## Philosophy 148

- Branden is in Europe this week (so no office hours for him).
- I am Kenny Easwaran and I am filling-in for him . . . and there will be another guest lecturer on Thursday . . .
- Grades for homework 2 are now on bspace, and Raul will be handing them back this week. Grades for homework 3 will be up later in the week.
- There will be a review session for homework 4 on April 15 at 6 pm, location TBA.





## Bridging Chances and Credences: Lewis's Principal Principle 1

- Joyce's argument operates on the level of the *truth-values* of propositions in possible worlds. So, his notion of accuracy is one of *closeness to truth-value*.
- This assumes that the propositions in question *have* truth-values in the worlds in question. But, since Joyce's argument aims to show that probabilistic $q$'s are closer to the truth *come what may*, it doesn't matter what these truth-values turn out to be. Nonetheless, one wonders about *future contingents*.
- In non-deterministic worlds, no amount of knowledge will allow one to know for certain whether future events will occur. In such worlds, the best we can do is know the *objective chance* that $p$. This is where Lewis comes in.
- Let the objective chance of $p$ at $t$ (as determined by the history of the world up to $t$) be a one-place function $Ch_t(p)$. Lewis assumes $Ch_t$ is a Pr function. He also assumes that rational credence functions $C$ are Pr functions.
- Lewis is concerned with bridge principles connecting chances and credences.





## Bridging Chances and Credences: Lewis's Principal Principle 2

- Lewis discusses several examples concerning the relationship between chance and credence (epistemically rational degree of belief), and then he abstracts from these a general principle that he calls the Principal Principle (PP).
- **Example #1**. A certain coin is scheduled to be tossed at noon today. You are sure that this chosen coin is fair: it has a 50% chance of falling heads and a 50% chance of falling tails. You have no other relevant information. Consider the proposition that the coin tossed at noon today falls heads. To what degree should you now believe that proposition? **Answer**: 50%, of course.
- **Example #2**. As before, except that you have plenty of seemingly relevant evidence tending to lead you to expect that the coin will fall heads. This coin is known to have a displaced center of mass, it has been tossed 100 times before with 86 heads, and many duplicates of it have been tossed thousands of times with about 90% heads. Yet you remain quite sure, despite all this evidence, that the chance of heads this time is 50%. To what degree should you believe the proposition that the coin falls heads this time? **Answer**: Still 50%. Such evidence is relevant to the outcome by way of its relevance to the proposition that the chance of heads is 50%, not in any other way. [Explain.]





- **Example #3**. As before, except that now it is afternoon and you have evidence that became available *after* the coin was tossed at noon. Maybe you know for certain that it fell heads; maybe some fairly reliable witness has told you that it fell heads; maybe the witness has told you that it fell heads in nine out of ten tosses of which the noon toss was one. You remain as sure as ever that the chance of heads, just before noon, was 50%. To what degree should you believe that the coin tossed at noon fell heads? **Answer**: Not 50%, but something not far short of 100%. Resiliency has its limits. If evidence bears in a direct enough way on the outcome – a way that may nevertheless fall short of outright implication – then it may bear on your beliefs about outcomes otherwise than by way of your beliefs about the chances of the outcomes. Resiliency under all evidence whatever would be extremely unreasonable. We can only say that degrees of belief about outcomes that are based on certainty about chances are resilient under admissible evidence. The previous question gave examples of admissible evidence; this question gave examples of inadmissible evidence.
- **Example #4**. You have no inadmissible evidence; if you have any relevant admissible evidence, it already has had its proper effect on your credence about the chance of heads. But this time, suppose you are not sure that the coin is fair. You divide your belief among three alternative hypotheses about the chance of heads, as follows:

You believe to degree 27% that the chance of heads is 50%.
You believe to degree 22% that the chance of heads is 35%.
You believe to degree 51% that the chance of heads is 80%.

Then to what degree should you believe that the coin falls heads?

**Answer**: (27% · 50%) + (22% · 35%) + (51% · 80%); that is, 62%.

Your degree of belief that the coin falls heads, conditionally on any one of the hypotheses about the chance of heads, should equal your unconditional degree of belief if you were sure of that hypothesis. That in turn should equal the chance of heads according to the hypothesis: 50% for the first hypothesis, 35% for the second, and 80% for the third. Given your degrees of belief that the coin falls heads, conditionally on the hypotheses, we need only apply the standard multiplicative and additive principles to obtain our answer. [This is where he assumes $C$ is a Pr-function!]

- **The Principal Principle**. Let $C$ be any reasonable initial credence function, $t$ be any time, $x$ be any real number in the unit interval, $Ch_t(p) = x$ be the proposition that the chance, at time $t$, of $p$'s holding equals $x$, and $E$ be any proposition compatible with $Ch_t(p) = x$ that is admissible at $t$. Then,

$$C(p \mid Ch_t(p) = x \,\&\, E) = x$$





- More generally, whether or not you are sure about the chance of heads, your unconditional degree of belief that the coin falls heads is given by summing over your alternative (partition of) hypotheses about chance:

$$C(p \mid E) = \sum_x C(H_x \mid E) \cdot C(p \mid Ch_t(p) = x \,\&\, E) = \sum_x C(H_x \mid E) \cdot x$$

where $H_x$ is the hypothesis that the chance at $t$ of $p$ equals $x$.

