

Philosophy 148 — Day 1: INTRODUCTION & ADMINISTRATION

- Administrative Stuff (*i.e.*, Syllabus)
 - Me & Raul (intros., personal data, office hours, etc.)
 - Prerequisites (Boolean logic, some simple algebra, no math phobia!)
 - Texts & Supplementary Readings (all online *via* website)
 - Requirements [Quiz (10), Assignments (30), Mid-Term (30), Final (30)]
 - Sections (determined this week, *via* index cards — meet next week)
 - * Index Cards: Name, email, section time ranking. The 8 possible times are: Tu or Th: 9-10, 10-11, 1-2, or 2-3. Give a *ranking* of those among the 8 that you *can* do. Indicate those you *cannot* do.
 - Website (main source of course information — stay tuned!)
 - Tentative Schedule (somewhat loose, time-wise, but all readings set)
- Next: Brief Overview/Outline of the Course

Philosophy 148 — Day 1: Fundamental Underlying Questions

- I am writing a book on inductive logic (*a.k.a.*, confirmation theory).
- My main focus is on “quantitative generalizations” of deductive logic.
- The notion of *validity* is the deductive ideal for “logical goodness”.
- But, some invalid arguments seem “better”/“stronger” than others:
 - P_1 . Someone is wise. P_2 . Someone is either wise or unwise.
 - $\therefore C_1$. Plato is wise. $\therefore C_2$. Socrates is wise.
- The argument from P_1 to C_1 seems “better” than the one from P_2 to C_2 .
- Is there a satisfying *explication* of this “better than” concept?
- And, if so, is this best understood a *logical* concept or an *epistemic* one or a *pragmatic* one, *etc.*? Moreover, if there is a *logical* “better than”, how is it related to *epistemology*? For that matter, how is *validity* related to epistemology? These are the sorts of questions in the air.

Philosophy 148 — Day 1: Course Overview/Outline

- The precise timing of the course is not fixed. But all readings are up.
- The *order* of topics in the course is also (more or less) set:
 - Review of Boolean Logic and Boolean Algebra [12A review + FBAs]
 - * Propositional Logic
 - * Monadic Predicate Logic
 - * Finite Boolean Algebras [general logical framework for course]
 - Introduction of the (formal) Probability Calculus
 - * Axiomatic Treatments
 - * Algebraic Treatments
 - “Personalistic” Interpretations/Kinds of Probability
 - * Pragmatic: betting odds / betting quotients / *rational* *dob*’s
 - * Epistemic: degrees of *credence* / *justified* degrees of belief

- Confirmation Theory and Inductive Logic
 - * Deductive Approaches to Confirmation
 - Hempelian
 - Hypothetico-Deductive
 - * Probabilistic Approaches to Confirmation
 - Logical (Carnapian)
 - Subjective/Personalistic (“Bayesian”)
- The Paradoxes of Confirmation
 - * The Raven Paradox
 - * The Grue Paradox
- Other Problems for Confirmation Theory (mainly, for “Bayesian” CT)
 - * Old Evidence/Logical Omniscience/maybe others
- Three *Psychological* Puzzles Involving Probability & Confirmation
 - * The Base Rate Fallacy
 - * The Conjunction Fallacy
 - * The Wason Selection Task

Syntax of Sentential Logic (SL)

- The syntax of SL is simple. Its lexicon contains the following symbols:
 - Upper-case letters 'A', 'B', ... which stand for *basic sentences*.
 - Five *sentential connectives* (or *sentential operators*):

Operator	Name	Logical Function	Used to translate
'~'	tilde	negation	not, it is not the case that
'&'	ampersand	conjunction	and, also, moreover, but
'∨'	vee	disjunction	or, either ... or ...
'→' ('⊃')	arrow	conditional	if ... then ..., only if
'↔' ('≡')	double arrow	biconditional	if and only if

- Parenteses '(', ')', brackets '[', ']', and braces '{', '}' for grouping.
- If a string of symbols contains anything other than these, it is not an SL sentence. And, only certain strings of these symbols are SL sentences.
- I assume you all know which SL strings are *sentences* and which are not...

Semantics of Sentential Logic: Truth Tables I

- Sentential Logic is *truth-functional* because the truth value of a compound *S* is a function of the truth values of *S*'s *atomic parts*.
- All statement forms *p* are defined by *truth tables*, which tell us how to determine the truth value of *p*'s from the truth values of *p*'s parts.
- Truth-tables provide a precise way of thinking about *logical possibility*. Each row of a truth-table can be thought of as a *logical possibility*. And, the actual world falls into *exactly one* of these rows/logical possibilities.
- In this sense, truth-tables provide a way to "see" logical space.
- Once we have an understanding of all the logically possible truth-values that an SL sentence can have (which truth-tables provide for us), testing the validity of SL arguments is easy — *inspection* of truth-tables!
- We just look for possible worlds (rows of the salient truth-table) in which all the premises are true and the conclusion is false.

