NEW FOUNDATIONS FOR IMPERATIVE LOGIC II:
Pure imperative inference

Peter B. M. Vranas
vranas@wisc.edu
University of Wisconsin-Madison

4th Formal Epistemology Workshop, 1 June 2007
INTRODUCTION

- Sign at a hotel: “don’t enter unless you are accompanied by a registered guest”.
- I say to someone about to enter: “don’t enter if you are an unaccompanied registered guest”.
  “Why?” “It follows from what the sign says.”
- But what is it in general for a pure imperative argument—whose premises and conclusion are prescriptions (i.e., commands, requests, instructions, suggestions, etc.)—to be valid?
Isomorphism: the corresponding pure declarative argument is valid. Problem: validates “if the sun shines, walk; so if you don’t walk, let the sun not shine” (contraposition).

Satisfaction-validity: satisfying the premises entails satisfying the conclusion. Problem: invalidates “(whether or not you smile) run; so if you smile, run”.

Bindingness-validity: the conclusion is binding if the premises are. Problem: unusable.
MY APPROACH

- We want a *usable* and *principled* approach (that goes beyond a mere appeal to intuitions).
- A desire for a *useful* definition of validity leads to a variant of *bindingness-validity*.
- Distinguish *strong* from *weak* bindingness, and thus strong from weak validity.
- Prove Equivalence Theorem rendering the definitions usable.
- Apply the theorem to specific arguments.
OVERVIEW

Part 1:
PURE IMPERATIVE VALIDITY

Part 2:
STRONG AND WEAK BINDINGNESS

Part 3:
AN EQUIVALENCE THEOREM

Part 4:
APPLYING THE THEOREM
General idea: If I should act according to the premises, I should act according to the conclusion.

(D1) If the premises are *pro tanto* (i.e., *prima facie*) binding, so is the conclusion.

(D2) If the premises are *all-things-considered* binding, so is the conclusion.

(D3) If the premises are *pro tanto* *morally* [or *legally*, etc.] binding, so is the conclusion.

(D4) If the premises are *all-moral-things-considered* binding, so is the conclusion.
Definition 1: A pure imperative argument is valid exactly if, necessarily, every reason that supports the conjunction of the premises of the argument also supports the conclusion.

This definition entails D1-D4:

(D1) If the premises are pro tanto (i.e., prima facie) binding, so is the conclusion.

What makes the derivations work is that the same reason that supports the premises also supports the conclusion.
PART 2

Part 1:
PURE IMPERATIVE VALIDITY

Part 2:
STRONG AND WEAK BINDINGNESS

Part 3:
AN EQUIVALENCE THEOREM

Part 4:
APPLYING THE THEOREM
REASONS AND SUPPORT

- Informally, a reason is a consideration that counts in favor of something.
- Formally, a noncomparative reason is a fact that favors some proposition.
- A comparative reason is a fact that favors some proposition over some other one.
- **Definition 2:** A (fact which is a comparative) reason supports a prescription exactly if it favors the satisfaction over the violation proposition of the prescription.
Definition 3: A (fact which is a comparative) reason strongly supports a prescription iff:

1. It favors every proposition which entails the satisfaction proposition of the prescription over every different proposition which entails the violation proposition (dominance condition);
2. It does not favor any proposition which entails the satisfaction proposition of the prescription over any other such possible proposition (satisfaction indifference condition).
WEAK BINDINGNESS

- The fact that I have promised to feed both the cat and the dog supports “feed the cat”.
- But not strongly, because it favors feeding both the cat and the dog over feeding the cat but not the dog, so satisfaction indifference fails.
- Feeding your cat is necessary for satisfying “feed both the cat and the dog”, which is strongly supported.
- **Definition 4**: A reason weakly supports a prescription $I$ iff it strongly supports some prescription $I^*$ such that $S^*$ entails $S$ and $C^* = C$. 
STRONG AND WEAK VALIDITY

- **Definition 1a**: A pure imperative argument is *strongly valid* exactly if, necessarily, every reason that *strongly* supports the conjunction of the premises of the argument also *strongly* supports the conclusion of the argument.

- **Definition 1b**: A pure imperative argument is *weakly valid* exactly if, necessarily, every reason that *weakly* supports the conjunction of the premises of the argument also *weakly* supports the conclusion of the argument.
PART 3

Part 1:
PURE IMPERATIVE VALIDITY

Part 2:
STRONG AND WEAK BINDINGNESS

Part 3:
AN EQUIVALENCE THEOREM

Part 4:
APPLYING THE THEOREM
Equivalence Theorem. Let $S$, $V$, and $C$ be respectively the satisfaction proposition, the violation proposition, and the context of the conjunction of the premises of a pure imperative argument, and define similarly $S'$, $V'$, and $C'$ for the conclusion of the argument.

