Influencing Behavior by Influencing Knowledge

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Knowledge in a Restaurant

Three people A, B, C walk into a coffee shop. One of them orders cappuccino, one orders tea, and one orders icecream. The waiter goes away and after ten minutes *another* waiter arrives with three cups. "Who has the cappuccino?" "I do," says A. "Who has the tea?" "I do," says C.

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Will the waiter ask a third question?"

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When C says that he has the tea, 1 is eliminated, leaving only 2.

2) CIT

Now the waiter knows that B has the icecream.

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When the butler said “sir”, he eliminated 1) and saved the lady from embarrassment.

Formalism

We create a language to talk about various knowledge properties in the following way.

- An atomic predicate P is a formula
- If A, B are formulas then so are $\neg A$ and $A \wedge B$
- If A is a formula and i is an agent then $K_i(A)$ is a formula
- We may also include formulas $C(A)$ if we wish to denote common knowledge

Intuition

Intuitively $K_i(A)$ means that the agent i knows the fact expressed by the formula A. $K_j K_i(A)$ means that j knows that i knows A. If i, j are the only agents, then $C(A)$ means that i knows A, j knows that i knows A, j knows that j knows that i knows A and so on forever.

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But we do not have $K_rK_u(Q)$.

Kripke structures

Kripke structures are used to interpret the language above.
Kripke structure M for knowledge for n knowers consists of a space W of states and for each knower i a relation $R_i \subseteq W \times W$.
There is a map π from $W \times A \longrightarrow \{0, 1\}$ which decides the truth value of atomic formulas at each state.

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Usually the R_i are taken to be equivalence relations, i.e., reflexive, symmetric and transitive.

We now define the truth values of formulas as follows:

1. $M, w \models P$ iff $\pi(w, P) = 1$
2. $M, w \models \neg A$ iff $M, w \not\models A$
3. $M, w \models A \wedge B$ iff $M, w \models A$ and $M, w \models B$
4. $M, w \models K_i(A)$ iff $(\forall t)(wR_i t \rightarrow M, t \models A)$

$K_i(A)$ holds at w, (i knows A at w) iff A holds at all states t which are R_i accessible from w.

Some Consequences

If R_i is reflexive then we will get $K_i(A) \rightarrow A$ (veridicality) as a consequence.
Moreover, regardless of the properties of R_i, we have,

1. If A is logically valid, then A is known
2. If A and $A \rightarrow B$ are known, then so is B

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Still, in small settings, such assumptions are reasonable.

Axiom system

1. All tautologies of the propositional calculus
2. $K_i(A \rightarrow B) \rightarrow (K_i(A) \rightarrow K_i(B))$
3. $K_i(A) \rightarrow A$
4. $K_i(A) \rightarrow K_i K_i(A)$
5. $\neg K_i(A) \rightarrow K_i(\neg K_i(A))$

Some of these axioms are controversial
but we will not discuss the controversy.

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The second rule does not say that if A is true than i knows it. Only that it A is a logical truth then i knows it.
These rules are complete. All valid formulas are provable using the axioms and rules.

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Suppose we are given a Kripke structure $\mathcal{M}$. Then some formula φ is announced publicly.

The new Kripke structure is then obtained by deleting all states in $\mathcal{M}$ where φ did not hold.

Theory of Mind

A group of children are told the following story:
Maxi goes out shopping with his mother and when they come back, Maxi helps mother put away the groceries, which include chocolate. There are two cupboards, red and blue. Maxi puts the chocolate in the red cupboard and goes out to play.

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While Maxi is gone, mother takes the chocolate out of the red cupboard, uses some of it to bake a cake, and then puts the rest in the blue cupboard.

Now Maxi comes back from play and wants the chocolate

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But children up to the age of three or four say, *Maxi will look in the blue cupboard.*

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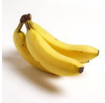
What three year old children lack, according to psychologists Premack and Woodruff is a Theory of Mind

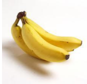
Animal Cognition

Do animals have a Theory of Mind?

BBBBBBBBBBBBBBBBBBBBBBBBBBBBBB



Food 1



Food 2

What chimps think about other chimps

In the last slide, the chimp at the bottom is subservient to the dominant chimp at the top and has to decide which group of bananas to go for.