Semantics of Sentential Logic: Truth Tables II

- We begin with negations, which have the simplest truth functions. The truth table for negation is as follows (we use T and F for true and false):

<i>p</i>	~ <i>p</i>
T	F
F	T

- In words, this says that if *p* is true then ~*p* is false, and if *p* is false, then ~*p* is true. This is quite intuitive, and corresponds well to the English meaning of 'not'. So, SL negation is like English negation.
- Examples:
 - It is not the case that Wagner wrote operas. (~*W*)
 - It is not the case that Picasso wrote operas. (~*P*)
- '~*W*' is false, since '*W*' is true, and '~*P*' is true, since '*P*' is false (like English).

Chapter 3 — Semantics of SL: Truth Tables III

<i>p</i>	<i>q</i>	<i>p</i> & <i>q</i>
T	T	T
T	F	F
F	T	F
F	F	F

- Notice how we have four (4) rows in our truth table this time (not 2). There are four possible ways of assigning truth values to *p* and *q*.
- The truth-functional definition of & is very close to the English 'and'. A SL conjunction is true if *both* conjuncts are true; it's false otherwise.
 - Monet and van Gogh were painters. (*M* & *V*)
 - Monet and Beethoven were painters. (*M* & *B*)
 - Beethoven and Einstein were painters. (*B* & *E*)
- '*M* & *V*' is true, since both '*M*' and '*V*' are true. '*M* & *B*' is false, since '*B*' is false. And, '*B* & *E*' is false, since '*B*' and '*E*' are both false (like English).

Semantics of Sentential Logic: Truth Tables IV

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- The truth-functional definition of \vee is not as close to the English ‘or’. A SL disjunction is true if *at least one* disjunct is true; it’s false otherwise.
- In English, ‘ A or B ’ often implies that ‘ A ’ and ‘ B ’ are *not both true*. That is called *exclusive or*. In SL, ‘ $A \vee B$ ’ is *not* exclusive; it is *inclusive* (it is true if both disjuncts are true). We *can* express exclusive or in SL. How?
 - Either Jane austen or René Descartes was novelist. ($J \vee R$)
 - Either Jane Austen or Charlotte Bronte was a novelist. ($J \vee C$)
 - Either René Descartes or David Hume was a novelist. ($R \vee D$)
- The first two disjunctions are true since at least one their disjuncts is true. The third disjunction is false, since both of its disjuncts are false.

Semantics of Sentential Logic: Truth Tables V

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

- The SL conditional (\rightarrow) is farther from the English ‘only if’. An SL conditional is false iff its antecedent is true and its consequent is false.
- Consider the following English conditionals. [M = the moon is made of green cheese, O = life exists on other planets, and E = life exists on Earth]
 - If the moon is made of green cheese, then life exists on other planets.
 - If life exists on other planets, then life exists on earth.
- The SL translations of these sentences are both true.
 - ‘ $M \rightarrow O$ ’ is true because its antecedent ‘ M ’ is false.
 - ‘ $O \rightarrow E$ ’ is true because its consequent ‘ E ’ is true.
- This does *not* capture the English ‘if’. Remember: $p \rightarrow q \equiv \sim p \vee q$.

Semantics of Sentential Logic: Truth Tables VI

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- The SL biconditional \leftrightarrow inherits similar problems. An SL biconditional is true iff both of its components have the same truth value.
- Consider these two biconditionals. [M = the moon is made of green cheese, U = there are unicorns, E = life exists on Earth, and S = the sky is blue]
 - The moon is made of green cheese if and only if there are unicorns.
 - Life exists on earth if and only if the sky is blue.
- The SL translations of these sentences are both true.
 - $M \leftrightarrow U$ is true because M and U are false.
 - $E \leftrightarrow S$ is true because E and S are true.
- This does *not* capture the English ‘iff’. [$p \leftrightarrow q \equiv (p \& q) \vee (\sim p \& \sim q)$]

Semantics of Sentential Logic: Truth Tables VII

- With the truth-table definitions of the five connectives in hand, we can now construct truth tables for arbitrary compound SL statements.
- A non-trivial example:

p	q	r	$(p \& (q \vee r))$	\rightarrow	$((p \& q) \vee (p \& r))$
T	T	T	T	T	T
T	T	F	T	T	F
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	F	T	F
F	T	F	F	T	F
F	F	T	F	T	F
F	F	F	F	T	F

- Thus, “ $(p \& (q \vee r)) \rightarrow ((p \& q) \vee (p \& r))$ ” is a *tautology*.

