Philosophy 148 — Announcements & Such

- Administrative Stuff
  - See the course website for all administrative information (also, note that lecture notes are posted the morning prior to each class):

    `http://socrates.berkeley.edu/~fitelson/148/`
  - Section times (II): Those of you who can't make Tu @ 10–11, please fill-out an index card. New times: Mon or Wed 9–10 or 10–11.
- Last Time: Review of Boolean (Truth-Functional) Sentential Logic
  - Truth-Table definitions of connectives
  - Semantical (Metatheoretic) notions
    - Individual Sentences: Logical Truth, Logical Falsity, etc.
    - Sets of Sentences: Entailment ($\vDash$), Consistency, etc.
- Today: Finite Propositional Boolean Algebras, Review of Boolean (Truth-Functional) Predicate Logic, and a General Boolean Framework

Finite Propositional Boolean Algebras

- Sentences express *propositions*. We individuate propositions according to their logical content. If two sentences are logically equivalent, then they express the same proposition. [*E.g.*, "$A \to B$" and "$\sim A \vee B$"]
- A *finite propositional Boolean algebra* is a finite set of *propositions* which is *closed* under the (Boolean) logical operations.
- A set S is *closed* under a logical operation λ if applying λ to a member (or pair of members) of S always yields a member of S.
- Example: consider a sentential language $\mathcal{L}$ with three atomic letters "X", "Y", and "Z". The set of propositions expressible using the logical connectives and these letters is a finite Boolean algebra of propositions.
- This Boolean algebra has $2^3 = 8$ *atomic propositions* or *states* (*i.e.*, the rows of a 3-atomic sentence truth-table!). Question: How many propositions does it contain *in total*? [A: $2^8 = 256$ — explanation later]

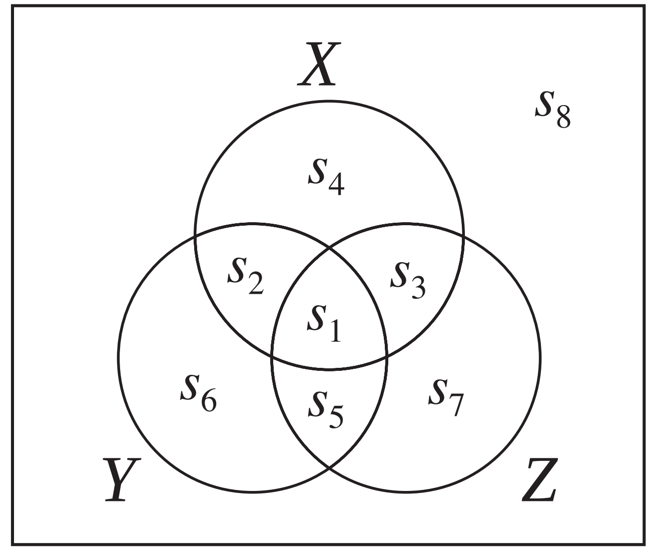
Propositional Boolean Algebras: States, Truth-Tables, and Venn Diagrams

- A *literal* is either an atomic sentence or the negation of an atomic sentence (*e.g.*, "A" and "$\sim A$" are the literals involving the atom "A").
- A *state* of a Boolean algebra $\mathcal{B}$ is a proposition expressed by a *state description* — a *maximal* conjunction of literals in a language $\mathcal{L}_{\mathcal{B}}$ describing $\mathcal{B}$ (maximal: having exactly one literal for each atom of $\mathcal{L}_{\mathcal{B}}$).
- Consider an algebra $\mathcal{B}$ described by a 3-atom language $\mathcal{L}_{\mathcal{B}}$ (X, Y, Z). The states of $\mathcal{B}$ are described by the $2^3 = 8$ *state descriptions* of $\mathcal{L}_{\mathcal{B}}$:

(s_1) $X \,\&\, Y \,\&\, Z$
(s_2) $X \,\&\, Y \,\&\, \sim Z$
(s_3) $X \,\&\, \sim Y \,\&\, Z$
(s_4) $X \,\&\, \sim Y \,\&\, \sim Z$
(s_5) $\sim X \,\&\, Y \,\&\, Z$
(s_6) $\sim X \,\&\, Y \,\&\, \sim Z$
(s_7) $\sim X \,\&\, \sim Y \,\&\, Z$
(s_8) $\sim X \,\&\, \sim Y \,\&\, \sim Z$

- We can "visualize" the states of $\mathcal{B}$ using a truth table or a Venn Diagram.

