

NEW FOUNDATIONS FOR IMPERATIVE LOGIC II: Pure imperative inference

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INTRODUCTION

- Sign at a hotel: “don’t enter unless you are accompanied by a registered guest”.
- I say to someone about to enter: “don’t enter if you are an unaccompanied registered guest”.
“Why?” “It follows from what the sign says.”
- But what is it in general for a *pure imperative argument*—whose premises and conclusion are *prescriptions* (i.e., commands, requests, instructions, suggestions, etc.)—to be *valid*?

PREVIOUS APPROACHES

- ① Isomorphism: the corresponding pure declarative argument is valid. Problem: validates “if the sun shines, walk; so if you don’t walk, let the sun not shine” (contraposition).
- ② Satisfaction-validity: satisfying the premises entails satisfying the conclusion. Problem: invalidates “(whether or not you smile) run; so if you smile, run”.
- ③ Bindingness-validity: the conclusion is binding if the premises are. Problem: unusable.

MY APPROACH

- We want a *usable* and *principled* approach (that goes beyond a mere appeal to intuitions).
- ① A desire for a *useful* definition of validity leads to a variant of bindingness-validity.
- ② Distinguish *strong* from *weak* bindingness, and thus strong from weak validity.
- ③ Prove Equivalence Theorem rendering the definitions usable.
- ④ Apply the theorem to specific arguments.

OVERVIEW

Part 1:

PURE IMPERATIVE VALIDITY

Part 2:

STRONG AND WEAK BINDINGNESS

Part 3:

AN EQUIVALENCE THEOREM

Part 4:

APPLYING THE THEOREM

DESIDERATA

General idea: If I should act according to the premises, I should act according to the conclusion.

(D1) If the premises are *pro tanto* (i.e., *prima facie*) binding, so is the conclusion.

(D2) If the premises are *all-things-considered* binding, so is the conclusion.

(D3) If the premises are *pro tanto morally* [or *legally*, etc.] binding, so is the conclusion.

(D4) If the premises are *all-moral-things-considered* binding, so is the conclusion.

THE DEFINITION

- Definition 1: *A pure imperative argument is valid exactly if, necessarily, every reason that supports the conjunction of the premises of the argument also supports the conclusion.*
- This definition entails D1-D4:
 - (D1) If the premises are *pro tanto* (i.e., *prima facie*) binding, so is the conclusion.
- What makes the derivations work is that the *same* reason that supports the premises also supports the conclusion.

PART 2

Part 1:

PURE IMPERATIVE VALIDITY

Part 2:

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REASONS AND SUPPORT

- Informally, a reason is a consideration that counts in favor of something.
- Formally, a *noncomparative* reason is a *fact* that favors some *proposition*.
- A *comparative* reason is a fact that favors some proposition *over* some other one.
- Definition 2: A (fact which is a comparative) reason **supports** a prescription exactly if it favors the satisfaction over the violation proposition of the prescription.

STRONG BINDINGNESS

Definition 3: *A (fact which is a comparative) reason **strongly** supports a prescription iff:*

- ① *It favors every proposition which entails the satisfaction proposition of the prescription over every different proposition which entails the violation proposition (dominance condition);*
- ② *It does not favor any proposition which entails the satisfaction proposition of the prescription over any other such possible proposition (satisfaction indifference condition).*

WEAK BINDINGNESS

- The fact that I have promised to feed both the cat and the dog supports “feed the cat”.
- But *not* strongly, because it favors feeding both the cat and the dog over feeding the cat but not the dog, so satisfaction indifference fails.
- Feeding your cat is *necessary* for satisfying “feed both the cat and the dog”, which is strongly supported.
- Definition 4: A reason *weakly* supports a prescription I iff it strongly supports some prescription I^* such that S^* entails S and $C^*=C$.

STRONG AND WEAK VALIDITY

- Definition 1a: *A pure imperative argument is **strongly valid** exactly if, necessarily, every reason that **strongly** supports the conjunction of the premises of the argument also **strongly** supports the conclusion of the argument.*
- Definition 1b: *A pure imperative argument is **weakly valid** exactly if, necessarily, every reason that **weakly** supports the conjunction of the premises of the argument also **weakly** supports the conclusion of the argument.*

PART 3

Part 1:

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THE EQUIVALENCE THEOREM

Equivalence Theorem. Let S , V , and C be respectively the satisfaction proposition, the violation proposition, and the context of the conjunction of the premises of a pure imperative argument, and define similarly S' , V' , and C' for the conclusion of the argument.