Logical Truth, Logical Falsity, and Contingency: Definitions

- A statement is **logically true** (or **tautologous**) if it is true regardless of the truth-values of its components. Example: $p \leftrightarrow p$ is a tautology.

p	$p \leftrightarrow p$
T	T
F	T

- A statement is **logically false** (or **self-contradictory**) if it is false regardless of the truth-values of its components. Example: $p \& \sim p$.

p	$p \& \sim p$
T	F
F	F

- A statement is **contingent** if its truth-value varies depending on the truth-values of its components. Example: A (or *any* atom) is contingent.

A	A
T	T
F	F

Interpretations and Logical Equivalence

- An *interpretation* of an SL formula p is an assignment of truth-values to all of the sentence letters in p .
- Each row of the truth-table of p is an *interpretation* of p . Sometimes, I will also refer to rows of SL truth-tables as (logically) *possible situations*, or *possible worlds*.
- A tautology (contradiction) is an SL sentence whose truth value is T (F) on *all* of its interpretations (*i.e.*, an SL sentence which is *true (false) in all (logically) possible worlds*).
- Two SL sentences are said to be *logically equivalent* iff they have the same truth-value on all (joint) interpretations.
- I'll abbreviate " p and q are logically equivalent" as " $p \models q$ " [*i.e.*, p follows from q ($q \models p$), and q follows from p ($p \models q$)].

Equivalence, Contradictoriness, Consistency, and Inconsistency

- Two statements are said to be **equivalent** (written $p \models q$) if they have the same truth-value in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, $A \rightarrow B \models \sim A \vee B$:

A	B	$A \rightarrow B$	$\sim A \vee B$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	F

- Two statements are **contradictory** if they have opposite truth-values in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, A and $\sim A$:

A	A	$\sim A$
T	T	F
F	F	T

- Two statements are **inconsistent** (**mutually exclusive**) if they cannot both be true (*i.e.*, no row of their simultaneous truth-table has them both being T). *E.g.*, $A \leftrightarrow B$ and $A \& \sim B$ are inconsistent (but *not* contradictory!):

A	B	$A \leftrightarrow B$	$A \& \sim B$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F

- Two statements are **consistent** if they are both true in at least one possible world (*i.e.*, in at least one row of a simultaneous truth-table of both statements). For instance, $A \& B$ and $A \vee B$ are consistent:

A	B	$A \& B$	$A \vee B$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

Logical Equivalence: Example #1

- I said that $p \rightarrow q$ is *logically equivalent* to $\sim p \vee q$.
- The following truth-table establishes this equivalence:

p	q	$\sim p$	\vee	q	$p \rightarrow q$
T	T	F	T	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	F	T

- The truth-tables of $\sim p \vee q$ and $p \rightarrow q$ are the same.

Logical Equivalence: Example #2

- $p \leftrightarrow q$ is an *abbreviation* for $(p \rightarrow q) \& (q \rightarrow p)$.
- The following truth-table shows it is a *legitimate* abbreviation:

p	q	$(p \rightarrow q)$	$\&$	$(q \rightarrow p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	T	F	F	F
F	F	T	T	T	T

- $p \leftrightarrow q$ and $(p \rightarrow q) \& (q \rightarrow p)$ have the same truth-table.

Some More Logical Equivalences

- Here is a simultaneous truth-table which establishes that

$$A \leftrightarrow B \models (A \& B) \vee (\sim A \& \sim B)$$

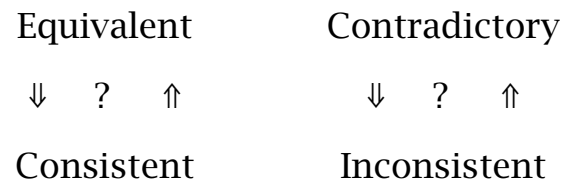
A	B	$A \leftrightarrow B$	$(A \& B) \vee (\sim A \& \sim B)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

- Can you prove the following equivalences with truth-tables?

- $\sim(A \& B) \models \sim A \vee \sim B$
- $\sim(A \vee B) \models \sim A \& \sim B$
- $A \models (A \& B) \vee (A \& \sim B)$
- $A \models A \& (B \rightarrow B)$
- $A \models A \vee (B \& \sim B)$

Logical Equivalence, Contradictoriness, etc.: Relationships

- What are the relationships between “ p and q are equivalent”, “ p and q are consistent”, “ p and q are contradictory”, “ p and q are inconsistent”?



- Answers:

1. Equivalent $\not\Rightarrow$ Consistent ($p \& \sim p$ and $q \& \sim q$)
2. Consistent $\not\Rightarrow$ Equivalent ($p \rightarrow q$ and $p \& q$)
3. Contradictory \Rightarrow Inconsistent (*why?*)
4. Inconsistent $\not\Rightarrow$ Contradictory (example?)

