Philosophy 148: HW #4 Solutions

Exercise 1

Part 1.

(i) E_1 Hempel-Confirms H. ($E_1 = Raa\&Rab\&Rba\&Rbb$)
$I = \{a, b\}$, $dev_I(H) = Raa\&Rab\&Rba\&Rbb = E_1$.
Therefore, $E_1 \models dev_I(H)$, so E_1 directly Hempel-Confirms H, so it Hempel Confirms it.

(ii) E_2= does not Hempel-Confirm H.($E_2 = Raa\&Rab\&Rba$)
$I = \{a, b\}$, $dev_I(H) = Raa\&Rab\&Rba\&Rbb$.

Claim 1: E_2 does not *directly* Hempel-Confirm H, i.e. $E_2 \not\models dev_I(H)$. In solving this problem, some people thought that this claim can be supported by saying things like: '$dev_I(H)$ but not E_2"contains" Rbb'. This is a sloppy way of putting things: if you want to show that $E_2 \not\models dev_I(H)$, you have to show that there are interpretations of our language $L = \{R, a, b\}$ that make E_2 true and $dev_I(H)$ false. This is, however, a relatively minor problem:we didn't need to get this sophisticated, since the relevant entailments are really trivial!

(*) Here is a very important point!
Almost everybody thought that establishing Claim 1 was enough to show that E_2 does not Hempel-Confirm H. Now, Hempel Confirmation is a weaker notion than direct Hempel Confirmation. In order to show that E_2 does not Hempel-Confirm $dev_I(H)$ it was necessary to show something like this: take any S such that E_2 directly Hempel-Confirms S, i.e. $Raa\&Rab\&Rba \models dev_I(S)$. Now suppose $S \models H$, therefore $S \models Rbb$. Therefore it follows that whenever b is in the class I, $dev_I(S) \models Rbb$. So $Raa\&Rab\&Rba \models Rbb$, but this is false. So there is no S such that E_2 directly Hempel Confirms S and $S \models H$. The same is true for the negative claims in Ex. 1 part 2.

(iii) Just analogous to (ii).

(iv) E_4 (i.e. Raa) Hempel Confirms H. I={a}, hence $dev_I(H) = Raa = E_4$, so we have entailment of the development, that is, direct Hempel confirmation.

Part 2

Let $H = \forall x(Ex \to Gx)$, let $H' = \forall x(Ex \to (Ox \equiv Gx))$, and let $C = Ea\&Oa\&Ga$.

Ea	Oa	Ga	Hempel Confirms H?	Hempel Confirms H'?
T	T	T	Yes	Yes
T	T	F	No	No
T	F	T	Yes	No
T	F	F	No	Yes

(i) Now, C Hempel Confirms H and H'. Note $dev_I(H) = Ea \to Ga$ and $dev_I(H') = Ea \to (Ga \equiv Oa)$. Obviously $C \models Ea \to Ga$ and $C \models E \to (Ga \equiv Oa)$. In fact, we have just shown that C directly Hempel Confirms H.

(ii) Moving to row 2, we deal with $C' = Ea\&Oa\& \sim Ga$. Here we have two choices: first, we could give an argument similar to the argument sketched in part 1, under (*). Otherwise, we could simply point out that C' *refutes* H and H' (that is to say $C' \models \sim H$ and $C' \models \sim H'$). In general it is *not* enough to just observe that $C' \not\models dev_I(H)$ and $C' \not\models dev_I(H')$, for the same reasons mentioned in (*).

(iii) In row 3 we deal with $C'' = Ea\& \sim Oa\&Ga$. Again, C'' refutes H'. For what concerns H we have: $I = \{a\}$. Therefore $dev_I(H) = Ea \to Ga$, and C'' is logically stronger than $Ea \to Ga$.

(iv) In row 4 we have $C''' = Ea\& \sim Oa\& \sim Ga$. Here C''' refutes H. With the usual argument we can show that $C''' \models dev_I(H')$, i.e. $C''' \models Ea \to (Ga \equiv Oa)$

Exercise 2.

Let $H = \forall x(Rx \rightarrow Bx)$

Assume,

(i) $Pr(\sim Ba) > Pr(Ra)$

(ii)$Pr(Ra|H) = Pr(Ra)$

(iii) $Pr(Ba|H) = Pr(Ba)$

Show: $Pr(H|Ra\&Ba) > Pr(H| \sim Ra\& \sim Ba)$

Note:

(iv) $H\&Ra \models Ba$

(v) $H\& \sim Ba \models\sim Ra$

Both follow by logic, given the content of H.

Also note that (ii) and (iii), imply all the usual ways of expressing independence. In particular, given (ii) we have:

(vi) $Pr(H|Ra) = Pr(H)$ (because independence is symmetric)

(vii) $Pr(Ra| \sim H) = Pr(Ra)$,

$Pr(\sim Ra|H) = Pr(\sim Ra)$,

$Pr(H| \sim Ba) = Pr(H)$, etc. (cf. Homework 1!!!)