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In the last slide, the chimp at the bottom is subservient to the dominant chimp at the top and has to decide which group of bananas to go for. In experiments, the sub-chimp tends to go for Food 1 which the dom-chimp cannot see. Is there use of epistemic logic by the sub-chimp? This is an issue of some controversy

Inducing false beliefs in the tigers of the Sundarbans

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Unfortunately the tigers eventually realized it was a hoax, and the attacks resumed.

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Knowledge means influence and power.

It is a commonplace that what we do depends on what we know. And given that most of us have at least the rudiments of a *theory of mind* (cf. Premack and Woodruff) we also know that what others do will depend on what *they* know.

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The play ends with Beatrice and Benedick getting married

Benedick's Decision problem

	love	nolove
propose	100	-20
nopropose	-10	0

Here love means "Beatrice loves me" and nolove the other possibility.

Applications of Epistemic Reasoning to Society

The peculiar character of the problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. The economic problem of society is thus not merely a problem of how to allocate "given" resources-if "given' is taken to mean given to a single mind which deliberately solves the problem set by these "data." It is rather a problem of how to secure the best use of resources known to any of the members of society, for ends whose relative importance only these individuals know.

F. Hayek, 1945

We consider three issues.

- How does an agent choose in the face of uncertainty? Suppose an agent wants to choose between actions L and R. Each has a number of possible consequences, Some consequences of L are better than some of R and vice versa. Then how to choose?
- We claim that there are various versions of rationality depending on whether an agent is cautious or aggressive or moderate. The famous experiment (reported by Kahneman) about the Asian Disease question can be explained as a switch from one version of rationality to another.
- Suppose agent M is in a position to control the knowledge that agents A and B (who are playing a game) have. Once their state of knowledge is determined they will play the game in a certain way. How should M pick the knowledge states of A and B so as to maximize the benefit to M herself?

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But if she prefers vanilla to chocolate and chocolate to strawberry, then we do not know if the utilities should be 10, 9, 1 or 10, 2, 1.

Cardinal Utilities

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We will look primarily at ordinal utilities in this talk.

Rationality

Sergei Artemov is concerned with rationality in the presence of uncertainty.[1] A rational player for him is one who makes a decision based on the highest *guaranteed* payoff, subject to the player's knowledge.

[1] His utilities are ordinal. The maxmin approach of von Neumann and Morgenstern is similar.

He shows that a rational player *in his sense* will follow the backward induction solution in the centipede game *even in the absence of common knowledge of rationality*. Thus Artemov generalizes Aumann's result, replacing common knowledge of rationality by plain rationality.

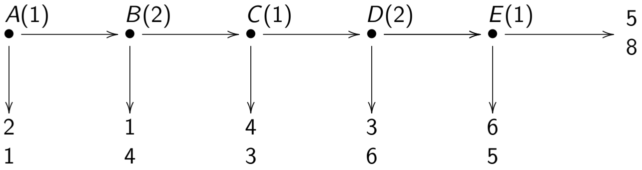

Figure: Centipede game

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Action *Across* will bring him a payoff among {1,3,4,5,6}.
A player who would choose down on the basis that its guaranteed payoff 2 is higher than 1, we will describe as *conservative*, rather than *rational*.
For a moderate (but rational) player may use, not the minima, but the medians as a way of evaluating the two sets {2} and {1,3,4,5,6} and prefer the second set whose median (4) is higher.

Consider two moderate players playing the centipede game. If both players are agnostic about the rest of the game, then the game will continue for quite a while, with both players going across and earning much larger payoffs.

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This agrees with the results of R. McKelvey and T. Palfrey, An experimental study of the centipede game, *Econometrica*, **60** (1992) 803-836.

Such points of view are often expressed by stockbrokers advising people on investments.

- A younger investor may prefer a stock with a high potential growth but significant risk.
- An investor close to retirement age may prefer a stock with less growth but also less risk.
- A middle aged investor may accept a moderate amount of risk.

Generalizing Rationality

If we are using ordinal utilities, and are in a state of uncertainty, then when comparing two actions, we are comparing two *sets* (or *sequences*, ordered by preference) of payoffs rather than individual payoffs.