X	Y	Z	States
T	T	T	s_1
T	T	F	s_2
T	F	T	s_3
T	F	F	s_4
F	T	T	s_5
F	T	F	s_6
F	F	T	s_7
F	F	F	s_8



- Every proposition expressible in the sentential language $\mathcal{L}_{\mathcal{B}}$ can be expressed as a *disjunction of state descriptions* (how does this work?).
- Thus, every proposition expressible in $\mathcal{L}_{\mathcal{B}}$ can be "visualized" simply by shading combinations of the 8 state-regions of the Venn Diagram of $\mathcal{B}$.
- How many ways of shading the above Venn diagram are there? $2^8 = 256 =$ the number of disjunctions of the s_i (255) — plus 1 for $\bot$.
- That's why there are $2^{2^3} = 2^8 = 256$ propositions (in total) in $\mathcal{B}$.

(Finite) Monadic Predicate Logic (MPL) I

- Consider the following two arguments:

① Socrates is wise. ∴ Someone is wise.

② Everyone is happy. ∴ Plato is happy.

- Intuitively, both ① and ② are *valid* (*why*?). But, if we try to translate these into sentential logic, we get the *in*valid SL forms:

$①_{SL}$ S $\therefore W$

$②_{SL}$ H $\therefore P$

- In SL, we are not able to express what the premises and conclusions of these kinds of arguments *have in common*.
- If it's not *atomic sentences* that the premises and conclusions of such arguments have in common, then what *is* it?
- This is what monadic predicate logic is about ...

(Finite) Monadic Predicate Logic II

- We need a *richer language* than SL — one which accurately captures the deeper *logical structure* of arguments like ① and ②. New Jargon:
- A **predicate** is something which *applies to* an object or *is true of* an object. *E.g.*, The predicate **(is) Wise** applies to Socrates.
- A **proper name** is a word or a phrase which *stands for*, or *refers to*, or *denotes* a person, place, or thing. *E.g.*, 'Socrates' is a proper name.
- **Quantifiers** specify *quantities*. *E.g.*, 'someone' means *at least one* person and 'everyone' means *all* people. [think: "Some" and "All"]
- The collection of objects to which the quantifiers in a statement are *relativized* is called the **domain of discourse** of the statement. *In this course, we'll only work with finite domains* (*e.g.*, the domain of ravens).
- As we'll explain soon, when we are restricted to finite domains, monadic predicate logic is *almost* "sentential logic in disguise".

(Finite) Mondacic Predicate Logic III

- Among the atomic sentences of MPL (*in addition to SL sentence letters*) are (new) strings of the form 'Xn', where 'X' is a (monadic) predicate, and 'n' is an individual constant (proper name).
- We use the lower-case letters 'a'–'s' as individual constants ('t'–'z' are used as *variables* — we won't say too much about variables).
- Some examples of these new kinds of atomic sentences:
  - 'Branden is tall.' $\mapsto$ 'Tb'.
  - 'Honda is an automobile manufacturer.' $\mapsto$ 'Ah'.
  - 'New York is a city.' $\mapsto$ 'Cn'.
- As in SL, we can *combine* different MPL atomic sentences using the sentential connectives to yield complex sentences. For instance:
  - 'Branden is tall, but Ruth is not tall.' $\mapsto$ '$Tb \,\&\, {\sim}Tr$'.

