

Philosophy 148 — Announcements & Such

- Branden will not be having office hours today (May 6).
- **New Plan for HW #5**
 - It will be due on the last day of class — this Thursday 5/8.
 - Our HW #5 discussion will be **Tonight 5/6 @ 6pm @ 110 Wheeler.**
- I will also be preparing some final extra-credit problems. They will be distributed Thursday, and due at the final exam (5/20 @ 8am).
- The final exam is **Tuesday, May 20 @ 8am @ 20 Barrows.**
 - I will hold a review session for the final exam — the day before the final (May 19). It will take place **May 19 @ 4-6pm @ 122 Wheeler.**
 - I will also be distributing a “practice final” later this week.
- Today’s Agenda (and next time too)
 - The Grue Paradox

- Carnapian confirmation (*i.e.*, *later* Carnapian theory [13] — see “Extras”) is based on *probabilistic relevance*, not entailment:
 - E confirms H , relative to K iff $\Pr(H \mid E \ \& \ K) > \Pr(H \mid K)$, for some “suitable” conditional probability function $\Pr(\cdot \mid \cdot)$.
 - Note how this is an *explicitly* 3-place relation. Hempel’s was only 2-place. This is because \Pr (unlike \models) is *non-monotonic*.
 - Carnap thought that “suitable \Pr ” meant “logical \Pr ” in a rather strong sense (see “Extras”). However, Goodman’s argument will work against *any* probability function \Pr .




Carnap’s theory implies *only 1* of our 3 Hempelian claims: (EQC). It does *not* imply (NC) *or* (M) (see “Extras” & [3]/[13]).

- This will allow Carnapian IL to avoid facing the full brunt of Goodman’s “grue” (but, it will still face a serious challenge).
- For Carnap, confirmation is a *logical* relation (akin to entailment). Like entailment, confirmation can be *applied*, but this requires *epistemic bridge principles* [akin to (2)].
- Carnap [1] discusses various bridge principles. The most well-known of these is the *requirement of total evidence*.

- **The Requirement of Total Evidence.** In the application of IL to a given knowledge situation, the total evidence available must be taken as a basis for determining the degree of confirmation.
- This *sounds* like a plausible principle. But, once it is made more precise, it will actually turn out to be subtly defective.
- More precisely, we have the following *bridge principle* connecting *confirmation* and *evidential support*:

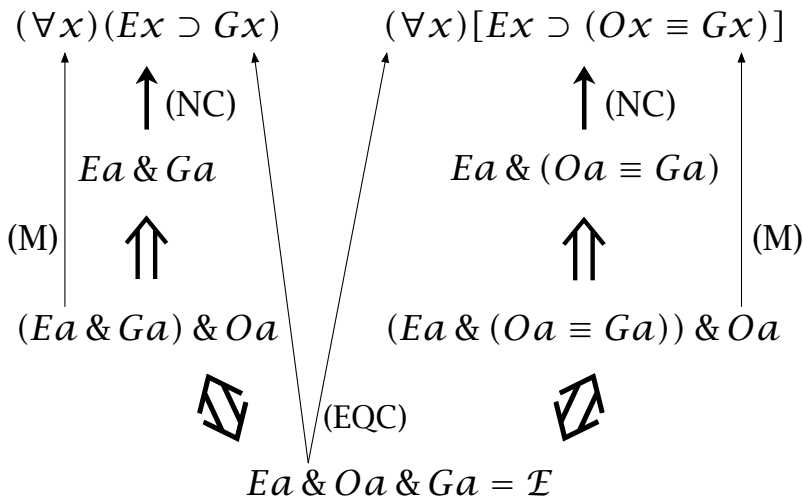
(RTE) *E* evidentially supports *H* for *S* in *C* iff *E* confirms *H*, relative to *K*, where *K* is *S*'s total evidence in *C*.

- The (RTE) has often been (implicitly) presupposed by Bayesian epistemologists (both subjective and objective).
 - However, as we will soon see, the (RTE) is not a tenable bridge principle, and for reasons independent of “grue”.
-  Moreover, Goodman's “grue” argument will rely *more heavily* on (RTE) than the relevantists' argument relies on (2). In this sense, Goodman's argument will be *even worse*.
- Before reconstructing the argument, a brief “grue” primer.

