

The No Alternatives Argument

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Notabene:

- This talk represents an intermediate stage of the paper!
- In the meantime (=after the conference), we have relabeled and weakened the assumptions in the proof of NAA (i.e., A6 and A7 are not required any more, A0 becomes A2).
- So if you are really interested, just email us or wait until we post the (re)submission version on the PhilSci-Archive. This will probably be in early July.

Motivation

Scientists often reason like this:

- 1 Theory H satisfies several desirable conditions: it incorporates various theoretical principles (e.g., certain symmetries), it coheres well with other theories, . . .
- 2 Despite a lot of effort, the scientific community has not yet found an alternative to H that also satisfies these conditions.
- 3 This lack of alternatives is in itself evidence for H.

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We ask: Under which conditions is a No Alternatives Argument valid? Skeptics might find it an argument from incompetence.

The paper investigates these questions in the framework of standard Bayesian epistemology.

- 1 The Conceptual Framework
- 2 The No Alternatives Argument
- 3 On the Significance of NAA
- 4 How Many Alternatives?
- 5 Conclusions

Examples from Fundamental Physics

We find many examples of No Alternatives Arguments (NAA) in Fundamental Physics, mainly because discriminating empirical evidence is hard to come by.

String Theory This theory is not empirically confirmed. What speaks in its favor are (unproven) coherence arguments and the NAA.

Cosmic Inflation This theory enjoys a very limited degree of empirical confirmation. Trust in the theory, however, crucially relies on the NAA.

Examples from Fundamental Physics (cont'd)

The Higgs Model This is not an independent theory. It is embedded in the Standard Model of Particle Physics, but it constitutes a 'module' of the Standard Model that may be replaced by another one. Trust in the Higgs Model is based on a NAA at two levels:

- 1 Scientists believe that no convincing alternative field theoretical account of mass generation exists.
- 2 Scientists believe that there is no adequate description of the phenomenology of the Standard Model that does not look like a field theory at the relevant energy scales.

Theory Individuation

Talking about No Alternative Arguments leads to the problem of **theory individuation**. In general, we leave this problem to the scientists (who usually have a good grip on it), but we make the following constraints:

- First, different theories make **different predictions**. If two theories make exactly the same predictions, then we consider them to be identical (e.g., different interpretations of QM).
- Second, different theories **provide different solutions** to a given scientific problem. That is, theories which only differ in a detail, say in the value of a parameter, or the existence of a physically meaningless dummy variable, do not count as different theories.

Then, it is **possible** that there are only **finitely many alternatives** to H.

Formalizing NAAs

We formalize the No Alternatives Argument by means of **propositional variables**.

- Let \mathcal{C} be a set of (theoretical) constraints, \mathcal{D} be a set of data, and \mathcal{E} be a set of relevant future experiments.
- The hypothesis H satisfies \mathcal{C} , accounts for \mathcal{D} and predicts the outcomes of \mathcal{E} .

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- We observe F_A : no alternative hypothesis has been found that has these properties, too.

Question: To what extent does this observation F_A confirm T ?

Number of Alternatives

We introduce another variable.

- Y has values in the natural numbers, viz. Y_k : There are exactly k hypotheses which fulfill \mathcal{C} , explain \mathcal{D} and predict \mathcal{E} . (H is one of them.)
- We claim that scientists have **degrees of belief about the number of alternatives to H**, that is, the values of the Y_k .

Difficulty of the Problem

And to complete the picture, we introduce yet another variable:

- Whether scientists find an alternative to H arguably depends on the complexity of the problem, the cleverness of the scientists, or the available computational, experimental, and mathematical resources.
- Call the variable that models the **difficulty of the problem** D , and let it take values in the natural numbers.

Relations between the Variables

- ① The variable F_A has two values:

F_A The scientific community has not yet found an alternative to H that fulfills \mathcal{C} , explains \mathcal{D} and predicts \mathcal{E} .

$\neg F_A$ The scientific community has found an alternative to H that fulfills \mathcal{C} , explains \mathcal{D} and predicts \mathcal{E} .

- ② The variable T has two values:

T The hypothesis H is empirically adequate.

$\neg T$ The hypothesis H is not empirically adequate.

