# Philosophy 148 — Day 1: Introduction & Administration

- Administrative Stuff (*i.e.*, Syllabus)
  - Me & Raul (intros., personal data, office hours, etc.)
  - Prerequisites (Boolean logic, some simple algebra, no math phobia!)
  - Texts & Supplementary Readings (all online *via* website)
  - Requirements [Quiz (10), Assignments (30), Mid-Term (30), Final (30)]
  - Sections (determined this week, *via* index cards — meet next week)
    * Index Cards: Name, email, section time ranking. The 8 possible times are: Tu or Th: 9–10, 10–11, 1–2, or 2–3. Give a *ranking* of those among the 8 that you *can* do. Indicate those you *cannot* do.
  - Website (main source of course information — stay tuned!)
  - Tentative Schedule (somewhat loose, time-wise, but all readings set)
- Next: Brief Overview/Outline of the Course

## Philosophy 148 — Day 1: Fundamental Underlying Questions

- I am writing a book on inductive logic (*a.k.a.*, confirmation theory).
- My main focus is on "quantitative generalizations" of deductive logic.
- The notion of *validity* is the deductive ideal for "logical goodness".
- But, some invalid arguments seem "better"/"stronger" than others:

| | |
|---|---|
| $P_1$. Someone is wise. | $P_2$. Someone is either wise or unwise. |
| $\therefore C_1$. Plato is wise. | $\therefore C_2$. Socrates is wise. |

- The argument from $P_1$ to $C_1$ seems "better" than the one from $P_2$ to $C_2$.
- Is there a satisfying *explication* of this "better than" concept?
- And, if so, is this best understood a *logical* concept or an *epistemic* one or a *pragmatic* one, *etc.*? Moreover, if there is a *logical* "better than", how is it related to *epistemology*? For that matter, how is *validity* related to epistemology? These are the sorts of questions in the air.

## Philosophy 148 — Day 1: Course Overview/Outline

- The precise timing of the course is not fixed. But all readings are up.
- The *order* of topics in the course is also (more or less) set:
  - Review of Boolean Logic and Boolean Algebra [12A review + FBAs]
    - Propositional Logic
    - Monadic Predicate Logic
    - Finite Boolean Algebras [general logical framework for course]
  - Introduction of the (formal) Probability Calculus
    - Axiomatic Treatments
    - Algebraic Treatments
  - "Personalistic" Interpretations/Kinds of Probability
    - Pragmatic: betting odds / betting quotients / *rational* dob's
    - Epistemic: degrees of *credence* / *justified* degrees of belief

- Confirmation Theory and Inductive Logic
  - ∗ Deductive Approaches to Confirmation
    - · Hempelian
    - · Hypothetico-Deductive
  - ∗ Probabilistic Approaches to Confirmation
    - · Logical (Carnapian)
    - · Subjective/Personalistic (“Bayesian”)
- The Paradoxes of Confirmation
  - ∗ The Raven Paradox
  - ∗ The Grue Paradox
- Other Problems for Confirmation Theory (mainly, for “Bayesian” CT)
  - ∗ Old Evidence/Logical Omniscience/maybe others
- Three *Psychological* Puzzles Involving Probability & Confirmation
  - ∗ The Base Rate Fallacy
  - ∗ The Conjunction Fallacy
  - ∗ The Wason Selection Task

## Syntax of Sentential Logic (SL)

- The syntax of SL is simple. Its lexicon contains the following symbols:
  - Upper-case letters '$A$', '$B$', ... which stand for *basic sentences*.
  - Five *sentential connectives* (or *sentential operators*):

| **Operator** | **Name** | **Logical Function** | **Used to translate** |
|---|---|---|---|
| '$\sim$' | tilde | negation | not, it is not the case that |
| '&' | ampersand | conjunction | and, also, moreover, but |
| '$\vee$' | vee | disjunction | or, either ... or ... |
| '$\rightarrow$' ('$\supset$') | arrow | conditional | if ... then ..., only if |
| '$\leftrightarrow$' ('$\equiv$') | double arrow | biconditional | if and only if |

  - Parentheses '(', ')', brackets '['. ']', and braces '{', '}' for grouping.
- If a string of symbols contains anything other than these, it is not an SL sentence. And, only certain strings of these symbols are SL sentences.
- I assume you all know which SL strings are *sentences* and which are not...