1. The argument is **strongly valid** iff: $V$ is necessary, or $S'$ entails $S$ and $V'$ entails $V$.

2. The argument is **weakly valid** iff: $C'$ entails $C$ and $V'$ entails $V$. 
SOME IMPLICATIONS

1. **Strong entails weak validity** (because, if $S'$ entails $S$ and $V'$ entails $V$, then $C'$ entails $C$).

2. An *unobeyable* prescription (with necessary violation proposition) entails *any* prescription.

3. For *unconditional* prescriptions:
   - Strong validity is trivial: it amounts to $<S, V> = <S', V'>$.
   - Weak validity amounts to *satisfaction*-validity (i.e., $S$ entails $S'$) and is thus isomorphic to pure *declarative* validity.
REDUNDANCY VALIDITY

- An argument is *redundancy valid* iff the conjunction of its conclusion with the conjunction of its premises is the conjunction of its premises: $<S', V'> \& <S, V> = <S, V>$. (The conclusion is redundant: adding it to the conjunction of the premises leaves that conjunction unchanged.)
- The *conjunction* of $<S, V>$ with $<S', V'>$ is $<(C \lor C') \& \sim (V \lor V'), V \lor V'>$.  
- *Weak validity amounts to redundancy validity.*
An argument is *non-conjunctively strongly valid* iff, necessarily, every reason that supports *every* premise supports the conclusion.

(D7) A multiple-premise argument is valid iff the corresponding single-premise argument is valid.

Non-conjunctive strong validity violates D7:

Run
Smile \textit{versus} Run and smile
Run
Run
PART 4

Part 1: PURE IMPERATIVE VALIDITY
Part 2: STRONG AND WEAK BINDINGNESS
Part 3: AN EQUVALENCE THEOREM
Part 4: APPLYING THE THEOREM
CLASSIFYING PURE IMPERATIVE ARGUMENTS

Classification 1: According to whether they are strongly or weakly valid. Three groups:
1. Both strongly and weakly valid.
2. Neither weakly nor strongly valid.
3. Weakly but not strongly valid.

Classification 2: According to whether they are intuitively valid. Three groups:
1. Intuitively valid.
2. Intuitively invalid.
3. Not intuitively valid & not intuitively invalid.
**BOTH STRONGLY AND WEAKLY VALID ARGUMENTS**

- **Strengthening the antecedent**: “If $A$ is true, let $B$ be true; so if $A \& A^*$ is true, let $B$ be true.”
- **Intuitively valid**: Premise is the conjunction of the conclusion with another prescription.
- **Objection**: “Don’t wake me up; so if the house is on fire, don’t wake me up” looks invalid.
- **My reply**: “Don’t wake me up” might express:
  1. “Don’t wake me up, no matter what.”
  2. “Don’t wake me up, unless there is an emergency.”
WEAKLY AND STRONGLY INVALID ARGUMENTS

1. **Negating the context**: “If you love him, marry him. So if you don’t love him, marry him.”

2. **Restricting the context to the consequent**: “Marry him. So if you marry him, kill him.”

3. **Strengthening the consequent**: “Marry him. So marry him and kill him.”

4. **Weakening the antecedent**: “If you see a burglar, call the police. So call the police.”

5. **Contraposition**: “If the volcano erupts, flee. So if you don’t flee, let the volcano not erupt.”
WEAKLY BUT NOT STRONGLY VALID ARGUMENTS

1. **Weakening the consequent:**
   - **Ross’s paradox:**
     “Mail the letter. So mail or burn the letter.”
   - **“Deontic” detachment:** “Read the book. If you read the book, come to discuss it. So come to discuss the book.”

2. **Hypothetical syllogism:**
   “If you take Physics I, take Physics II. If you take Physics II, take Physics III. So if you take Physics I, take Physics III.”
FUTURE RESEARCH

- New foundations for imperative logic III: Mixed imperative inference.
- New foundations for imperative logic IV: Soundness and completeness.
- New foundations for deontic logic I: Unconditional deontic propositions.
- New foundations for deontic logic II: Conditional deontic propositions.
- Imperative and deontic logic: New foundations.