Quantifiers in (Finite) Mondaic Predicate Logic

- In finite domains, we can (almost) think of a universal claim $(\forall v)\phi v$ as a *conjunction* which asserts that the predicate expression ϕ applies to *each object* in the domain [*i.e.*, as $\phi a \,\&\, (\phi b \,\&\, (\phi c \,\&\, \ldots))$].
- Analogously, in finite domains, we can (almost) think of an existential claim $(\exists v)\phi v$ as a *disjunction* which asserts that ϕ applies to *at least one object* in the domain [*i.e.*, as $\phi a \vee (\phi b \vee (\phi c \vee \ldots))$].
- Upshot: when the size of the domain is finite (and, *known*), we can say everything we need to say about the domain using *sentential* logic.
- For each ϕ and each constant τ, we can construct an atomic sentence $\phi\tau$. And, if we think of these as the atomic sentences of a sentential language, then we can express every claim we need to express using conjunctions, disjunctions, *etc.* of these atomic sentences.
- So, we can use finite Boolean algebras of propositions for FMPL too…

Finite Boolean Algebras and Finite Monadic Predicate Logic

- Consider the language $\mathcal{L}_2^2$, with two monadic predicates F and G and two individual constants a and b. $\mathcal{L}_2^2$ has 16 *state descriptions*:

$Fa \,\&\, Ga \,\&\, Fb \,\&\, Gb$	$Fa \,\&\, Ga \,\&\, Fb \,\&\, {\sim}Gb$	$Fa \,\&\, Ga \,\&\, {\sim}Fb \,\&\, Gb$
$Fa \,\&\, Ga \,\&\, {\sim}Fb \,\&\, {\sim}Gb$	$Fa \,\&\, {\sim}Ga \,\&\, Fb \,\&\, Gb$	$Fa \,\&\, {\sim}Ga \,\&\, Fb \,\&\, {\sim}Gb$
$Fa \,\&\, {\sim}Ga \,\&\, {\sim}Fb \,\&\, Gb$	$Fa \,\&\, {\sim}Ga \,\&\, {\sim}Fb \,\&\, {\sim}Gb$	${\sim}Fa \,\&\, Ga \,\&\, Fb \,\&\, Gb$
${\sim}Fa \,\&\, Ga \,\&\, Fb \,\&\, {\sim}Gb$	${\sim}Fa \,\&\, Ga \,\&\, {\sim}Fb \,\&\, Gb$	${\sim}Fa \,\&\, Ga \,\&\, {\sim}Fb \,\&\, {\sim}Gb$
${\sim}Fa \,\&\, {\sim}Ga \,\&\, Fb \,\&\, Gb$	${\sim}Fa \,\&\, {\sim}Ga \,\&\, Fb \,\&\, {\sim}Gb$	${\sim}Fa \,\&\, {\sim}Ga \,\&\, {\sim}Fb \,\&\, Gb$
	${\sim}Fa \,\&\, {\sim}Ga \,\&\, {\sim}Fb \,\&\, {\sim}Gb$	

- This characterizes a Boolean algebra $\mathcal{B}$ with 16 *states*. We cannot easily visualize $\mathcal{B}$ with Venn diagrams. But, we can (and will) use truth-tables.
- Note: the total number of propositions in $\mathcal{B}$ is very large ($2^{16} = 65536$). There are 65535 disjunctions of state-descriptions of $\mathcal{L}_2^2$ (plus 1 for $\perp$).