- Let $Gx \stackrel{\text{def}}{=} x$ is green, $Ox \stackrel{\text{def}}{=} x$ is examined prior to t , and $Ex \stackrel{\text{def}}{=} x$ is an emerald. Goodman introduces a predicate “grue”

$$Gx \stackrel{\text{def}}{=} x \text{ is grue} \stackrel{\text{def}}{=} Ox \equiv Gx.$$
- Consider the following two universal generalizations
 - (H_1) All emeralds are green. $[(\forall x)(Ex \supset Gx)]$
 - (H_2) All emeralds are grue. $[(\forall x)[Ex \supset (Ox \equiv Gx)]]$
- And, consider the following instantial evidential statement
 - (\mathcal{E}) $Ea \ \& \ Oa \ \& \ Ga$
- Hempel’s confirmation theory [(EQC) & (NC) & (M)] entails:
 - (\dagger) \mathcal{E} confirms H_1 , and \mathcal{E} confirms H_2 . [▶ proof]
- As a result, his theory entails the following weaker claim
 - (\ddagger) \mathcal{E} confirms H_1 if and only if \mathcal{E} confirms H_2 .
- What about (later) Carnapian theory? Does *it* entail even (\ddagger)?
- 👉 Interestingly, NO! There are (later) Carnapian Pr-models in which \mathcal{E} confirms H_1 but \mathcal{E} *disconfirms* H_2 (see “Extras”).
- In this sense, Hempel was an easier target for Goodman than Carnap (Goodman claims to be attacking both).
- Now, we’re ready to reconstruct Goodman’s argument.

A Proof of (\dagger) From Hempel's (NC), (M), and (EQC)



- There is just one more ingredient in Goodman's argument:
 - The agent S who is assessing the evidential support that \mathcal{E} provides for H_1 vs H_2 in a Goodmanian "grue" context C_G has Oa as part of their total evidence in C_G . (e.g., [14].)
- Now, we can run the following Goodmanian *reductio*:
 - (i) E confirms H , relative to K iff $\Pr(H | E \& K) > \Pr(H | K)$.
 - (ii) E evidentially supports H for S in C iff E confirms H , relative to K , where K is S 's total evidence in C .
 - (iii) The agent S who is assessing the evidential support \mathcal{E} provides for H_1 vs H_2 in a Goodmanian "grue" context C_G has Oa as part of their total evidence in C_G [i.e., $K \models Oa$].
 - (iv) If $K \models Oa$, then—c.p.— \mathcal{E} confirms H_1 relative to K iff \mathcal{E} confirms H_2 relative to K , for **any** \Pr [i.e., (\ddagger) holds, $\forall \Pr$'s].
 - (v) Therefore, \mathcal{E} evidentially supports H_1 for S in C_G if and only if \mathcal{E} evidentially supports H_2 for S in C_G .
 - (vi) \mathcal{E} evidentially supports H_1 for S in C_G , but \mathcal{E} does *not* evidentially support H_2 for S in C_G .
- \therefore (i)–(vi) lead to an absurdity. Hence, our initial assumption (i) must have been false. Carnapian inductive logic refuted?

- Premise (vi) is based on Goodman's *epistemic intuition* that, in “grue” contexts, \mathcal{E} evidentially supports H_1 but *not* H_2 .
- Premise (v) follows logically from premises (i)–(iv).
- Premise (iv) is a theorem of probability calculus (**any** Pr!).
 - The *c.p.* clause needed is $\Pr(Ea \mid H_1 \ \& \ K) = \Pr(Ea \mid H_2 \ \& \ K)$, which is assumed in all probabilistic renditions of “grue”.
- Premise (iii) is an assumption about the agent's background knowledge K that's implicit in Goodman's set-up. See [14].
- Premise (ii) is (RTE). It's the *bridge principle*, akin to (2) in the relevantists' *reductio*. This is the premise I will focus on.
- Here are my two main points about Goodman's argument:
 - (ii) must be rejected by Bayesians for independent reasons.
 - Carnapian confirmation theory *doesn't even entail* (\nexists).
[Hempel's theory does, just as deductive logic entails (1).]
- This suggests Goodman's argument is *even less* a *reductio* of (i) than the relevantists' argument is a *reductio* of (1).
- Next, I will explain why Carnapians/Bayesians should reject (ii) on *independent* grounds: The Problem of Old Evidence.