## Semantics of Sentential Logic: Truth Tables I

- Sentential Logic is *truth-functional* because the truth value of a compound $S$ is a function of the truth values of $S$'s *atomic parts*.
- All statement forms $p$ are defined by *truth tables*, which tell us how to determine the truth value of $p$'s from the truth values of $p$'s parts.
- Truth-tables provide a precise way of thinking about *logical possibility*. Each row of a truth-table can be thought of as a *logical possibility*. And, the actual world falls into *exactly one* of these rows/logical possibilities.
- In this sense, truth-tables provide a way to "see" logical space.
- Once we have an understanding of all the logically possible truth-values that and SL sentence can have (which truth-tables provide for us), testing the validity of SL arguments is easy — *inspection* of truth-tables!
- We just look for possible worlds (rows of the salient truth-table) in which all the premises are true and the conclusion is false.

## Semantics of Sentential Logic: Truth Tables II

- We begin with negations, which have the simplest truth functions. The truth table for negation is as follows (we use T and F for true and false):

| $p$ | $\sim p$ |
|---|---|
| T | F |
| F | T |

- In words, this says that if $p$ is true than $\sim p$ is false, and if $p$ is false, then $\sim p$ is true. This is quite intuitive, and corresponds well to the English meaning of 'not'. So, SL negation is like English negation.
- Examples:
  - It is not the case that Wagner wrote operas. ($\sim W$)
  - It is not the case that Picasso wrote operas. ($\sim P$)
- '$\sim W$' is false, since '$W$' is true, and '$\sim P$' is true, since '$P$' is false (like English).

## Chapter 3 — Semantics of SL: Truth Tables III

| $p$ | $q$ | $p \& q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

- Notice how we have four (4) rows in our truth table this time (not 2). There are four possible ways of assigning truth values to $p$ and $q$.
- The truth-functional definition of & is very close to the English 'and'. A SL conjunction is true if *both* conjuncts are true; it's false otherwise.
  - Monet and van Gogh were painters. ($M \& V$)
  - Monet and Beethoven were painters. ($M \& B$)
  - Beethoven and Einstein were painters. ($B \& E$)
- '$M \& V$' is true, since both '$M$' and '$V$' are true. '$M \& B$' is false, since '$B$' is false. And, '$B \& E$' is false, since '$B$' and '$E$' are both false (like English).

# Semantics of Sentential Logic: Truth Tables IV

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

- The truth-functional definition of $\vee$ is not as close to the English 'or'. A SL disjunction is true if *at least one* disjunct is true; it's false otherwise.
- In English, '$A$ or $B$' often implies that '$A$' and '$B$' are *not both true*. That is called *exclusive* or. In SL, '$A \vee B$' is *not* exclusive; it is *inclusive* (it is true if both disjuncts are true). We *can* express exclusive or in SL. How?
  - Either Jane austen or René Descartes was novelist. ($J \vee R$)
  - Either Jane Austen or Charlotte Bronte was a novelist. ($J \vee C$)
  - Either René Descartes or David Hume was a novelist. ($R \vee D$)
- The first two disjunctions are true since at least one their disjuncts is true. The third disjunction is false, since both of its disjuncts are false.