Also note that in all the proofs that follow, we implicitly use the fact that probabilities are always non-negative. In some cases we will make the stronger assumption that the probabilities we are dealing with are non-zero (I will explicitly signal the one step where this is really essential to the proof).

Lemma 1. $Pr(\sim Ra\& \sim Ba) > Pr(Ra\&Ba)$

Proof We show that:

(#) $Pr(\sim Ra\& \sim Ba) - Pr(Ra\&Ba) = Pr(\sim Ba) - Pr(Ra)$.

Together with assumption (i), (#) implies $Pr(\sim Ra\& \sim Ba) > Pr(Ra\&Ba)$.

Here is the proof of (#):

$Pr(\sim Ra\& \sim Ba) - Pr(Ra\&Ba) = Pr(\sim (Ra \vee Ba)) - Pr(Ra\&Ba) =$

$= 1 - Pr(Ra \vee Ba)) - Pr(Ra\&Ba) =$

$= 1 - (Pr(Ra) + Pr(Ba) - Pr(Ra\&Ba)) - Pr(Ra\&Ba) =$

$= 1 - Pr(Ra) - Pr(Ba) = Pr(\sim Ba) - Pr(Ra)$.

The first equality holds by logic, the second, third and fifth by the probability calculus (respectively: negation theorem, general disjunction rule, and negation again), the fourth just by simplifying.

Also note that, from this, it immediately follows :

$$\frac{1}{Pr(Ba\&Ra)} > \frac{1}{Pr(\sim Ba\& \sim Ra)}$$

Lemma 2.

$$Pr(H\& \sim Ba) > Pr(H\&Ra)$$

This follows at once from assumption (i), multiplying both sides by Pr(H) (which we can do because multiplication is positive over the non-negative reals) and then appealing to the independence facts determined by (ii) and (iii).

Lemma 3.

$$\frac{Pr(H\&Ra)}{Pr(Ra\&Ba)} > \frac{Pr(H\& \sim Ba)}{Pr(\sim Ba\& \sim Ra)}$$

Proof Let: $Pr(H\&Ra) = a$, $Pr(H\& \sim Ba) = b$, $Pr(Ra\&Ba) = c$, $Pr(\sim Ra\& \sim Ba) = d$.
We know: $b > a$ and $d > c$. We want: $\frac{a}{c} > \frac{b}{d}$.

We will prove equivalently that $\frac{a}{b} > \frac{c}{d}$.

Notice:

$$\frac{a}{b} = \frac{Pr(H\&Ra)}{Pr(H\& \sim Ba)} = \frac{Pr(H) \cdot Pr(Ra)}{Pr(H) \cdot Pr(\sim Ba)} = \frac{Pr(Ra)}{Pr(\sim Ba)} = \frac{Pr(Ra\&Ba) + Pr(Ra\& \sim Ba)}{Pr(\sim Ba\& \sim Ra) + Pr(Ra\& \sim Ba)}$$

The second equality holds by the independence of H and Ra and of H and $\sim Ba$, the third by canceling out $Pr(H)$, the fourth by the law of total probability.

Now, let: $Pr(Ra\& \sim Ba) = k$. We also assume that $Pr(Ra\& \sim Ba) \neq 0$. (Hence $k \neq 0$). Then we as a result of our equalities we can write

$$\frac{a}{b} = \frac{Pr(Ra\&Ba) + Pr(Ra\& \sim Ba)}{Pr(\sim Ba\& \sim Ra) + Pr(Ra\& \sim Ba)} = \frac{Pr(Ra\&Ba) + k}{Pr(\sim Ba\& \sim Ra) + k} = \frac{c + k}{d + k}$$

Now since k is positive, by simple algebra

$$\frac{a}{b} = \frac{c + k}{d + k} > \frac{c}{d}$$

as desired.

Theorem $(i)\&(ii)\&(iii) \Rightarrow Pr(H|Ra\&Ba) > Pr(H|\sim Ra\&\sim Ba)$

Proof We have practically done all the work. From assumptions (i)-(iii) we have Lemma 3, i.e.

$$\frac{Pr(H\&Ra)}{Pr(Ra\&Ba)} > \frac{Pr(H\&\sim Ba)}{Pr(\sim Ba\&\sim Ra)}$$

By assumption (iv), $Pr(H\&Ra) = Pr(H\&Ra\&Ba)$.
Similarly by assumption (v), $Pr(H\&\sim Ba) = Pr(H\&\sim Ra\&\sim Ba)$.

Hence,

$$\frac{Pr(H\&Ra\&Ba)}{Pr(Ra\&Ba)} > \frac{Pr(H\&\sim Ba\&\sim Ra)}{Pr(\sim Ba\&\sim Ra)}$$

which is our goal.