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Since the basic comparison is between individual points rather than sets, one method is to choose representative elements from the two sets and compare these elements.

- Suppose that a player must choose between actions L and R but does not know precisely what will transpire as a result of these choices.
- Suppose that based on her knowledge, L will yield possible outputs $a_1 > ... > a_k$
- and R will yield possible outputs $b_1 > b_2 > ... > b_m$.

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- and R will yield possible outputs $b_1 > b_2 > ... > b_m$.
- Suppose that a_k is better than b_m and b_1 is better than a_1
- Then a conservative player will choose L.
- And an aggressive player will choose R.
- But a moderate player may do something in between, say compare the medians of the two sequences (which can be defined *solely* from the ordering).

Suppose a player uses some selection function f to choose the representative element. Then he will prefer set X to set Y if $f(X) > f(Y)$.

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This requirement imposes restrictions on what f can be like.

Definition: A function f is suitable if it has the propeties below.

1. If ordered sets A and B are isomorphic by an order preserving map g, then $g(f(A)) = f(B)$
2. If set A is augmented to a set B by adding an element x which exceeds all elements of A, then $f(B) \geq f(A)$
3. If set A is augmented to a set B by adding an element x which is less than all elements of A, then $f(B) \leq f(A)$
4. If sets A and B overlap but all elements in B - A exceed those in $A \cap B$ which exceeds all elements in A - B then $f(A) \leq f(B)$.

Note that conditions 2 and 3 together imply (for finite sets) condition 4 which implies both 2 and 3.

Definition

Given an SCF f, An f-rational agent is an agent who, when uncertain between sets X and Y of alternatives, always picks X if $f(X) > f(Y)$.

The following monotonicity property is a consequence of properties 1-4 above.

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If a set X is changed to Y by moving up one (or more) elements, then $f(X)$ should increase, i.e., $f(X) \leq f(Y)$.
All of min, max and median satisfy monotonicity.

Lemma

The minimum, the median and the maximum are all suitable functions in the sense above (and the corresponding notions of f-rationality are equivalent to being conservative, moderate, and aggressive respectively).

Is there a characterization of the class of all suitable functions?

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Let us define $s(n) = f(\{1, ..., n\})$
Because of the order isomorphism property 1) above, the function s completely characterizes f. For any finite ordered set is order isomorphic to some set $\{1,...,n\}$. By abuse of language we will say thar s is suitable iff the corresponding f is.

Proposition: s is suitable iff $s(n) \leq s(n+1) \leq s(n)+1$ for all n

Proof: Necessity: Note that $f(\{1,..,n\}) \leq f(\{1,...,n+1\})$ since the second set is obtained from the first by adding a larger element. This yields $s(n) \leq s(n+1)$
Note again that $f(\{1,..,n+1\}) \leq f(\{2,...,n+1\})$ since the first set is obtained from the second by adding a smaller element.
But the second value (using isomorphism) is just $s(n)+1$. So we get $s(n+1) \leq s(n)+1$
Before proving sufficiency we remark that this yields $s(n+m) \leq s(n)+m$.

Sufficiency: Suppose that s satisfies $s(n) \leq s(n+m) \leq s(n) + m$ for all m, n.
It is easy to see that the first three conditions above will hold for the corresponding f.
To see the last, suppose that set A is {1,...,n} and set B is {m+1,...,n,...,m+r} so that the elements 1,,.,m are in A and below B, and elements n+1,,.m+r are elements of B above A. Since we assume overlap as in 4), we may assume that m+r is at least n.
Now $f(B) = s(r) + m$ since B consists of r elements *in order* but shifted rightward from 1,...,r by an amount of m. Since $n \leq r + m$, $s(n) \leq s(r+m) \leq s(r) + m$. Now $f(A) = s(n)$, and $f(B) = s(r) + m$. So indeed $f(A) \leq f(B)$.
$\square$

This shows that there are uncountably many suitable choice functions since the transition from $s(n)$ to $s(n+1)$ can be zero or one, infinitely many times. Since there are not that many human beings, the conservative humans with $s(n) = 1$, aggressive humans with $s(n) = n$ and moderate humans with $s(n)$ approximately equal to $n/2$ are the typical cases to appear in practice.