# Semantics of Sentential Logic: Truth Tables V

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

- The SL conditional ($\rightarrow$) is farther from the English 'only if'. An SL conditional is false iff its antecedent is true and its consequent is false.
- Consider the following English conditionals. [$M$ = the moon is made of green cheese, $O$ = life exists on other planets, and $E$ = life exists on Earth]
  - If the moon is made of green cheese, then life exists on other planets.
  - If life exists on other planets, then life exists on earth.
- The SL translations of these sentences are both true.
  - '$M \rightarrow O$' is true because its antecedent '$M$' is false.
  - '$O \rightarrow E$' is true because its consequent '$E$' is true.
- This does *not* capture the English 'if'. Remember: $p \rightarrow q \dashv\vDash \sim p \vee q$.

# Semantics of Sentential Logic: Truth Tables VI

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

- The SL biconditional $\leftrightarrow$ inherits similar problems. An SL biconditional is true iff both of its components have the same truth value.
- Consider these two biconditionals. [$M$ = the moon is made of green cheese, $U$ = there are unicorns, $E$ = life exists on Earth, and $S$ = the sky is blue]
  - The moon is made of green cheese if and only if there are unicorns.
  - Life exists on earth if and only if the sky is blue.
- The SL translations of these sentences are both true.
  - $M \leftrightarrow U$ is true because $M$ and $U$ are false.
  - $E \leftrightarrow S$ is true because $E$ and $S$ are true.
- This does *not* capture the English 'iff'. [$p \leftrightarrow q \dashv\vDash (p \,\&\, q) \vee (\sim p \,\&\, \sim q)$]

# Semantics of Sentential Logic: Truth Tables VII

- With the truth-table definitions of the five connectives in hand, we can now construct truth tables for arbitrary compound SL statements.
- A non-trivial example:

| $p$ | $q$ | $r$ | $(p$ | $\&$ | $(q \vee r))$ | $\rightarrow$ | $((p \& q)$ | $\vee$ | $(p \& r))$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | **T** | T | T | T |
| T | T | F | T | T | T | **T** | T | T | F |
| T | F | T | T | T | T | **T** | F | T | T |
| T | F | F | T | F | F | **T** | F | F | F |
| F | T | T | F | F | T | **T** | F | F | F |
| F | T | F | F | F | T | **T** | F | F | F |
| F | F | T | F | F | T | **T** | F | F | F |
| F | F | F | F | F | F | **T** | F | F | F |

- Thus, "$(p \& (q \vee r)) \rightarrow ((p \& q) \vee (p \& r))$" is a *tautology*.

## Logical Truth, Logical Falsity, and Contingency: Definitions

- A statement is logically true (or tautologous) if it is true regardless of the truth-values of its components. Example: $p \leftrightarrow p$ is a tautology.

| $p$ | $p$ | $\leftrightarrow$ | $p$ |
|---|---|---|---|
| T | T | T | T |
| F | F | T | F |

- A statement is logically false (or self-contradictory) if it is false regardless of the truth-values of its components. Example: $p \,\&\, {\sim} p$.

| $p$ | $p$ | $\&$ | $\sim$ | $p$ |
|---|---|---|---|---|
| T | T | F | F | T |
| F | F | F | T | F |

- A statement is contingent if its truth-value varies depending on the truth-values of its components. Example: $A$ (or *any* atom) is contingent.

| $A$ | $A$ |
|---|---|
| T | T |
| F | F |

# Interpretations and Logical Equivalence

- An *interpretation* of an SL formula $p$ is an assignment of truth-values to all of the sentence letters in $p$.
- Each row of the truth-table of $p$ is an *interpretation* of $p$. Sometimes, I will also refer to rows of SL truth-tables as (logically) *possible situations*, or *possible worlds*.
- A tautology (contradiction) is an SL sentence whose truth value is T (F) on *all* of its interpretations (*i.e.*, an SL sentence which is *true (false) in all (logically) possible worlds*).
- Two SL sentences are said to be *logically equivalent* iff they have the same truth-value on all (joint) interpretations.
- I'll abbreviate "$p$ and $q$ are logically equivalent" as "$p \dashv\vDash q$" [*i.e.*, $p$ follows from $q$ ($q \vDash p$), and $q$ follows from $p$ ($p \vDash q$)].