Truth-Tables and Deductive Validity I

- Remember, an argument is **valid** if it is *impossible* for its premises to be true while its conclusion is false. Let p_1, \dots, p_n be the premises of a SL argument, and let q be the conclusion of the argument. Then, we have:

$$\frac{p_1 \quad \vdots \quad p_n}{\therefore q}$$
 is valid if and only if there is no row in the simultaneous truth-table (*interpretation*) of p_1, \dots, p_n , and q which looks like:

atoms		premises		conclusion
...	p_1	...	p_n	q
...	T	T	T	F

Finite Propositional Boolean Algebras I

- A *finite propositional Boolean algebra* is a finite set of *propositions* which is *closed* under the logical operations and satisfies the laws of SL.
- Propositions* are the things expressed by sentences (abstract entities, distinct from sentences). If two sentences are logically equivalent, then they express the same proposition. *E.g.*, " $A \rightarrow B$ " and " $\sim A \vee B$ ".
- A set S is *closed* under logical operations if applying a logical operation to a member (or pair of members) of S always yields a member of S .
- Example: consider a sentential language with three atomic letters " X ", " Y ", and " Z ". The set of propositions expressible using the logical connectives and these three atomic letters forms a finite Boolean algebra.
- This Boolean algebra has $2^3 = 8$ *atomic propositions* or *states* (*i.e.*, rows of a 3-sentence truth-table!). Question: How many propositions does it contain *in total*? Answer: $2^8 = 256$ (255 plus the contradiction). *Why?*

Truth-Tables and Deductive Validity II

$$\frac{A \quad A \rightarrow B}{\therefore B}$$
 is *valid*:

atoms		premises			conclusion
A	B	A	$A \rightarrow B$	B	
T	T	T	T	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	F	T	F	F

$$\frac{B \quad A \rightarrow B}{\therefore A}$$
 is *invalid*:

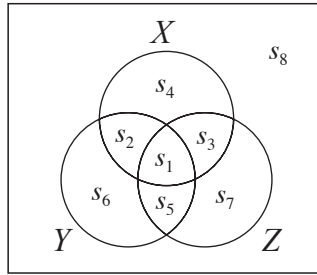
atoms		premises			conclusion
A	B	B	$A \rightarrow B$	A	
T	T	T	T	T	T
T	F	F	F	F	T
F	T	T	T	F	F
F	F	F	T	F	F

Finite Propositional Boolean Algebras II

- A *literal* is either an atomic sentence or the negation of an atomic sentence (*e.g.*, " A " and " $\sim A$ " are literals involving the atom " A ").
- A *state* of a Boolean algebra \mathcal{B} is a proposition expressed by a *maximal* conjunction of literals in a language $\mathcal{L}_{\mathcal{B}}$ describing \mathcal{B} ("maximal": "containing exactly one literal for each atomic sentence in \mathcal{B} ").
- Consider an algebra \mathcal{B} described by a 3-atom language $\mathcal{L}_{\mathcal{B}}$ (" X ", " Y ", " Z "). The states of \mathcal{B} are described by the $2^3 = 8$ *state descriptions* of $\mathcal{L}_{\mathcal{B}}$:
 - (s_1) $X \& Y \& Z$
 - (s_2) $X \& Y \& \sim Z$
 - (s_3) $X \& \sim Y \& Z$
 - (s_4) $X \& \sim Y \& \sim Z$
 - (s_5) $\sim X \& Y \& Z$
 - (s_6) $\sim X \& Y \& \sim Z$
 - (s_7) $\sim X \& \sim Y \& Z$
 - (s_8) $\sim X \& \sim Y \& \sim Z$

- We can “visualize” the states of \mathcal{B} using a truth table or a Venn Diagram.

X	Y	Z	States
T	T	T	s_1
T	T	F	s_2
T	F	T	s_3
T	F	F	s_4
F	T	T	s_5
F	T	F	s_6
F	F	T	s_7
F	F	F	s_8



- Everything that can be expressed in the sentential language $\mathcal{L}_{\mathcal{B}}$ can be expressed as a *disjunction of state descriptions* (think about why).
- Thus, every proposition expressible in $\mathcal{L}_{\mathcal{B}}$ can be “visualized” simply by shading combinations of the 8 state-regions of the Venn Diagram of \mathcal{B} . It is because of this that we can use Venn Diagrams to establish Boolean Laws.
- $p \models q$ (in \mathcal{B}) iff every shaded region in the Venn Diagram representation of p (in \mathcal{B}) is also shaded in the Venn Diagram representation of q (in \mathcal{B}).