- Note: The assumption that $Ch_t$ is a probability function together with (PP) does not suffice to ensure that $C$ is a probability function. This is because (1) propositions about chance are just *some* of the propositions $C$ operates on, and (2) Lewis assumes $C$ is a Pr-function in his *development* of (PP) itself.
- This is why an argument like Joyce's is needed, even for Lewis. If we assume that the truth is a probability distribution, then don't we get probabilism for free by requiring accuracy with respect to the true probability distribution?
- Lewis assumes *only* that "reasonable" initial credence functions are *regular* probability functions – assigning extreme probability *only* to logical truths/falsehoods. This allows him to use the ratio definition of CP without worrying about the zero-denominator case. And, he thinks it's "conservative".





## Epistemic Probability 3 – Lewis, The Principal Principle, and Accuracy Again 3

- The key notion here is that of *admissibility*. We can't allow arbitrary evidence to be included in the (PP), since some $E$'s will make the advice the (PP) gives incorrect. The (PP) is a restricted epistemic norm. A more general principle is:
- **The Requirement of Total Evidence** (RTE). $C$ should take into account one's *total* evidence $K$. That us, *in general*, an agent's credence for $p$ (in a context $C$) should be $\Pr(p \mid K)$, for some Pr and the agent's total evidence $K$ (in $C$).
- But, the (PP) is not a fully general epistemic principle. It's not trying to tell you what your $C(p)$ should be like, for arbitrary $p$, relative to arbitrary kinds of background knowledge – it's just forging a connection between $C$ and $Ch_t$.
- Admissible Evidence:
  - Historical information [say, past observations about this coin]
  - Theoretical information about the dependence of chance on history
  - Conditionals with chance conseqeuents [Railtonian Laws]
  - Boolean combinations of the above





- Inadmissible Evidence [Lewis adds another case for "Sleeping Beauty" – next...]
  - Information relevant to the occurrence of $p$ (*e.g.*, after it happens or fails).

Recall the case of the labyrinth: you enter at 11:00, choosing your turns by chance, and hope to reach the center by noon, Your subsequent chance of success depends on the point you have reached. The proposition that at 11:30 your chance of success has fallen to 26% is not admissible information at 11:00; it is a giveaway about your bad luck in the first half hour. What is admissible at 11:00 is a conditional version: if you were to reach a certain point at 11:30, your chance of success would then be 26%. But even some conditionals are tainted: for instance, any conditional that could yield inadmissible information about future chances by modus ponens from admissible historical propositions. Consider also the truth-functional conditional that if history up to 11:30 follows a certain course, then you will have a 98% chance of becoming a monkey's uncle before the year is out. This conditional closely resembles the denial of its antecedent, and is inadmissible at 11:00 for the same reason.

## Carnap and Lewis: An Interesting Analogy

- Carnap proposed the following bridge principle for logical and epistemic Pr:

  If $S$ knows that $P$ – *and $S$ knows nothing else a posteriori* – and $S$ knows that the logical probability of $C$ given $P$ is $r$, then $S$'s degree of credence (epsitemic dob) in $C$ on the supposition of $P$ should be $r$.

- Let $\mathfrak{c}$ be logical probability, and $\Pr_{S_T}$ be *a priori* credence. Carnap *doesn't* say:

  $\Pr_{S_T}(C \mid P \;\&\; \mathfrak{c}(C \mid P) = r) = r$

- Why not? What does it *mean* to "*conditionalize* on a *probability* claim"? This sort of "second-order probability" talk is something Carnap wanted to avoid (next slide). This is why he talks instead about *knowing* $P$ and $\mathfrak{c}(C \mid P) = r$.
- For Carnap, *nothing* (*a posteriori*) is "admissible" in a Lewisian sense here. That is, you can't add *anything a posteriori* to your background knowledge, because then the logical bridge principle could fail. Nice analogy here.
- Lewis (roughly): nothing but your knowledge about $Ch_t$ is admissible. Carnap: nothing but your knowledge about $\mathfrak{c}$ (or the *a priori*) is admissible.





## Miller's Paradox and "Second Order" Probabilities

- We must be careful when asserting second order probability statements like "the probability that $x$ has probability $r$ is $s$" — *even if* we make sure that the two occurrences of "probability" have *different meanings* in such assertions.
- David Miller argues that all principles like (PP) are absurd. Let Pr be any initial credence function (at $t$), and $p$ be any physical probability function (at $t$). Assume Pr conditions on nothing inadmissible, and Pr, $p$ are *probabilities*.
  1. For all $x$ and $r$, $\Pr(x \mid p(x) = r) = r$ [Assumption: like the (PP)]
  2. $\Pr(x \mid p(x) = \frac{1}{2}) = \frac{1}{2}$ [Instance of (1)]
  3. $\Pr(x \mid p(x) = p(\sim x)) = p(\sim x)$ [Instance of (1)]
  4. $\Pr(x \mid p(x) = \frac{1}{2}) = \Pr(x \mid p(x) = p(\sim x))$ [$p(x) = \frac{1}{2}$ iff $p(x) = p(\sim x)$]
  5. $\therefore p(x) = p(\sim x) = \frac{1}{2}$, for all $x$. [(2),(3),(4)]
- We seem to have just shown that all events $x$ have a physical probability of $\frac{1}{2}$, which is absurd! Miller thinks this argument is valid, and he rejects (1).
- Is this a fallacy? Some say so. Carnap's wording avoids the paradox. Why?