## Equivalence, Contradictoriness, Consistency, and Inconsistency

- Two statements are said to be equivalent (written $p \dashv\vDash q$) if they have the same truth-value in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, $A \to B \dashv\vDash \sim A \vee B$:

| $A$ | $B$ | $A$ | $\to$ | $B$ | $\sim$ | $A$ | $\vee$ | $B$ |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | F | T | T | T |
| T | F | T | F | F | F | T | F | F |
| F | T | F | T | T | T | F | T | T |
| F | F | F | T | F | T | F | T | F |

- Two statements are contradictory if they have opposite truth-values in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, $A$ and $\sim A$:

| $A$ | $A$ | $\sim$ | $A$ |
|---|---|---|---|
| T | T | F | T |
| F | F | T | F |

- Two statements are inconsistent (mutually exclusive) if they cannot both be true (*i.e.*, no row of their simultaneous truth-table has them both being T). *E.g.*, $A \leftrightarrow B$ and $A \,\&\, {\sim}B$ are inconsistent (but *not* contradictory!):

| $A$ | $B$ | $A$ | $\leftrightarrow$ | $B$ | $A$ | $\&$ | $\sim$ | $B$ |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | F | F | T |
| T | F | T | F | F | T | T | T | F |
| F | T | F | F | T | F | F | F | T |
| F | F | F | T | F | F | F | T | F |

- Two statements are consistent if they are both true in at least one possible world (*i.e.*, in at least one row of a simultaneous truth-table of both statements). For instance, $A \,\&\, B$ and $A \vee B$ are consistent:

| $A$ | $B$ | $A$ | $\&$ | $B$ | $A$ | $\vee$ | $B$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | F | T | F | F | T | T | F |
| F | T | F | F | T | F | T | T |
| F | F | F | F | F | F | F | F |

## Logical Equivalence: Example #1

- I said that $p \to q$ *is logically equivalent to* $\sim p \lor q$.
- The following truth-table establishes this equivalence:

| $p$ | $q$ | $\sim p$ | $\lor$ | $q$ | $p \to q$ |
|---|---|---|---|---|---|
| T | T | F | **T** | T | **T** |
| T | F | F | **F** | F | **F** |
| F | T | T | **T** | T | **T** |
| F | F | T | **T** | F | **T** |

- The truth-tables of $\sim p \lor q$ and $p \to q$ are the same.

# Logical Equivalence: Example #2

- $p \leftrightarrow q$ is an *abbreviation* for $(p \rightarrow q)\,\&\,(q \rightarrow p)$.
- The following truth-table shows it is a *legitimate* abbreviation:

| $p$ | $q$ | $(p \rightarrow q)$ | **&** | $(q \rightarrow p)$ | $p \leftrightarrow q$ |
|---|---|---|---|---|---|
| T | T | T | **T** | T | **T** |
| T | F | F | **F** | T | **F** |
| F | T | T | **F** | F | **F** |
| F | F | T | **T** | T | **T** |

- $p \leftrightarrow q$ and $(p \rightarrow q)\,\&\,(q \rightarrow p)$ have the same truth-table.

# Some More Logical Equivalences

- Here is a simultaneous truth-table which establishes that

$$A \leftrightarrow B \dashv\vDash (A \,\&\, B) \vee (\sim A \,\&\, \sim B)$$

| $A$ | $B$ | $A$ | $\leftrightarrow$ | $B$ | $(A$ | $\&$ | $B)$ | $\vee$ | $(\sim$ | $A$ | $\&$ | $\sim$ | $B)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T | T | F | T | F | F | T |
| T | F | T | F | F | T | F | F | F | F | T | F | T | F |
| F | T | F | F | T | F | F | T | F | T | F | F | F | T |
| F | F | F | T | F | F | F | F | T | T | F | T | T | F |