- As Tim Willimson points out [16, ch. 9], Carnap's (RTE) must be rejected, because of the problem of old evidence [2].
- If S 's total evidence in C (K) entails E , then, according to (RTE), E cannot evidentially support *any* H for S in C .
- As a result, one cannot (in all contexts) use $\text{Pr}(\cdot | K)$ — for *any* Pr — when assessing the *evidential import of E*.
- There are (basically) two kinds of strategies for revising (RTE). Carnap [1, p. 472] & Williamson [16, ch. 9] propose:

(RTE₊) E evidentially supports H for S in C iff S possesses E as evidence in C and $\text{Pr}_T(H | E \& K_T) > \text{Pr}_T(H | K_T)$. [K_T is *a priori*, Pr_T is “inductive” [13]/“evidential” [16]/“logical” [1].]
- Note: Hempel explicitly *required* that confirmation be taken “*relative to* K_T ” in all treatments of the paradoxes [9, 10]. (RTE₊) is a charitable Carnapian reconstruction of Hempel.
- A more “standard” way to revise (RTE) is [(RTE’)] to use $\text{Pr}_{S'}(\cdot | K')$, where $K \models K' \neq E$, and $\text{Pr}_{S'}$ is the credence function of a “counterpart” S' of S with total evidence K' .

- Carnap never re-wrote the part of LFP [1] that discusses the (RTE), in light of a probabilistic *relevance* (“increase in firmness” [1]) notion of confirmation. This is too bad.
- If Carnap had discussed this (“old evidence”) issue, I suspect he would have used something like Williamson’s (RTE_⊥) as his bridge principle connecting confirmation and evidence.
- Various other philosophers have proposed similar accounts of “support” as some probabilistic relation, taken relative to an “informationless” or “*a priori*” background/probability.
 - Richard Fumerton (who, unlike Williamson, is an epistemological *internalist*) proposes such a view in his [4].
 - Patrick Maher [13] applies such relations extensively in his recent (neo-Carnapian) work on confirmation theory.
 - Brian Weatherson [15] uses a similar, “Keynesian” [11] inductive-probability approach to evidential support.
- So, many Bayesians *already* reject (RTE). [Of course, “grue” gives Bayesians another important reason to reject (RTE).]

- So far, I have left open (precisely) what I think Bayesian confirmation theorists *should* say (*logically & epistemically*) in light of Goodman’s “grue” paradox (but, see “Extras”).
- Clearly, BCTs will need to revise (RTE) in light of “grue”. But, the standard (RTE’) way of doing this to cope with “old evidence” isn’t powerful enough to avoid *both* problems.
- Williamson’s (RTE₊) revision of (RTE) — also suggested by Carnap — avoids both problems, from a *logical* point of view (*if* “inductive”/“logical”/“evidential” probabilities *exist!*). But, what should BCTs say on the *epistemic* side?
- I don’t have a fully satisfactory answer to this question (yet). But, I remain unconvinced that the epistemic problem (if there is one) is caused by the “non-naturalness” of “grue”.
- The problem, I suspect, may involve an *observation selection effect*: we know something about the “grue” observation process that *undermines* (or *defeats*) evidence it produces.
- I hope we can discuss this (and IL) in the Q&A (see “Extras”).

- [1] R. Carnap, *Logical Foundations of Probability*, 2nd ed., Chicago Univ. Press, 1962.
- [2] E. Eells, *Bayesian problems of old evidence*, in C. Wade Savage (ed.) *Scientific theories*, *Minnesota Studies in the Philosophy of Science* (Vol. X), 205-223, 1990.
- [3] B. Fitelson, *The Paradox of Confirmation*, *Philosophy Compass* (online publication), Blackwell, 2006. URL: <http://fitelson.org/ravens.htm>.
- [4] R. Fumerton, *Metaepistemology and Skepticism*, Rowman & Littlefield, 1995.
- [5] C. Glymour, *Theory and Evidence*, Princeton University Press, 1980.
- [6] I.J. Good, *The white shoe is a red herring*, *BJPS* **17** (1967), 322.
- [7] N. Goodman, *Fact, Fiction, and Forecast*, Harvard University Press, 1955.
- [8] G. Harman, *Change in View: Principles of Reasoning*, MIT Press, 1988.
- [9] C. Hempel, *Studies in the logic of confirmation*, *Mind* **54** (1945), 1-26, 97-121.
- [10] ———, *The white shoe: no red herring*, *BJPS* **18** (1967), 239-240.
- [11] J. Keynes, *A Treatise on Probability*, Macmillan, 1921.
- [12] J. MacFarlane, *In what sense (if any) is logic normative for thought?*, 2004.
- [13] P. Maher, *Probability captures the logic of scientific confirmation*, *Contemporary Debates in the Philosophy of Science* (C. Hitchcock, ed.), Blackwell, 2004.
- [14] E. Sober, *No model, no inference: A Bayesian primer on the grue problem*, in *grue! The New Riddle of Induction* (D. Stalker ed.), Open Court, Chicago, 1994.
- [15] B. Weatherson, *The Bayesian and the Dogmatist*, manuscript, 2007. URL: <http://brian.weatherson.org/tbatd.pdf>.
- [16] T. Williamson, *Knowledge and its Limits*, Oxford University Press, 2000.