Fa	Ga	Fb	Gb	State Descriptions (s_i)
T	T	T	T	$Fa \& Ga \& Fb \& Gb$
T	T	T	F	$Fa \& Ga \& Fb \& {\sim}Gb$
T	T	F	T	$Fa \& Ga \& {\sim}Fb \& Gb$
T	T	F	F	$Fa \& Ga \& {\sim}Fb \& {\sim}Gb$
T	F	T	T	$Fa \& {\sim}Ga \& Fb \& Gb$
T	F	T	F	$Fa \& {\sim}Ga \& Fb \& {\sim}Gb$
T	F	F	T	$Fa \& {\sim}Ga \& {\sim}Fb \& Gb$
T	F	F	F	$Fa \& {\sim}Ga \& {\sim}Fb \& {\sim}Gb$
F	T	T	T	${\sim}Fa \& Ga \& Fb \& Gb$
F	T	T	F	${\sim}Fa \& Ga \& Fb \& {\sim}Gb$
F	T	F	T	${\sim}Fa \& Ga \& {\sim}Fb \& Gb$
F	T	F	F	${\sim}Fa \& Ga \& {\sim}Fb \& {\sim}Gb$
F	F	T	T	${\sim}Fa \& {\sim}Ga \& Fb \& Gb$
F	F	T	F	${\sim}Fa \& {\sim}Ga \& Fb \& {\sim}Gb$
F	F	F	T	${\sim}Fa \& {\sim}Ga \& {\sim}Fb \& Gb$
F	F	F	F	${\sim}Fa \& {\sim}Ga \& {\sim}Fb \& {\sim}Gb$

(Finite) Relational Predicate Logic (RPL)

- We won't work much (hardly at all) with relational predicate logic in this course, but a similar trick can be done to use finite relational logical languages to characterize finite Boolean algebras of propositions:
- Consider a language $\mathcal{L}$, which has one 2-place predicate "R" and two individual constants "a" and "b". $\mathcal{L}$ also has 16 *state descriptions*:

$Raa \,\&\, Rab \,\&\, Rba \,\&\, Rbb$	$Raa \,\&\, Rab \,\&\, Rba \,\&\, {\sim}Rbb$	$Raa \,\&\, Rab \,\&\, {\sim}Rba \,\&\, Rbb$
$Raa \,\&\, Rab \,\&\, {\sim}Rba \,\&\, {\sim}Rbb$	$Raa \,\&\, {\sim}Rab \,\&\, Rba \,\&\, Rbb$	$Raa \,\&\, {\sim}Rab \,\&\, Rba \,\&\, {\sim}Rbb$
$Raa \,\&\, {\sim}Rab \,\&\, {\sim}Rba \,\&\, Rbb$	$Raa \,\&\, {\sim}Rab \,\&\, {\sim}Rba \,\&\, {\sim}Rbb$	${\sim}Raa \,\&\, Rab \,\&\, Rba \,\&\, Rbb$
${\sim}Raa \,\&\, Rab \,\&\, Rba \,\&\, {\sim}Rbb$	${\sim}Raa \,\&\, Rab \,\&\, {\sim}Rba \,\&\, Rbb$	${\sim}Raa \,\&\, Rab \,\&\, {\sim}Rba \,\&\, {\sim}Rbb$
${\sim}Raa \,\&\, {\sim}Rab \,\&\, Rba \,\&\, Rbb$	${\sim}Raa \,\&\, {\sim}Rab \,\&\, Rba \,\&\, {\sim}Rbb$	${\sim}Raa \,\&\, {\sim}Rab \,\&\, {\sim}Rba \,\&\, Rbb$
	${\sim}Raa \,\&\, {\sim}Rab \,\&\, {\sim}Rba \,\&\, {\sim}Rbb$	

- The truth-table for $\mathcal{L}$ looks very much like the one for $\mathcal{L}_2^2$, above, since each language ($\mathcal{L}$ and $\mathcal{L}_2^2$) has four atomic sentences. Here's the table:

Raa	Rab	Rba	Rbb	State Descriptions (s_i)
T	T	T	T	$Raa \mathbin{\&} Rab \mathbin{\&} Rba \mathbin{\&} Rbb$
T	T	T	F	$Raa \mathbin{\&} Rab \mathbin{\&} Rba \mathbin{\&} {\sim}Rbb$
T	T	F	T	$Raa \mathbin{\&} Rab \mathbin{\&} {\sim}Rba \mathbin{\&} Rbb$
T	T	F	F	$Raa \mathbin{\&} Rab \mathbin{\&} {\sim}Rba \mathbin{\&} {\sim}Rbb$
T	F	T	T	$Raa \mathbin{\&} {\sim}Rab \mathbin{\&} Rba \mathbin{\&} Rbb$
T	F	T	F	$Raa \mathbin{\&} {\sim}Rab \mathbin{\&} Rba \mathbin{\&} {\sim}Rbb$
T	F	F	T	$Raa \mathbin{\&} {\sim}Rab \mathbin{\&} {\sim}Rba \mathbin{\&} Rbb$
T	F	F	F	$Raa \mathbin{\&} {\sim}Rab \mathbin{\&} {\sim}Rba \mathbin{\&} {\sim}Rbb$
F	T	T	T	${\sim}Raa \mathbin{\&} Rab \mathbin{\&} Rba \mathbin{\&} Rbb$
F	T	T	F	${\sim}Raa \mathbin{\&} Rab \mathbin{\&} Rba \mathbin{\&} {\sim}Rbb$
F	T	F	T	${\sim}Raa \mathbin{\&} Rab \mathbin{\&} {\sim}Rba \mathbin{\&} Rbb$
F	T	F	F	${\sim}Raa \mathbin{\&} Rab \mathbin{\&} {\sim}Rba \mathbin{\&} {\sim}Rbb$
F	F	T	T	${\sim}Raa \mathbin{\&} {\sim}Rab \mathbin{\&} Rba \mathbin{\&} Rbb$
F	F	T	F	${\sim}Raa \mathbin{\&} {\sim}Rab \mathbin{\&} Rba \mathbin{\&} {\sim}Rbb$
F	F	F	T	${\sim}Raa \mathbin{\&} {\sim}Rab \mathbin{\&} {\sim}Rba \mathbin{\&} Rbb$
F	F	F	F	${\sim}Raa \mathbin{\&} {\sim}Rab \mathbin{\&} {\sim}Rba \mathbin{\&} {\sim}Rbb$

☞ Assuming finitely many constant symbols, predicate logics (monadic or relational) characterize finite Boolean algebras of propositions.

Overview of Deductive Logic I

- Deductive Logic provides *formal theories* of *validity* (*following-from*). The logician thus *theoretically grounds* our *informal* validity notion(s).
- In English, there are various argument forms or patterns that are intuitively or informally valid. Here's a simple, *sentential* example:

Dr. Ruth is a man.
(1) If Dr. Ruth is a man, then Dr. Ruth is 10 feet tall.
∴ Dr. Ruth is 10 feet tall.

- Intuitively, the conclusion of (1) *follows-from* its premises, since *if* the premises of (1) *were* true, then (1)'s conclusion would *have to be* true.
- Our simplest logical theory (SL) correctly classifies this argument form (and many other valid English forms) as *valid* — an SL "success story":

p.
(1_{SL}) If p, then q.
$\therefore q$.

Overview of Deductive Logic II

- However, there are many English arguments that are (intuitively, or "absolutely") valid, but their SL forms are *not* valid. For instance:

(2)
Socrates is wise.
∴ Someone is wise.

- Intuitively, argument (2) is ("absolutely") *valid*. But, if we try to translate this argument into SL, we get the following *in*valid SL form:

$$(2_{\text{SL}}) \quad \begin{array}{l} p \\ \therefore q \end{array}$$

- This motivates the richer logical language MPL, which subsumes SL, and which adds additional structure that allows us to "see" its validity:

$$(2_{\text{MPL}}) \quad \begin{array}{l} Ws \\ \therefore (\exists x)Wx \end{array}$$

Overview of Deductive Logic III

- The relational predicate logical language is even richer than either SL or MPL. Indeed, it subsumes both SL and MPL. This language (RPL) can formalize many more English validities. Example:

(3) Everybody loves John.
∴ Someone is loved by everyone.