- ③ Y has \mathbb{N} values, viz.

Y_k There are exactly k hypotheses which fulfill \mathcal{C} , explain \mathcal{D} and predict \mathcal{E} . (H is one of them.)

- ④ Let D denote the difficulty of the problem.

D_j The problem has difficulty rank j .

A Bayesian Network Representation

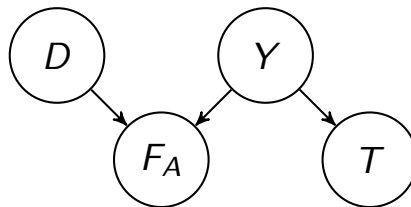


Figure: The Bayesian Network representation of the four-propositions scenario.

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A2. The prior probabilities

$$y_k := P(Y_k)$$

are smaller than 1, that is, $0 \leq y_k < 1$.

A2 reflects the fact that we do not know the number of viable alternatives a priori.

Mathematical Assumptions (cont'd)

A3. The conditional probabilities

$$f_{kj} := P(F_A | Y_k, D_j)$$

are monotonically decreasing in k for all $j \in \mathbb{N}$ and monotonically increasing in j for all $k \in \mathbb{N}$.

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A5. There is at least one pair (i, k) with $i < k$ for which (i) $y_i y_k > 0$, (ii) $f_{ij} > f_{kj}$ for some $j \in \mathbb{N}$, and (iii) $t_i > t_k$.

The Main Result

With these five assumptions at hand, we can show our main result:

The No Alternative Argument

Theorem: If assumptions **A1** to **A5** hold, then F_A confirms T , that is, $P(T|F_A) > P(T)$.

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This seems to show the possibility of **non-empirical theory confirmation** – where non-empirical evidence F_A for a theory H is neither deductively nor probabilistically implied by H .

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- “Thus, NAA does not go through any more.”

Actually, we can refute the skeptic if she concedes the following two assumptions.

NAA: The Infinite Case

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- A7** The probability that theory H is empirically adequate, given an infinite number of alternative theories, is zero:

$$P(T|Y = \infty) = 0$$

As regards this last assumption, we believe that there is **no good reason for preferring H to its unconceived alternatives**.

NAA: The Infinite Case

Given these additional assumptions and the natural modification of **A3** for the case of $\{Y = \infty\}$, the previous theorem can be extended as follows:

The No Alternatives Argument (infinite case)

Theorem: If Y may also take the value $Y = \infty$ and assumptions **A1** to **A7** hold, then F_A confirms T , that is, $P(T|F_A) > P(T)$.

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Thus, an undogmatic skeptic has to **acknowledge the validity of the NAA**.

Discussion

- Note that the assumptions of the theorem are rather weak.
- Only if an agent believes with certainty that the number of alternatives is infinite (i.e. that $y_\infty = 1$), then F_A does not confirm T and the NAA has no pull.

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Refined skeptical position: The **degree of confirmation** depends on the specific values of the parameters that occur in **A2-A5**. So the argument is, as it stands, only **qualitatively**, not **quantitatively** convincing.

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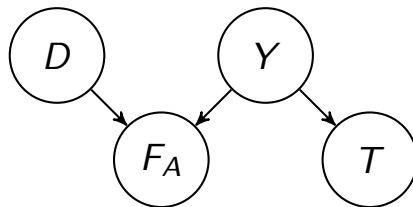


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- Thus, F_A typically confirms the claim that the problem at hand is rather complicated and typically disconfirms the claim that it is not particularly complicated.
- To show the practical relevance of NAA, one has to show that F_A confirms T more than a high value of D , but such a claim is sensitive to the specific parameter assignments and therefore hard to prove in general.
- In particular, such claims depend on beliefs over the number of alternative theories, that is, the values of y_k .

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- Scientists have often succeeded at identifying a theory that makes the correct **predictions**, rather than just accommodating existing data (Kahn et al. 1992, Hitchcock and Sober 1994).
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However, such a **meta-inductive argument (MIA)** still needs to be formalized! (→ future work) How else can we assess the number of alternatives?