The IIA condition

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Now Nash's condition need not apply to choice functions in our sense, since we are choosing a typical element and not the 'best' one. Still it is worth asking which suitable functions do satisfy Nash's condition.

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Proof: Suppose that f is a suitable choice function other than the max or the min. Then for some set $\{1, ..., n\}$ it must be the case that $1 < f(n) < n$. Now consider $f(n-1)$. It must be either $f(n)$ or $f(n) - 1$. If it is the latter then it is chosen from $\{1, .., n-1\}$ even though $f(n)$ (which is $< n$) is still available. If it is $f(n)$ then (by IIA) it is chosen from $\{2, ..., n\}$ but by the isomorphism property $f(n-1) + 1$, i.e., $f(n) + 1$ should be chosen from $\{2, ..., n\}$. Either way we have a contradiction.
So the only suitable choice functions satisfying the IIA condition are max and min. $\square$

Other Approaches

The extension of preference orderings from pairs of elements to pairs of sets need not proceed via representative elements as we have done. Nonetheless, if there is preference ordering $>$ on elements and we extend it to a relation $\succ$ on pairs of sets, then certain rationality conditions have been proposed.
It is generally assumed of course that $\{a\} \succ \{b\}$ in the set ordering iff $a > b$ in the element ordering.

In the following the set of elements is X.

Dominance: for all $x \in X$ and for all $A \subseteq X$

$$[x > y \text{ for all } y \in A] \quad \rightarrow A \cup \{x\} \succ A$$
$$[x < y \text{ for all } y \in A] \quad \rightarrow A \succ A \cup \{x\}$$

Independence: for all $A, B \subseteq X$ and $x \in X - (A \cup B)$

$$A \succ B \rightarrow A \cup \{x\} \succeq B \cup \{x\}$$

Kannai and Peleg attribute the first property to Gärdenfors.

Teddy Seidenfeld has shown that the principle of independence is incompatible with choosing by subjective expected utility

To be precise, he has shown that if we assume that the ordinal utilities correspond to some unknown cardinal utilities, and that we are choosing by comparing expected utilities, again relative to unknown subjective probabilities, then the independence priciple can lead to choosing the option which has lower expected utility.

Theorem

(Kannai and Peleg) If X has at least six elements, then dominance and independence are incompatible

Hint of proof They show first that given dominance and indepence, a set is equivalent to the two element set consisting of its smallest and largest elements.

Weak dominance: for all $x \in X$ and for all $A \subseteq X$

$$[x > y \text{ for all } y \in A] \quad \rightarrow A \cup \{x\} \succeq A$$

$$[x < y \text{ for all } y \in A] \quad \rightarrow A \succeq A \cup \{x\}$$

Theorem

Let f be one of the functions, max, min or median. Define $A \succ B$ by $A \succ B \leftrightarrow f(A) > f(B)$

Then the $\succ$ defined this way satisfies weak dominance and independence if f is max or min. Median satisfies (weak?) dominance but not independence.

Example: let $A = \{1, 4, 7, 10, 13\}$ and $B = \{1, 4, 8, 9, 13\}$
Them $\text{med}(A) = 7 < 8 = \text{med}(B)$
However, after adding elements 11 and 12 we get
$A' = \{1, 4, 7, 10, 11, 12, 13\}$ and $B' = \{1, 4, 8, 9, 11, 12, 13\}$
Now the medians are 10 and 9 respectively so the sets have interchanged places.

Dominated Strategies

Suppose that an agent considers a set $X = \{a_1, ..., a_n\}$ of possibilities and the two actions L and R result in payoffs x_i, y_i respectively in case a_i turns out to be the case. We will say that L dominates R if $x_i > y_i$ for all i.

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Then for the three f's we consider, an f-rational agent will never choose a dominated strategy.

The notion of dominated strategy that we consider is subjective. The elements of X might not all be actually possible, though of course if only part of X is actually possible, a dominated strategy will remain dominated if X shrinks.

