

Focused Correlation and Confirmation

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coherence theory and the 'truth connection'

- Bonjour's 'cognitively spontaneous' beliefs (1985)
- Klein and Warfield's observation (1994-96)
 - *One can increase coherence of a set without increasing likelihood of truth.*
- March of the Probabilists (late '90s)
 - Shogenji, Fitelson, Olsson ...
 - *Under what conditions is coherence 'truth-conducive'?*
- Impossibility results (early '00s)
 - Bovens & Hartmann: *There is no coherence measure that induces an ordering on sets of variables.*
 - Olsson: *There is no informative link between coherence and likelihood of truth*
- 'Irrelevancy' arguments (lately)
 - Schupbach, *ceteris paribus* conditions (FEW 2007)
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- ★ Olsson: *There is no informative link between coherence and likelihood of truth*

'Irrelevancy' arguments (lately)

Schupbach, *ceteris paribus* conditions (FEW 2007)

- ★ —, artifact of testimonial systems.

coherence measures

the mini-series:

- *Some good news*: Focused Correlation, coherence and the 'truth connection'
- *Bad news*: Coherence measures and the dynamics of belief change.
- *More bad news*: Bayesian coherentism and anti-realism (with a surprise guest).
- *Finale*: An alternative to Bayesian coherentism (with a surprise guest).

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what this talk is about

Good news

- There is an informative relationship between 'coherence' and 'truth', when
 - *coherence* =_{df} association (deviation from independence), and
 - *truth conduciveness* =_{df} boost of incremental confirmation.

'coherence'

- is a *distance from independence* property of sets of binary variables, interpreted as sets of propositions.

'truth-conduciveness'

- is an increase in posterior probability;
- an increase in incremental confirmation.

some machinery

incremental convergence (Cohen 1977)

- Evidence A and B converge upon h if $\Pr(h|A \cap B) > \Pr(h|A)$.
- $i_c(\cdot) =_{df} \Pr(h|A \cap B) / \Pr(h|A)$ measures *incremental convergence* from A to $\{A, B\}$ on h :

$$i_c(h, A, \{A, B\}) = r \text{ is } \begin{cases} \text{positive,} & \text{if } r > 1 \\ \text{neutral,} & \text{if } r = 1 \\ \text{negative,} & \text{if } r < 1 \end{cases}$$

the Wayne-Shogenji measure

$$S(A, B) = \frac{\Pr(A \cap B)}{\Pr(A) \Pr(B)} = s \text{ is } \begin{cases} \text{positive,} & \text{if } s > 1 \\ \text{neutral,} & \text{if } s = 1 \\ \text{negative,} & \text{if } s < 1 \end{cases}$$

Is coherence truth conducive?

- *In order for coherence to be truth conducive, more coherence must imply higher posterior probability (Olsson 2005, 136).*

Answers

Is coherence truth conducive?

- *In order for coherence to be truth conducive, more coherence must imply higher posterior probability (Olsson 2005, 136).*

Answers

- Klein and Warfield: (not always).
- Olsson: No.

example 1 (− cor, + conf)



h_1 : The drawn ball is the 2 ball. $\Pr(h_1) = 0.2$
 A_1 : The drawn ball is solid. $\Pr(A_1) = 0.6$
 A_2 : The drawn ball is even. $\Pr(A_2) = 0.4$

$$S(A_1, A_2) = \frac{\Pr(A \cap B)}{\Pr(A) \Pr(B)} \approx 0.833$$

$$\frac{\Pr(h_1|A_1)}{\Pr(h_1)} \approx 1.667, \quad \frac{\Pr(h_1|A_2)}{\Pr(h_1)} = 2.5$$

but

$$\frac{\Pr(h_1|A_1 \cap A_2)}{\Pr(h_1|A_1)} = 3, \quad \frac{\Pr(h_1|A_1 \cap A_2)}{\Pr(h_1|A_2)} = 2.$$

example 2 (+ cor, + conf)



h_2 : The drawn ball is striped.

A_3 : The drawn ball is odd.

A_4 : The drawn ball is an even or an odd stripe.

$$S(A_3, A_4) = 1.25 \quad \Pr(h_2) = \Pr(A_3) = \Pr(A_4) = 0.4$$

$$\frac{\Pr(h_2|A_3)}{\Pr(h_2)} = 1.25 = \frac{\Pr(h_2|A_4)}{\Pr(h_2)}$$

$$\frac{\Pr(h_2|A_3 \cap A_4)}{\Pr(h_2|A_3)} = 2 = \frac{\Pr(h_2|A_3 \cap A_4)}{\Pr(h_2|A_4)}$$

example 3 (– cor, – conf)



h_3 : The drawn ball is solid.