Motivation

Assessing the number of alternatives is a worthwhile project for a variety of reasons:

- More information about the values of the y_k helps to evaluate the significance of the NAA.
- Allows for a more rigorous study of the **problem of theoretical underdetermination** (in our particular perspective on theory individuation) and its belief dynamics.

An Epistemic St. Petersburg Paradox

Epistemic Tension

Proposition 1: For any $N \in \mathbb{N}$ and any $1 > \varepsilon > 0$, an agent's belief function P may jointly satisfy (i) $P(Y = \infty) = 0$, (ii) $P(Y < N) \geq 1 - \varepsilon$, and (iii) $\langle Y \rangle = \sum_{k=0}^{\infty} k P(Y_k) = \infty$.

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- The agent can be certain that there are only finitely many alternatives to H, but her best guess about the number of alternatives is “indefinitely large”.
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- Gives an interesting twist to the **problem of theoretical underdetermination**.

From $\langle Y \rangle = \infty$ to $\langle Y \rangle < \infty$?

Question: Can an agent whose best guess about the number of alternatives is “indefinitely large” change this opinion?

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Shifting to finitely many alternatives

Theorem: Assume that $\langle Y \rangle = \infty$. Then the following conditions on evidence E with $P(E) \neq 0$ are **individually sufficient** for $\langle Y \rangle_E < \infty$.

- 1 The sequence $(k \cdot P(E|Y_k))_{k \in \mathbb{N}}$ is bounded.
- 2 $\sum_{k=0}^{\infty} P(E|Y_k) < \infty$ and there is a $N_0 \in \mathbb{N}$ such that $(P(Y_k))_{k \in \mathbb{N}}$ is, for all $k \geq N_0$, monotonically decreasing.
- 3 ...

From $\langle Y \rangle = \infty$ to $\langle Y \rangle < \infty$? (cont'd)

- Main idea: $P(E|Y_k)$ converges fast enough to zero.
- For evidence E that is related to an empirical test of H , this assumption is reasonable: if there are more and more alternatives, why should H , instead of an unconceived alternative, survive empirical tests?

From $\langle Y \rangle = \infty$ to $\langle Y \rangle < \infty$

Actually, the converse direction is **not** possible:

Finitely many alternatives as an “annihilating state”

Proposition 2: If $\langle Y \rangle < \infty$, then for any evidence E (empirical or non-empirical) with $P(E) \neq 0$, $\langle Y \rangle_E < \infty$.

That is, learning additional evidence (by means of Bayesian Conditionalization) cannot change our guess that the number of alternatives is finite.

From $\langle Y \rangle = \infty$ to $\langle Y \rangle < \infty$

Finally, we give a **criterion** for when empirical evidence **lowers** the expected number of alternatives:

Lowering the expected number of alternative

Theorem: Let Y_k^+ denote the proposition that there are at least k alternatives to theory H , and let Y_k^- denote the proposition that there are at most $k - 1$ alternatives to H . Then, if

$$P(E|Y_k^+) \leq P(E|Y_k^-) \quad \forall k \in \mathbb{N}; \quad \exists l > 0 : P(E|Y_1^+) < P(E|Y_1^-)$$

it will also be the case that $\langle Y \rangle > \langle Y \rangle_E$, the latter expression denoting the expectation value of Y under $P(\cdot|E)$.

Conclusions and Outlook

- ① We have provided a Bayesian account of the No Alternatives Argument and analyzed under which conditions this argument is valid.
- ② In particular, we have extended NAA to the infinite case and are now able to rebut a non-dogmatic skeptical position about the validity of NAA.
- ③ Given that various assumptions have to be fulfilled, the strength of a proposed No Alternatives Argument (that is, the degree of confirmation it provides) will often be controversial.

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The **validity of IBE** under certain assumption is a potential application of NAA:

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Question: Can this kind of reasoning be used to **respond to van Fraassen's best of a bad lot argument?**

\Rightarrow An NAA analysis helps us to better appreciate when IBE is valid.

Conclusions and Outlook

Open questions:

- Further weakening of the assumptions (→ Frederik Herzberg's commentary)
- Detailed case studies from science (e.g., string theory)
- No-Alternatives Arguments in philosophy (e.g., Inference to the Best Explanation)
- Formalization of the meta-inductive argument

Thanks a lot for your attention!