- ① The argument is *strongly* valid iff: V is necessary, or S' entails S and V' entails V .
- ② The argument is *weakly* valid iff: C' entails C and V' entails V .

SOME IMPLICATIONS

- ① *Strong entails weak validity* (because, if S' entails S and V' entails V , then C' entails C).
- ② An *unobeyable* prescription (with necessary violation proposition) entails *any* prescription.
- ③ For *unconditional* prescriptions:
 - Strong validity is trivial: it amounts to $\langle S, V \rangle = \langle S', V' \rangle$.
 - Weak validity amounts to *satisfaction-validity* (i.e., S entails S^\wedge) and is thus isomorphic to pure *declarative* validity.

REDUNDANCY VALIDITY

- An argument is *redundancy valid* iff the conjunction of its conclusion with the conjunction of its premises is the conjunction of its premises: $\langle S', V' \rangle \& \langle S, V \rangle = \langle S, V \rangle$. (The conclusion is redundant: adding it to the conjunction of the premises leaves that conjunction unchanged.)
- The *conjunction* of $\langle S, V \rangle$ with $\langle S', V' \rangle$ is $\langle (C \vee C') \& \sim (V \vee V'), V \vee V' \rangle$.
- *Weak validity amounts to redundancy validity.*

NON-CONJUNCTIVE VALIDITY

- An argument is *non-conjunctively strongly valid* iff, necessarily, every reason that supports *every* premise supports the conclusion.
- (D7) A multiple-premise argument is valid iff the corresponding single-premise argument is valid.
- Non-conjunctive strong validity violates D7:

Run

Smile

Run

versus

Run and smile

Run

PART 4

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CLASSIFYING PURE IMPERATIVE ARGUMENTS

- Classification 1: According to whether they are strongly or weakly valid. Three groups:
 - ① Both strongly and weakly valid.
 - ② Neither weakly nor strongly valid.
 - ③ Weakly but not strongly valid.
- Classification 2: According to whether they are intuitively valid. Three groups:
 - ① Intuitively valid.
 - ② Intuitively invalid.
 - ③ Not intuitively valid & not intuitively invalid.

BOTH STRONGLY AND WEAKLY VALID ARGUMENTS

- Strengthening the antecedent: “If A is true, let B be true; so if $A \& A^*$ is true, let B be true.”
- Intuitively valid: Premise is the conjunction of the conclusion with another prescription.
- Objection: “Don’t wake me up; so if the house is on fire, don’t wake me up” looks invalid.
- My reply: “Don’t wake me up” might express:
 - ① “Don’t wake me up, no matter what.”
 - ② “Don’t wake me up, unless there is an emergency.”

WEAKLY AND STRONGLY INVALID ARGUMENTS

- ① Negating the context: “If you love him, marry him. So if you don’t love him, marry him.”
- ② Restricting the context to the consequent: “Marry him. So if you marry him, kill him.”
- ③ Strengthening the consequent: “Marry him. So marry him and kill him.”
- ④ Weakening the antecedent: “If you see a burglar, call the police. So call the police.”
- ⑤ Contraposition: “If the volcano erupts, flee. So if you don’t flee, let the volcano not erupt.”

WEAKLY BUT NOT STRONGLY VALID ARGUMENTS

① Weakening the consequent:

■ Ross's paradox:

“Mail the letter. So mail or burn the letter.”

■ “Deontic” detachment: “Read the book.

If you read the book, come to discuss it.

So come to discuss the book.”

② Hypothetical syllogism:

“If you take Physics I, take Physics II.

If you take Physics II, take Physics III.

So if you take Physics I, take Physics III.”

FUTURE RESEARCH

- New foundations for imperative logic III: Mixed imperative inference.
- New foundations for imperative logic IV: Soundness and completeness.
- New foundations for deontic logic I: Unconditional deontic propositions.
- New foundations for deontic logic II: Conditional deontic propositions.
- *Imperative and deontic logic: New foundations.*