Non-classical logic and probability.

Basics

Reasons to be consistent?

- **Philosophers of logic:**
  What is the normative significance of logic, and where does it come from?
- **Metaethicists:**
  Are there reasons to be consistent?

  "Simply put, it seems outlandish that the kind of psychic tidiness that [consistency], or any other requirement of formal coherence, enjoins should be set alongside such final ends as pleasure, friendship, and knowledge"
  Kolodny 2007

Formal epistemology offers...

- **Formulation** of the rational constraints logic requires (probabilism).
- **Reasons** to meet these constraints (dutch books, accuracy domination).

P1c. (Non-negativity) $\forall S \in L, P(S) \in \mathbb{R}_{\geq 0}$

P2c. (Normalization) For T a (classical) logical truth, $P(T) = 1$

P3c. (Additivity) $\forall R, S \in L$ with R and S (classically) inconsistent, $P(R \vee S) = P(R) + P(S)$.

P4c. (Zero) For F a (classical) logical falsehood, $P(F) = 0$;

P5c. (Monotonicity) If S is a (classical) logical consequence of R, $P(S) \geq P(R)$;

Reasons to be rational, redux.

- **Dutch book.**
  Simple interpretation: (monetary) reasons to be rational. Sophisticated interpretation: intra-rationality arg.
- **Accuracy domination.**
  If B is improbabilistic, there is a probabilistic C which is more accurate than B at every possible world.

Metaphysics vs. epistemology.

- Metaphysically necessary loss
  Betting that Hesperus isn't Phosphorus.
- Kolodny-style bridge from reasons-to-rationality
  If you believe X is better than Y, than don't Y!
- Epistemically necessary loss.

The non-classical challenge

Problems with the formulation

- Normalization: Kleene.
- Additivity: Supervaluationism.
- Zero: Dialethists.
- Monotonicity: Kleene and dialethists object.
- Non-negativity: ?? Non-linear credal states ??

Three non-classical challenges

- The true believer.
- The empiricist.
- The pessimist (practical or ideal).

Expectational formulation.

$$f(S) = \sum_{w} c(w)|S|_w$$

w the set of classical truth value distributions.
|| function to truth value of S at w (1 or 0).

Paris's proposal

- **Generalized probabilities:**
  expectations of (non-classical) truth-values.
- **Generalized axiomatizations:**
  these can be uniformly characterized in terms of "guaranteed no drop in truth value".

Choquet-Paris

$$(\mathcal{T}2) \quad V(A) = 1 \wedge V(B) = 1 \iff V(A \wedge B) = 1$$
$$(\mathcal{T}3) \quad V(A) = 0 \wedge V(B) = 0 \iff V(A \vee B) = 0.$$

P2x. (Normalization) If $\vdash_x T$, then $P(T) = 1$

P4x. (Zero) If $F \vdash_x$, then $P(F) = 0$;

P5x. (Monotonicity) If $R \vdash_x S$, then $P(S) \geq P(R)$;

P3x+. (IncExc) $\forall R, S \in L, P(R) + P(S) = P(R \vee S) + P(R \wedge S)$

Generalized justifications

- **Extended dutch book** (De Finetti, Paris). Numerical values represent pragmatic loading: returns from a bet on p scaled by p's truth value.
- **Extended accuracy domination** (De Finetti,...) Numerical values represent cognitive loading: accuracy of credence k is distance from truth value.
- [Note restriction to real-valued truth values. What of lattice-values? Expectations of non-real valued RVs].

Three Extensions

Conditionalization

- The picture.
- The justification.
- Consequences and limits

cf. Williams "Generalized probabilism", RSL 2012.

Conditional probabilities

- The picture.
- The justification.
- Consequences.

$$P_A(X) = \sum_{w \in W} \frac{c_A(w)}{P(A)} |X|_w$$

$$= \frac{P(X \circ A)}{P(A)}$$

1. **Generalized Lemma.** $P(C) = \sum_{\gamma \in \Gamma} P(C \circ \gamma)$, so long as Γ is a partition.
2. **Generalized Corollary.** $P(C) = \sum_{\gamma \in \Gamma} P(C|\gamma)P(\gamma)$, so long as Γ is a partition.

cf. Milne 2008.

Jeffrey-style desirability

$$D(A) := \sum_{w} P(w|A)v(w)$$

$$D(A) = \sum_{S \in \Gamma} P(S|A)D(S \circ A)$$

Nonclassical challenges, redux.

The true believer

- Has the beginnings of a package to rival the classicist.
- The identification of cognitive/pragmatic loadings vital.
- Problems if these are non-real.
- Problems if they diverge (and why not?)
- Otherwise, straightforward challenge to extend theory.

The empiricist

- The space of empirically open possibilities is wide open.
- Vindications of expectational-probabilism go through.
- (Caveat: non-real valued empirical possibilities).
- Can define a "logic" over this space of truth value distributions, but likely to be very weak.
- More traction: conditionalization by logical setting.

The pessimist

- Much like the empiricist, but with less controversial presuppositions.
- Can we avoid pessimism?
- Is pessimism really so bad?

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