Suppose that Ingrid invites three guests for dinner and the options are d and d' for the menu. Guests 1 and 2 prefer d to d' but guest 3 is allergic to d.
If Ingrid is conservative, she will choose d' and avoid the worst outcome.

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If Ingrid is conservative, she will choose d' and avoid the worst outcome.
But suppose guest 3 is unable to come. Then d dominates d'.
Nonetheless, Ingrid is not being irrational. d is not a dominated startegy from her point of view.

The Asian Disease – Tversky and Kahneman 1981

Imagine that the United States is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people.

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If Program B is adopted, there is a one-third probability that 600 people will be saved and a two-thirds probability that no people will be saved

Which of the two programs would you favor?

In this version of the problem, a substantial majority of respondents favor program A, indicating risk aversion.

Other respondents, selected at random, receive a question in which the same cover story is followed by a different description of the options:
If Program A' is adopted, 400 people will die

Other respondents, selected at random, receive a question in which the same cover story is followed by a different description of the options:
If Program A' is adopted, 400 people will die
If Program B' is adopted, there is a one-third probability that nobody will die and a two-thirds probability that 600 people will die

A clear majority now favor B'

One way to understand this phenomenon is that the way the problem is stated causes a change from *min*-rationality to *max*-rationality

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Option A, or A' results in 400 deaths. Options B and B' amount to no deaths if we are lucky and to 600 deaths if we are unlucky.

But both forms of rationality are 'rational'.

Three aspects of rationality

1. Making the best of the choices one believes oneself to have.
2. Fully using logic to eliminate impossible choices
3. Using psychology to predict the choices of other people

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Perhaps we should keep these three aspects separate and reason about them separately.

Knowledge in Game Theory

We will usually assume that the temperament of the players conservative or moderate or aggressive is common knowledge.

We know that (given perfect knowledge) backward induction yields a unique way in which the game is played and according to Aumann, that will indeed be the way the game will be played if there is common knowledge of rationality.

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- This matters because someone who can manipulate the knowledge of others can also affect the way they play some particular game.
- If the game has payoffs not only for the active players, but also for the knowledge manipulator, then the manipulator will seek to manipulate the active players' knowledge in such a way as to maximize his own payoff.

Is Knowledge always beneficial?

Kamien, Tauman and Zamir consider the following example. A black or white card is chosen from a deck and player 1 is invited to guess its color. After 1 makes her choice, which is announced, player 2 is invited to make a choice. The payoffs are as follows:

- If both players guess correctly, then both get 2.
- If neither player guesses correctly, then both get 0.
- If only one player guesses correctly, then the correct player gets 5 and the other player gets 0.

- Suppose neither player knows the color then player 1 should choose randomly, player 2 should choose a different color and the expected payoff for both is 2.5 (half of 5+0)
- If player 1 is allowed to see the card, then the dominant strategy for her is to announce the correct color, player 2 should choose the same color and the expected payoff for both (a certain payoff in fact) is 2.

So the knowledge of player 1 makes her worse off.

However, player 1 is not harmed by the fact that she knows the color but by the fact that player 2 knows that 1 knows the color. Neyman shows that if we can make one player know more but prevent other players from having more knowledge, then the player who knows more cannot lose.

We now consider how the same game may be played differently depending on the information available to the agents and their temperaments.

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If there is a knowledge manipulator - KM who can control how much information the various agents can have then that agent can influence the way the game is played and the outcome.

Wife and husband

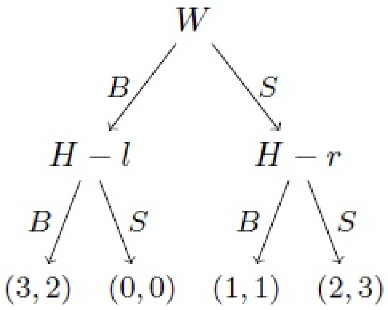

Fig. 1

In the last figure we assume that the wife moves first and the husband after.

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We consider various scenarios involving the husband's knowledge and temperament. We assume that the wife knows the husband's payoffs and temperament and he does not know hers.