- Can you prove the following equivalences with truth-tables?
  - $\sim(A \,\&\, B) \dashv\vDash \sim A \vee \sim B$
  - $\sim(A \vee B) \dashv\vDash \sim A \,\&\, \sim B$
  - $A \dashv\vDash (A \,\&\, B) \vee (A \,\&\, \sim B)$
  - $A \dashv\vDash A \,\&\, (B \to B)$
  - $A \dashv\vDash A \vee (B \,\&\, \sim B)$

## Logical Equivalence, Contradictoriness, *etc.*: Relationships

- What are the relationships between "$p$ and $q$ are equivalent", "$p$ and $q$ are consistent", "$p$ and $q$ are contradictory", "$p$ and $q$ are inconsistent"?

$$\begin{array}{ccc} \text{Equivalent} & \quad & \text{Contradictory} \\ \Downarrow \;\; ? \;\; \Uparrow & & \Downarrow \;\; ? \;\; \Uparrow \\ \text{Consistent} & & \text{Inconsistent} \end{array}$$

- Answers:

1. Equivalent $\not\Rightarrow$ Consistent ($p \mathbin{\&} {\sim}p$ and $q \mathbin{\&} {\sim}q$)
2. Consistent $\not\Rightarrow$ Equivalent ($p \to q$ and $p \mathbin{\&} q$)
3. Contradictory $\Rightarrow$ Inconsistent (*why*?)
4. Inconsistent $\not\Rightarrow$ Contradictory (example?)

# Truth-Tables and Deductive Validity I

- Remember, an argument is valid if it is *impossible* for its premises to be true while its conclusion is false. Let $p_1, \ldots, p_n$ be the premises of a SL argument, and let $q$ be the conclusion of the argument. Then, we have:

$$\begin{array}{l} p_1 \\ \vdots \\ p_n \\ \hline \therefore\ q \end{array}$$

is valid if and only if there is no row in the simultaneous truth-table (*interpretation*) of $p_1, \ldots, p_n$, and $q$ which looks like:

| atoms | premises | | | conclusion |
|---|---|---|---|---|
| $\cdots$ | $p_1$ | $\cdots$ | $p_n$ | $q$ |
| $\cdots$ | T | T | T | F |

# Truth-Tables and Deductive Validity II

$A$

$A \to B$ is *valid*:

$\therefore B$

| atoms | | premises | | | | conclusion |
|---|---|---|---|---|---|---|
| $A$ | $B$ | $A$ | $A$ | $\to$ | $B$ | $B$ |
| T | T | T | T | T | T | T |
| T | F | T | T | F | F | F |
| F | T | F | F | T | T | T |
| F | F | F | F | T | F | F |

$B$

$A \to B$ is *in*valid:

$\therefore A$

| | atoms | | premises | | | | conclusion |
|---|---|---|---|---|---|---|---|
| | $A$ | $B$ | $B$ | $A$ | $\to$ | $B$ | $A$ |
| | T | T | T | T | T | T | T |
| | T | F | F | T | F | F | T |
| ☞ | F | T | T | F | T | T | F |
| | F | F | F | F | T | F | F |

# Finite Propositional Boolean Algebras I

- A *finite propositional Boolean algebra* is a finite set of *propositions* which is *closed* under the logical operations and satisfies the laws of SL.
- *Propositions* are the things expressed by sentences (abstract entities, distinct from sentences). If two sentences are logically equivalent, then they express the same proposition. *E.g.*, "$A \rightarrow B$" and "$\sim A \vee B$".
- A set $S$ is *closed* under logical operations if applying a logical operation to a member (or pair of members) of $S$ always yields a member of $S$.
- Example: consider a sentential language with three atomic letters "$X$", "$Y$", and "$Z$". The set of propositions expressible using the logical connectives and these three atomic letters forms a finite Boolean algebra.
- This Boolean algebra has $2^3 = 8$ *atomic propositions* or *states* (*i.e.*, rows of a 3-sentence truth-table!). Question: How many propositions does it contain *in total*? Answer: $2^8 = 256$ (255 plus the contradiction). *Why*?