“Carnapian” Counterexamples to (NC) and (M)

(K) Either: (H) there are 100 black ravens, no nonblack ravens, and 1 million other things, or ($\sim H$) there are 1,000 black ravens, 1 white raven, and 1 million other things.

- Let $E \stackrel{\text{def}}{=} Ra \ \& \ Ba$ (a randomly sampled from universe). Then:

$$\Pr(E \mid H \ \& \ K) = \frac{100}{1000100} \ll \frac{1000}{1001001} = \Pr(E \mid \sim H \ \& \ K)$$

- \therefore This K/\Pr constitute a counterexample to (NC), assuming a “Carnapian” theory of confirmation. This model can be emulated in the later Carnapian λ/γ -systems [13].

- Let $Bx \stackrel{\text{def}}{=} x$ is a black card, $Ax \stackrel{\text{def}}{=} x$ is the ace of spades, $Jx \stackrel{\text{def}}{=} x$ is the jack of clubs, and $K \stackrel{\text{def}}{=} a$ card a is sampled at random from a standard deck (where \Pr is also standard):

- $\Pr(Aa \mid Ba \ \& \ K) = \frac{1}{26} > \frac{1}{52} = \Pr(Aa \mid K).$
- $\Pr(Aa \mid Ba \ \& \ Ja \ \& \ K) = 0 < \frac{1}{52} = \Pr(Aa \mid K).$

A “Carnapian” Counterexample to (\ddagger)

- (K) Either: (H_1) there are 1000 green emeralds 900 of which have been examined before t , no non-green emeralds, and 1 million other things in the universe, or (H_2) there are 100 green emeralds that have been examined before t , no green emeralds that have not been examined before t , 900 non-green emeralds that have not been examined before t , and 1 million other things.
- Imagine an urn containing true descriptions of each object in the universe ($\text{Pr} \stackrel{\text{def}}{=} \text{urn model}$). Let $\mathcal{E} \stackrel{\text{def}}{=} “Ea \& Oa \& Ga”$ be drawn. \mathcal{E} confirms H_1 but \mathcal{E} *disconfirms* H_2 , relative to K :

$$\text{Pr}(\mathcal{E} \mid H_1 \& K) = \frac{900}{1001000} > \frac{100}{1001000} = \text{Pr}(\mathcal{E} \mid H_2 \& K)$$

- This K/Pr constitute a counterexample to (\ddagger) , assuming a “Carnapian” theory of confirmation. This probability model can be emulated in the later Carnapian λ/γ -systems [13].

Is “Grue” an Observation Selection Effect? Part I

- **Canonical Example of an OSE:** I use a fishing net to capture samples of fish from various (randomly selected) parts of a lake. Let E be the claim that all of the sampled fish were over one foot in length. Let H be the hypothesis that all the fish in the lake are over one foot $[(\forall x)((Fx \ \& \ Lx) \supset O x)]$.
- Intuitively, one might think E should evidentially support H . This may be so for an agent who knows *only* the above information (K) about the observation process. That is, it seems plausible that $\Pr(E \mid H \ \& \ K) > \Pr(E \mid \sim H \ \& \ K)$, where \Pr is taken to be “evidential” (or “epistemic”) probability.
- But, what if I *also* tell you that (D) the net I used to sample the fish from the lake (which generated E) has holes that are all over one foot in diameter? It seems that D *defeats* the support E provides for H (relative to K), because D *ensures* O . Thus, intuitively, $\Pr(E \mid H \ \& \ D \ \& \ K) = \Pr(E \mid \sim H \ \& \ D \ \& \ K)$.

