n)}{P(R_1) \times \ldots \times P(R_n)} > \frac{P(R_1', \ldots, R_n')}{P(R_1') \times \ldots \times P(R_n')} = C(S') \) iff \( P(R_1, \ldots, R_n) > P(R_1', \ldots, R_n') \) - and more generally, ceteris paribus, relative values of coherence between sets will be directly proportional to those of the expectedness of the information.

Putting result 1 and result 2 together, we get separability \((BC_1)\): For all information sets \( S, S' \in S \), if \( S \) is no less coherent than \( S' \) \((S \succeq S')\), then our degree of confidence that the content of \( S \) is true is no less than our degree of confidence that the content of \( S' \) is true, ceteris paribus.
Utilizing Shogenji’s measure, and by ceteris paribus condition (1):

**Result 2**

Ceteris paribus, \( C(S) = \frac{P(R_1,\ldots,R_n)}{P(R_1)\times\ldots\times P(R_n)} > \frac{P(R'_1,\ldots,R'_n)}{P(R'_1)\times\ldots\times P(R'_n)} = C(S') \) iff \( P(R_1,\ldots,R_n) > P(R'_1,\ldots,R'_n) \) - and more generally, ceteris paribus, relative values of coherence between sets will be directly proportional to those of the expectedness of the information.

Putting result 1 and result 2 together, we get *separability* \((BC_1)\): For all information sets \( S, S' \subseteq S \), if \( S \) is no less coherent than \( S' \) \((S \succeq S')\), then our degree of confidence that the content of \( S \) is true is no less than our degree of confidence that the content of \( S' \) is true, ceteris paribus.

**Fact**

*Thus, there exist sets of ceteris paribus conditions that are intuitively appealing and do allow one to avoid B&H’s impossibility result.*
Morals of the Story:

1. Crucially, the set of ceteris paribus conditions that one needs to enforce in the testing for separability will change depending upon the putative measure of coherence that one is considering.

B&H’s mistake then is to rely on a specific set of ceteris paribus conditions in deriving their impossibility result, which in turn purports to apply across the board to all proposed measures of coherence. One simply cannot stipulate such conditions that will be appropriate to all such measures. If one changes the measure being considered, he or she may very well have to change the ceteris paribus conditions considered.

Before their impossibility result can be deemed successful or unsuccessful, B&H need to convince us that their ceteris paribus conditions - as opposed to other seemingly plausible options - are the appropriate ones. Such convincing must go beyond the typical intuition-based taxonomies of epistemically relevant factors.
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