A_5 : The drawn ball is odd.

A_6 : The drawn ball is even.

$$S(A_5, A_6) = 0 \quad \Pr(h_3) = \Pr(A_5) = \Pr(A_6) = 0.4$$

$$\frac{\Pr(h_3|A_5)}{\Pr(h_3)} = 1.25 = \frac{\Pr(h_3|A_6)}{\Pr(h_3)}$$

$$\frac{\Pr(h_3|A_5 \cap A_6)}{\Pr(h_3|A_5)} = 0 = \frac{\Pr(h_3|A_5 \cap A_6)}{\Pr(h_3|A_6)}$$

example 4 (+ cor, - conf)



h_4 : The drawn ball is even.

A_7 : The drawn ball is the 2 ball.

A_8 : The drawn ball is solid.

$$S(A_7, A_8) = 2.5 \qquad \Pr(h_4) = \Pr(A_8) = 0.4,$$

$$\Pr(A_7) = 0.2$$

$$\frac{\Pr(h_4|A_7)}{\Pr(h_4)} = 2.5 = \frac{\Pr(h_4|A_8)}{\Pr(h_4)}$$

$$\frac{\Pr(h_4|A_7 \cap A_8)}{\Pr(h_4|A_7)} = 1 = \frac{\Pr(h_4|A_7 \cap A_8)}{\Pr(h_4|A_8)}.$$

Formal question

- Is there an informative relationship between a *d_{fi}*-measure and confirmation?

Implementation question

- What is the mechanism for ensuring independent 'cognitively spontaneous beliefs' ?

Formal question

- Is there an informative relationship between a *d_{fi}*-measure and confirmation? (YES)

Implementation question

What is the mechanism for ensuring independent 'cognitively spontaneous beliefs' ?

the Wayne-Shogenji measure

$$S(A, B) = \frac{\Pr(A \cap B)}{\Pr(A) \Pr(B)} = s \text{ is } \begin{cases} \text{positive,} & \text{if } s > 1 \\ \text{neutral,} & \text{if } s = 1 \\ \text{negative,} & \text{if } s < 1 \end{cases}$$

Conditional W-S measure (Wayne 1995)

$$S(A, B|h) = \frac{\Pr(A \cap B|h)}{\Pr(A|h) \Pr(B|h)} = t \text{ is } \begin{cases} \text{positive,} & \text{if } t > 1 \\ \text{neutral,} & \text{if } t = 1 \\ \text{negative,} & \text{if } t < 1 \end{cases}$$

Focused Correlation

Suppose

$$S(A, B) = \frac{\Pr(A \cap B)}{\Pr(A) \Pr(B)}, \text{ and}$$

$$S(A, B|h) = \frac{\Pr(A \cap B|h)}{\Pr(A|h) \Pr(B|h)}. \text{ Then,}$$

$$\text{For}_h(A, B) := \frac{S(A, B|h)}{S(A, B)} = \frac{\Pr(h|A \cap B)}{\Pr(h)} \times \frac{\Pr(h)}{\Pr(h|A)} \times \frac{\Pr(h)}{\Pr(h|B)}.$$

Interpreting Focused Correlation

$$\frac{\Pr(h|A \cap B)}{\Pr(h)} = For_h(A, B) \times \frac{\Pr(h|A)^*}{\Pr(h)} \times \frac{\Pr(h|B)^*}{\Pr(h)}.$$

if $For_h(A, B) > 1$, then combining *increases* $\Pr_{post}(h)$.

if $For_h(A, B) < 1$, combining does **not** increase $\Pr_{post}(h)$.

if $For_h(A, B) = 1$, then combining yields no difference.

if $For_h(A, B) = 0$, $\{A, B\}$ gives no information on $\Pr_{post}(h)$.

* *only if* $\Pr(h|\cdot) > \Pr(h)$.

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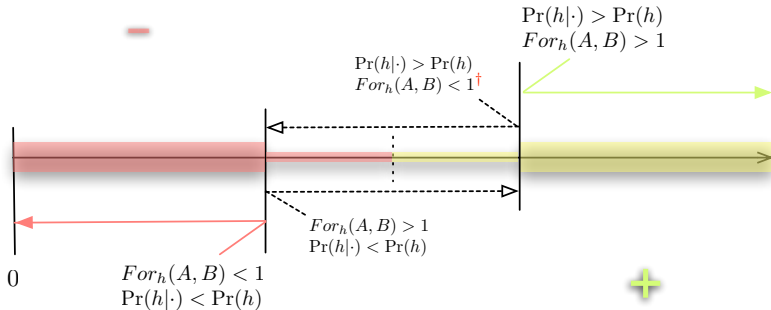
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* *only if* $\Pr(h|\cdot) < \Pr(h)$.

interpreting focused correlation



† when $For_h(A, B) \neq 0$.

example 1 (− cor, + conf)



h_1 : The drawn ball is the 2 ball.