- Adequately formalizing this argument requires the use of a *two-place predicate/relation*: 'Lxy', which reads 'x loves y' or 'y is loved by x'.

$$(3_{\text{RPL}}) \quad \begin{array}{c} (\forall x)Lxj \\ \therefore\ (\exists y)(\forall x)Lxy \end{array}$$

- Exercise: try symbolizing (3) in MPL. You'll get something like this:

$$(3_{\text{MPL?}}) \quad \begin{array}{c} (\forall x)Jx \\ \therefore\ (\exists x)Ex \end{array}$$

- We will not use RPL much in this course. Mainly, we'll use just SL/MPL.

Overview of Deductive Logic IV

- Logical theories replace imprecise, informal notions of following-from with precise, theoretical validity concepts (in formal logical languages).
  - **One Informal Validity Notion.**
    - If $\mathscr{A}$'s premises are true, then $\mathscr{A}$'s conclusion must also be true.
  - **Some Corresponding Theoretical Validity Concepts.**
    - An English argument $\mathscr{A}$ is *SL*-valid if there is no SL interpretation in which all of the premises of $\mathscr{A}_{\text{SL}}$ are T and the conclusion of $\mathscr{A}_{\text{SL}}$ is F, where $\mathscr{A}_{\text{SL}}$ is the *SL form* of the English argument $\mathscr{A}$.
    - An English argument $\mathscr{A}$ is *MPL*-valid if there is no MPL interpretation in which all of the premises of $\mathscr{A}_{\text{MPL}}$ are T and the conclusion of $\mathscr{A}_{\text{MPL}}$ is F, where $\mathscr{A}_{\text{MPL}}$ is the *MPL form* of $\mathscr{A}$.
    - Similarly for *RPL*-validity ...
- The story of formal deductive logic does not end with RPL...

Overview of Deductive Logic V

- The full theory of first-order logic (LFOL) includes RPL, plus n-place predicates, the identity relation $=$, and also function symbols.
- LFOL can capture even more valid arguments than RPL. For instance, LFOL can capture arguments like the following mathematical one:

$$2 + 4 = 6$$

(4) $$3 \times 2 = 6$$

$$\therefore 2 + 4 = 3 \times 2$$

- Indeed, LFOL can capture just about any argument in just about any branch of modern mathematics. That's a lot of expressive power.
- In PHIL 140A, we study the full theory of first-order logic (LFOL). There, we give a semantics for LFOL, and we show that there is a sound and complete proof theory for LFOL (but, no decision procedure for $\vDash$!).
- Of course, even full first-order logic (LFOL) has its limitations. . .

Overview of Deductive Logic VI

- Some arguments involve quantification over not only objects but *properties*. These arguments are *second-order* and ∴ beyond LFOL.
- Leibniz (sometimes) talked as if the following argument were valid:

(5) a and b have exactly the same (monadic) properties.
∴ a and b are identical.

- In second-order logic (SOL), (5) would be formalized as follows:

(5_{SOL}) $(\forall P)(Pa \leftrightarrow Pb)$.
$\therefore a = b$.