Case 1) Husband does not know wife's move (and she knows this).
a) He is aggressive. Then being aggressive, he will choose S (Stravinsky) for his move since the highest possible payoff is 3. Anticipating his move, she will also choose S, and they will end up with payoffs of (2,3).

b) If the husband is conservative, then not knowing what his wife chose, he will choose B since the minimum payoff of 1 is better than the minimum payoff of 0. Anticipating this, the wife will also choose B and they will end up with (3,2).

2) Finally if the husband will know what node he will be at, then the wife will choose B, the husband will also choose B and they will end up at (3,2).

We consider now the question of how KM can create these various knowledge scenarios of the last example.
KM is capable of creating all these three situations by means of signals, as well as the one we did not mention where the husband does not know but the wife does not know that he will not.
For case 1a), $s(H - l) = (l, a)$ and $s(H - r) = (r, a)$. The wife knows (if she did not already) which node they are at, but the husband will not.

For case 2, $s(H - l) = (l, l)$ and $s(H - r) = (r, r)$. Both will know which node they are at.
Finally if KM wants the wife to be in doubt whether the husband knows, he could make $s(H - l) = \{(l, l), (l, a)\}$ and $s(H - r) = \{(r, r), (r, a)\}$. Then if the wife chose left and receives an l, she will not know if the husband got an l or the neutral a. If KM does send (l, l) then the husband will know, but will also know that his wife did not know whether he *would* know.

Limits to the power of the manipulator

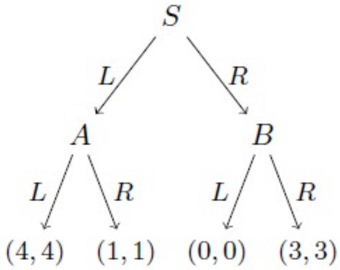

Fig. 3

Now suppose that when it is his turn to play, 2 (who is conservative) does not know whether he is at node A or B. Then he will choose Right which gives him one of $\{1,3\}$, safer than $\{4,0\}$, which he would get with Left. 1 will anticipate this and also choose Right. So they end up at (3,3).

However, 2 might start his reasoning by trying to figure out 1's move. 1 will get one of $\{4,1\}$ if she plays Left, and one of $\{0,3\}$ if she plays Right. So she will play Left. 2 will anticipate this and will play Left. So they end up at (4,4).

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Clearly the KM (whose payoffs we have not included) cannot count on any particular play.

Learning from Communication

Observation (Lewis, Aumann): Suppose a group of people are commonly aware of a number of possibilities (states) among which they are uncertain. They commonly know some fact ψ if ψ is true of all these possibilities.

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What are the consequences for the candidate of this fact?

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Major parties, Democrats, Republicans

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Important political liability for Obama: His association with Jeremiah Wright, a fiery black preacher who had made anti-America comments.

Our concern in this talk will be to study the way in which communication is used to change voter views

An illustrative example

- Hillary Clinton (while campaigning in Indiana):

$$D = \texttt{As a child, I shot a duck.}$$

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  - Indiana is a conservative state. So most of her immediate audience V_1 will be conservatives.
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  - Conservatives tend to **dis**favor gun control.
  - Hearing D is likely to improve HC in the eyes of V_1
    – say by amount u_1.
  - But (virtually) all statements a candidate makes are **public announcements**.
  - So another group of voters V_2 (say liberals in Massachusetts) also hear HC say D.
  - This is likely to make her go down for V_2
    – say by amount u_2.
  - But we likely have $|u_1| > |u_2|$ since
    i) $|V_1| > |V_2|$, or at least V_1 cares more passionately about the issue than V_2.
    ii) D merely **implicates** that HC will not impose gun control.

Towards a formal model: languages and theories

- We begin by considering a single candidate C.
- C's views about the issues are formulated in a proposition language $\mathcal{L}$ containing **finitely many** atomic propositions $At = \{P_1, \ldots, P_n\}$.
- For instance:
  - $P_1 =$ `We should withdraw from Iraq.`
  - $P_2 =$ `I will impose no new taxes.`
  - $\ldots$
  - $P_n =$ `We should bail out the banks.`
- $T_a = C$'s **actual theory** (i.e. the entirety of her views)
- $T_c = C$'s **current theory** (i.e. what's she's said thus far)
- Typically (but not always) $T_c \subseteq T_a$.