A_1 : The drawn ball is solid.

A_2 : The drawn ball is even.

$$S(A_1, A_2) \approx 0.833 \text{ For } h(A_1, A_2) = \frac{S(A_1, A_2|h)}{S(A_1, A_2)} \approx 1.20$$

$$\frac{\Pr(h_1|A_1)}{\Pr(h_1)} \approx 1.667, \quad \frac{\Pr(h_1|A_2)}{\Pr(h_1)} = 2.5$$

$$\frac{\Pr(h_1|A_1 \cap A_2)}{\Pr(h_1|A_1)} = 3, \quad \frac{\Pr(h_1|A_1 \cap A_2)}{\Pr(h_1|A_2)} = 2.$$

example 2 (+ cor, + conf)



h_2 : The drawn ball is striped.

A_3 : The drawn ball is odd.

A_4 : The drawn ball is an even or an odd stripe.

$$S(A_3, A_4) = 1.25$$

$$For_{h_2}(A_3, A_4) = 1.6$$

$$\frac{\Pr(h_2|A_3)}{\Pr(h_2)} = 1.25 = \frac{\Pr(h_2|A_4)}{\Pr(h_2)}$$

$$\frac{\Pr(h_2|A_3 \cap A_4)}{\Pr(h_2|A_3)} = 2 = \frac{\Pr(h_2|A_3 \cap A_4)}{\Pr(h_2|A_4)}$$

example 3 (– cor, – conf)



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A_6 : The drawn ball is even.

$$S(A_5, A_6) = 0$$

$$For_{h_3}(A_5, A_6) = 0$$

$$\frac{\Pr(h_3|A_5)}{\Pr(h_3)} = 1.25 = \frac{\Pr(h_3|A_6)}{\Pr(h_3)}$$

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example 4 (+ cor, - conf)



h_4 : The drawn ball is even.

A_7 : The drawn ball is the 2 ball.

A_8 : The drawn ball is solid.

$$S(A_7, A_8) = 2.5$$

$$\text{For}_{h_4}(A_7, A_8) = 0.4$$

$$\frac{\Pr(h_4|A_7)}{\Pr(h_4)} = 2.5 = \frac{\Pr(h_4|A_8)}{\Pr(h_4)}$$

$$\frac{\Pr(h_4|A_7 \cap A_8)}{\Pr(h_4|A_7)} = 1 = \frac{\Pr(h_4|A_7 \cap A_8)}{\Pr(h_4|A_8)}.$$

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Answers

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* *Within a testimonial system.*

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Answers

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- Olsson: NO*
- Wheeler: YES†

* *Within a testimonial system.*

† *if you don't mind talking this way.*

features of focused correlation

- The results do not depend upon conditional independence assumptions (Cf. Bovens, Hartmann, Olsson, Shogenji).
- No restrictions on logical relationships between evidence and hypothesis variables.
- Highlights that linking dfi of n evidence variables to confirmation of h is an $n + 1$ relation.
- Reveal important asymmetries in incremental confirmation; ordering of evidence variables.

some limitations of focused correlation

- How do you model 'cognitively spontaneous' belief?
- Do we really want to drag h into this?
- Does the For_h answer to Olsson obstruct giving an answer to Bovens & Hartmann?
- For_h ignores caveats and precautions that attend drawing inferences from correlation measures (just like everybody else.)

summary

For_h offers some good news:

- ★ Focused Correlation resolves a purely formal question raised by the ‘truth connection’ problem for coherence measures.
- ★ Locates the action in Olsson’s impossibility result within his testimonial model and its conditional independence assumptions.
- ★ Identifying the parameters necessary for *For_h* can locate the most informative evidence in a distribution for a given hypothesis variable. Evidence variables can then be ordered by coarse to fine-grained impact upon a hypothesis.
- ★ Values for *For_h* can reveal whether evidence is (positively or negatively) monotone.

future work

For_h in context:

- ★ Some good news and many open questions . . .
- *Bad news*: Coherence measures and the dynamics of belief change.
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Focused correlation

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thank you!