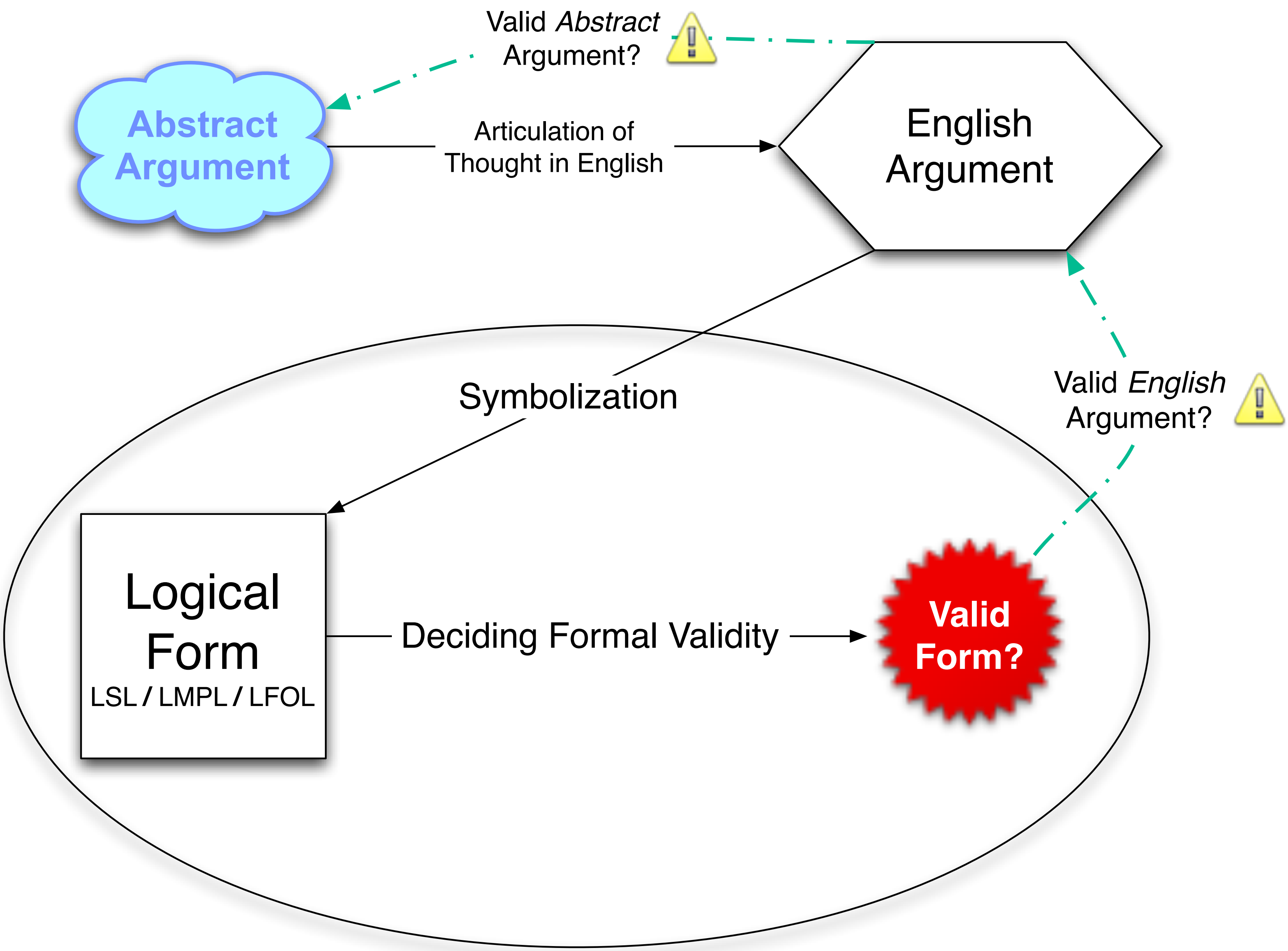
- Note that the premise of (5) quantifies over (monadic) *predicates*.
- This is something that LFOL is not designed to do.
- We could also have an SOL which allows quantification over *relations*.
- Second-order logic is beyond 140A. It is touched upon (a little) in 140B.

Overview of Deductive Logic VII

- All the logics I've mentioned are *classical* deductive logics. Not all logicians think classical logics capture our intuitive validity notions.
- Classical logics all share the following two properties:
  (i) All arguments with contradictory premises (*e.g.*, $p \,\&\, {\sim}p$) are valid.
  (ii) All arguments with tautological conclusions (*e.g.*, $p \vee {\sim}p$) are valid.
- Some logicians think (i) and/or (ii) are *counterexamples* to the classical theory of validity (as an explication of our informal "following-from").
- Such logicians propose alternative formal theories of validity ($\vDash^\star$).
- Usually, non-classical logicians reject the classical (truth-functional) theory of the *conditional*. They adopt a non-classical conditional ($\to^\star$) which obeys constraints like the deduction theorem (relative to $\vDash^\star$).

$$p \vDash^\star q \text{ if and only if } \vDash^\star p \to^\star q$$

- These and other foundational questions about deductive logic are addressed in Philosophical Logic (142). We'll see *some* overlap here...