Given the theory generated by a candidate's statements, there is a set of possible worlds which are all compatible with that theory.

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Let us call that set X_c. Then $X_c = \{w | w \models T_c\}$

X_c is the set of those worlds which are compatible with what the candidate has said.

Worlds and preferences

- We conflate propositional valuations and worlds $w \in 2^{At}$.
- We also define $w[i] = \begin{cases} 1 & \textit{if } w \models P_i \\ -1 & \textit{if } w \not\models P_i \end{cases}$
- We initially consider a single group of voters V (think of this as a constituency).
- The voters in V are characterized by their preference for an **ideal world**.
- This is formalized via two functions p_v, x_v:
  - $p_v(i) = \begin{cases} 1 & V \text{ would prefer } P_i \text{ to be true} \\ 0 & V \text{ is neutral about } P_i \\ -1 & V \text{ would prefer } P_i \text{ to be false} \end{cases}$
  - $x_v : At \to [0, 1]$ the **weight** which V assigns to P_i s.t. $\sum w_v(i) \leq 1$.

Utilities of worlds and theories

- The utility of a world for V is defined as

$$u(w) = \sum_{1 \leq i \leq n} p_v(i) \cdot x_v(i) \cdot w[i]$$

- Note that a candidate's current theory T_c is likely to be **incomplete** – i.e. she may not express a view on some P_i.
- To calculate the utility of an arbitrary T we need to know how V will "fill in the blanks."
- That is, extend the evaluation from a single world to a **set** of worlds.

Voter types

- We postulate that there are three types of voters:
  - **Optimistic voters** (assume the best about C given T_c)
  - **Pessimistic voters** (asssume the worst about C given T_c)
  - **Expected value voters** (average across possibilities compatible with T_c).
  - We will use a flat probability distribution, but only to simplify our treatment.

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  - We will use a flat probability distribution, but only to simplify our treatment.

How will the different kinds of voters evaluate the candidate's theory T?

Voter types

- optimistic voters: $ut^o(T) = \max\{u(w) : w \models T\}$
- pessimistic voters: $ut^p(T) = \min\{u(w) : w \models T\}$
- expected value voters: $ut^e(T) = \frac{\sum_{w \models T} u(w)}{|\{w : w \models T\}|}$

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Above, if we assume the probability p to be uniform, then $p(w)$ will be just $\frac{1}{|X|}$

Note that with ut^e we will have a convexity property. If X, Y are disjoint, then $ut^e(X \cup Y)$ will be in the closed interval whose endpoints are $ut^e(X)$ and $ut^e(Y)$

The value of a message

- Suppose T is the logical closure C of T_c .
- What's the best thing for her to say next?

The value of a message

- Suppose T is the logical closure C of T_c .
- What's the best thing for her to say next?
- Roughly: $val(A, T) = ut(T \circ A) - ut(T)$
- $T \circ A$ is what T becomes after A is added,
- and $val(A, T)$ is the value of uttering A when her current theory (as seen by voters) is T.

- But the precise definition will depend on
  - the kind of voter we're assuming (i.e. **o** vs. **p** vs. **e**)

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  - the set from which A is selected
- Wrt the latter, consider A from
  - $\mathcal{X}_a = T_a$ (i.e. only "true convictions")
  - $\mathcal{X}_t = \mathfrak{L} - \{\neg A : T_a \vdash A\}$ (i.e. anything consistent with "true convictions" = **tactical**)
  - $\mathcal{X}_m = \mathfrak{L} - \{\neg A : T_c \vdash A\}$ (i.e. anything consistent with the current theory = **Machiavellian**)
  - $\mathcal{X}_\ell = \mathfrak{L}$ (i.e. any sentence in the language, allowing for contradictions and lying)
- Note: $\mathcal{X}_a \subseteq \mathcal{X}_t \subseteq \mathcal{X}_m \subseteq \mathcal{X}_\ell$

The value of a message (cont.)