# Finite Propositional Boolean Algebras II

- A *literal* is either an atomic sentence or the negation of an atomic sentence (*e.g.*, "$A$" and "$\sim A$" are literals involving the atom "$A$").
- A *state* of a Boolean algebra $\mathcal{B}$ is a proposition expressed by a *maximal* conjunction of literals in a language $\mathcal{L}_{\mathcal{B}}$ describing $\mathcal{B}$ ("maximal": "containing exactly one literal for each atomic sentence in $\mathcal{B}$").
- Consider an algebra $\mathcal{B}$ described by a 3-atom language $\mathcal{L}_{\mathcal{B}}$ ("$X$", "$Y$", "$Z$"). The states of $\mathcal{B}$ are described by the $2^3 = 8$ *state descriptions* of $\mathcal{L}_{\mathcal{B}}$:

($s_1$) $X \,\&\, Y \,\&\, Z$

($s_2$) $X \,\&\, Y \,\&\, {\sim}Z$

($s_3$) $X \,\&\, {\sim}Y \,\&\, Z$

($s_4$) $X \,\&\, {\sim}Y \,\&\, {\sim}Z$

($s_5$) ${\sim}X \,\&\, Y \,\&\, Z$

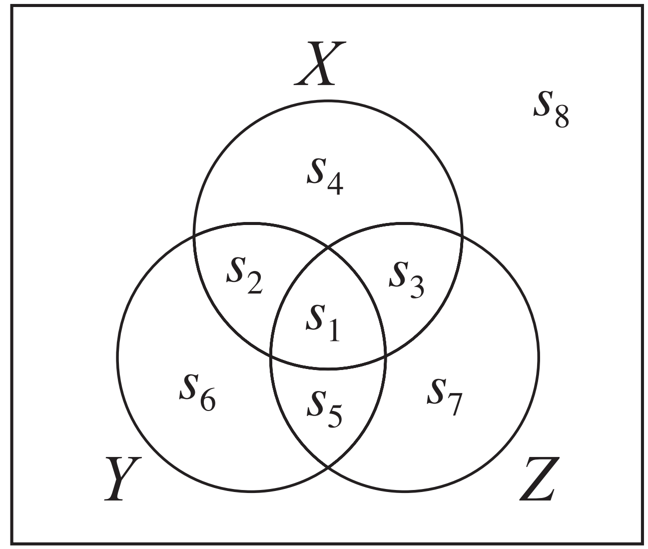
($s_6$) ${\sim}X \,\&\, Y \,\&\, {\sim}Z$

($s_7$) ${\sim}X \,\&\, {\sim}Y \,\&\, Z$

($s_8$) ${\sim}X \,\&\, {\sim}Y \,\&\, {\sim}Z$

- We can "visualize" the states of $\mathcal{B}$ using a truth table or a Venn Diagram.

| $X$ | $Y$ | $Z$ | States |
|---|---|---|---|
| T | T | T | $s_1$ |
| T | T | F | $s_2$ |
| T | F | T | $s_3$ |
| T | F | F | $s_4$ |
| F | T | T | $s_5$ |
| F | T | F | $s_6$ |
| F | F | T | $s_7$ |
| F | F | F | $s_8$ |




- Everything that can be expressed in the sentential language $\mathcal{L}_{\mathcal{B}}$ can be expressed as a *disjunction of state descriptions* (think about why).
- Thus, every proposition expressible in $\mathcal{L}_{\mathcal{B}}$ can be "visualized" simply by shading combinations of the 8 state-regions of the Venn Diagram of $\mathcal{B}$. It because of this that we can use Venn Diagrams to establish Boolean Laws.
- $p \vDash q$ (in $\mathcal{B}$) iff every shaded region in the Venn Diagram representation of $p$ (in $\mathcal{B}$) is also shaded in the Venn Diagram representation of $q$ (in $\mathcal{B}$).