Valid *Abstract* Argument?
Abstract Argument
Articulation of Thought in English
English Argument
Symbolization
Valid *English* Argument?
Logical Form
LSL / LMPL / LFOL
Deciding Formal Validity
Valid Form?

Why study logic *formally* or *symbolically*?

- In ordinary contexts, we want to know if arguments expressed *in English* are valid or invalid. But, in formal logic, we only study arguments expressible in *formal* languages (SL, MPL, *etc.*). *Why?*
- Analogous question: What we want from natural science is to understand natural systems. But, our theories (strictly) apply only to systems faithfully describable in mathematical terms.
- Although formal models are *idealizations* which abstract away some aspects of natural systems, they are *useful idealizations* that help us understand *many* natural relationships and regularities.
- Studying arguments expressible in formal languages allows us to develop and use powerful tools for testing validity, etc. We can't capture *all* valid arguments this way. But, we can grasp *many*.
- We will take the same attitude toward inductive logic as well . . .

Inductive Logic — Basic Motivation and Ideas

- Intuitively, not all "logically good" arguments are deductively valid. Some invalid arguments seem (intuitively) logically *better than* others:

(6) p. Someone is wise. $\therefore q$. Socrates is wise.

(7) r. Someone is either wise or unwise. $\therefore q$. Socrates is wise.

- *Inductive* logic should *theoretically ground* our intuition that (6) is a *logically stronger* argument than (7) is. Neither argument is *valid*.
- More ambitiously, an inductive logician might aim for a theory of "the *degree* to which the premises of an argument *confirm* its conclusion".
- This ambitious project would aim to characterize a *function* $\mathfrak{c}(\mathscr{C}, \mathscr{P})$. And, an intuitive requirement would be that this function be such that:

$$\mathfrak{c}(q, p) > \mathfrak{c}(q, r)$$

- This course is (mainly) about *inductive logic*. We will examine how *probabilities* might be used to *quantitatively generalize* deductive logic.

Logic and Epistemology — A Prelude I

- As I mentioned, some have worried about the adequacy of classical logic as a formal explication of our informal "following-from" relation.
- Here's a fact about classical deductive logic that may seem "odd":

(†) If p and q are (classically) logically inconsistent, then the argument from p and q to r is (classically) valid — *for **any** r*.

- There's *something* "odd" about the fact that *everything follows-from inconsistent premises*, according to the classical formal explication of following-from. But, what, exactly, is supposed to be "odd" about it?
- Here's an *epistemological* principle that is downright *crazy*:

(‡) If one's beliefs are inconsistent (and one knows that they are), then one should believe everything (*i.e.*, every proposition).

- It is clear that (‡) is false. There are things I *know* to be false, and I shouldn't believe those things — no matter what else is true of me.

Logic and Epistemology — A Prelude II

- OK, (‡) is clearly false. So? What does that have to do with (†)?
- After all, (†) is a about *logic*, and (‡) is about *epistemology*.
- Perhaps those worried about (†) are assuming that logic and epistemology are connected, or bridged by something like:

  (*) If an agent S's belief set B is such that $B \vDash p$ (and S knows that $B \vDash p$), then it would be reasonable for S to infer/believe p.

- If (*) were true, then (†) would imply (‡), and — as a result — classical logicians who accepted (*) would seem to be stuck with (‡) too.
- More precisely, classical logicians who believe (*) should find it reasonable to believe (‡). But, they don't (at least, they shouldn't!).

☞ But, *this* doesn't *force* classical logicians to give up (†). They could give up (*) instead. In such contexts, logic (alone) doesn't seem to tell us whether to infer something new, or reject something we already believe.