- If we have $\mathcal{X} = \mathcal{X}_\ell$ then T_c may become **inconsistent**.
- In this case, $\circ = *$ (i.e. an AGM-like update operation).
- In the other cases, $\circ = \dot{+}$
  addition of A followed by logical closure.
- If $\mathcal{X} = \mathcal{X}_a, \mathcal{X}_t$ or $\mathcal{X}_m$, then we let

  $$val(A, T) = ut(T \dot{+} A) - ut(T)$$

  where ut is one of ut^o, ut^p or ut^e.
- We can now define **best statements** for C given T from $\mathcal{X}$ as follows:

  $$best(T, \mathcal{X}) = argmax_A val(A, T) : A \in \mathcal{X}$$

Complex statements

Proposition (1)

Assume **e**-voters. For all A, B s.t. $A, B, A \wedge B \in \mathcal{X}_m$, (i.e., $A, B, A \wedge B$ consistent with T_c) there exist $a, ..., f \in [0, 1]$ s.t.

1) $a \cdot val(A, T) + b \cdot val(\neg A, T) = 0$
2) $val(A \wedge B, T) = val(A, T) + val(B, T \dot{+} A) = val(B, T) + val(A, T \dot{+} B)$
3) $c \cdot val(A \vee B) + d \cdot val(A \wedge B, T) = e \cdot val(A, T) + f \cdot val(B, T)$

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Proof: For 1), $ut(T) = a \cdot ut(T + A) + (1 - a) \cdot ut(T + \neg A)$ where $a = \frac{\{w \mid w \models T \dot{+} A\}}{\{w \mid w \models T\}}$.

Moving to complete theories

Corollary

There is a **complete** $T \supseteq T_c$ *s.t.* $ut^e(T) \geq ut^e(T_c)$.

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Proof: Form the above above, we must have exactly one of

i) $val(P_i, T) = val(\neg P_i, T) = 0$

ii) $val(P_i, T) > 0$ and $val(\neg P_i, T) < 0$

iii) $val(P_i, T) < 0$ and $val(\neg P_i, T) > 0$

Suppose $Q_i, \ldots, Q_k$ ($k \leq n$) are all the atoms not in T_c.

Let $T_0 = T_c$ and $T_{i+1} = \begin{cases} T_i \cup Q_i & val(Q_i, T_i) \geq 0 \\ T_i \cup \neg Q_i & \text{else} \end{cases}$

Let $T = Cn(T_k)$.

Moving to complete theories (cont.)

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One of the best extensions of T_c is a complete theory $T \supseteq T_c$

Moving to complete theories (cont.)

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Proof:

- Suppose T' is a best extension of T_c and T' is incomplete.
- By the previous corollary, there is $T'' \supseteq T'$ which is a complete extension of T' (and thus of T_c) such that $ut^e(T'') \geq ut^e(T')$.
- T'' is complete and among the best extensions.

Moving to complete theories (cont.)

- The previous result suggests that if C assumes **e**-voters, then it will never be to C's disadvantage to move towards a complete theory.
- This will also be the case if the voters are either **e**-voters or pessimistic voters.
- But why then do we have the *Onion* phenomenon?
- I.e. why do candidates state vacuities like "`God bless America`" or "`9/11 was a tragedy.`"

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- But why then do we have the *Onion* phenomenon?
- I.e. why do candidates state vacuities like "`God bless America`" or "`9/11 was a tragedy`."
- Conjecture: They must be assuming that there are at least some **o**-voters (who 'always assume the best').
- $T \supseteq T' \Longrightarrow \max\{u(w) \mid w \models T'\} \leq \max\{u(w) \mid w \models T\}$
- I.e. $T \supseteq T' \Longrightarrow ut^o(T') \leq ut^o(T)$

Conclusions

When we are programming people, the task is much more complex than it is with computers.

- People have their own motivations.
- Information which people need in order to act properly must be made available to them
  and sometimes, some information may need to be hidden.
- When different people have opposing motives, then conflicts can arise.

Nonetheless, issues arise in Social Software which are similar to the issues which arise in programming.

- We need to be clear about the desired post conditions of the procedure we propose.
- We need to make sure what are the preconditions needed for the procedure to work.
- Exchange of information, and preserving the proper order of actions must be attended to just as it is in Distributed Computing.

Some of my recent papers can be downloaded from the site

http://cuny.academia.edu/RohitParikh

See also

http://www.sci.brooklyn.cuny.edu/cis/parikh/