Is “Grue” an Observation Selection Effect? Part II

- Note: the “grue” hypothesis (H_2) entails the following claim, which is not entailed by the green hypothesis (H_1):
 - (H') All green emeralds have been (or will have been) examined prior to t . $[(\forall x)((Ex \ \& \ Gx) \supset O_x)]$.
- Now, consider the following two observation processes:
 - **Process 1.** For each green emerald in the universe, a slip of paper is created, on which is written a true description of that object as to whether it has property O . All the slips are placed in an urn, and one slip is sampled at random from the urn. By *this* process, we learn (\mathcal{E}) that $Ea \ \& \ Ga \ \& \ Oa$.
 - **Process 2.** Suppose all the green emeralds in the universe are placed in an urn. We sample an emerald (a) at random from this urn, and we examine it. [We know *antecedently* that the examination of a will take place prior to t , *i.e.*, that Oa is true.] By *this* process, we learn (\mathcal{E}) that $Ea \ \& \ Ga \ \& \ Oa$.
- Goodman seems to presuppose Process 2 in his set-up.

What Could “Carnapian” Inductive Logic Be? Part I

- The early Carnap dreamt that probabilistic inductive logic (confirmation theory) could be formulated in such a way that it *supervenes* on deductive logic in a *very strong* sense.
 - **Strong Supervenience (SS).** All confirmation relations involving sentences of a first-order language \mathcal{L} supervene on the deductive relations involving sentences *of* \mathcal{L} .
- Hempel clearly saw (SS) as a *desideratum* for confirmation theory. The early Carnap also seems to have (SS) in mind.
- I think it is fair to say that Carnap’s project — understood as requiring (SS) — was unsuccessful. [I think *this* is true for reasons that are *independent* of “grue” considerations.]
- The later Carnap seems to be aware of this. Most commentators interpret this shift as the later Carnap simply *giving up* on inductive logic (*qua logic*) altogether.
- I want to resist this “standard” reading of the history.

What Could “Carnapian” Inductive Logic Be? Part II

- I propose a different reading of the later Carnap, which makes him much more coherent with the early Carnap.
- I propose *weakening* the supervenience requirement in such a way that it (a) ensures this coherence, and (b) maintains the “logicality” of confirmation relations in Carnap’s sense.
- Let \mathcal{L} be a formal language strong enough to express the fragment of probability theory Carnap needs for his later, more sophisticated confirmation-theoretic framework.
 - **Weak Supervenience (WS).** All confirmation relations involving sentences of a first-order language \mathcal{L} supervene on the deductive relations involving sentences *of* \mathcal{L} .
- As it turns out, \mathcal{L} needn’t be very strong (in fact, one can get away with PRA!). So, even by early (*logician*) Carnapian lights, satisfying (WS) is all that is *really* required for “logicality”.
- The specific (WS) approach I propose takes confirmation to be a *four*-place relation: between E , H , K , and a function Pr .

What Could “Carnapian” Inductive Logic Be? Part III

- Consequences of moving to a 4-place confirmation relation:
 - We need not try to “construct” “logical” probability functions from the syntax of \mathcal{L} . This is a dead-end anyhow.
 - Indeed, on this view, inductive logic has nothing to say about the *interpretation/origin* of Pr. That is *not* a *logical* question, but a question about the *application* of logic.
 - Analogy: Deductive logicians don’t owe us a “logical interpretation” of the truth value assignment function v .
 - Moreover, this leads to a vast increase in the *generality* of inductive logic. Carnap was stuck with an impoverished set of “logical” probability functions (in his λ/γ -continuum).
 - On my approach, *any* probability function can be part of a confirmation relation. Which functions are “suitable” or “appropriate” or “interesting” will depend on *applications*.
 - So, some confirmation relations will not be “interesting”, *etc.* But, this is (already) true of *entailments*, as Harman showed.
 - Questions: Now, what *is* the job of the inductive logician, and how (if at all) do they interact with *epistemologists*?

What Could “Carnapian” Inductive Logic Be? Part IV

- The inductive logician must explain how it is that inductive logic can satisfy the following Carnapian *desiderata*.
 - The confirmation function $c(H, E | K)$ quantifies a *logical* (in a Carnapian sense) relation among statements E , H , and K .
 - (\mathcal{D}_1) One aspect of “logicality” is ensured by moving from (SS) to (WS) [from an \mathcal{L} -determinate to an \mathcal{L} -determinate concept].
 - (\mathcal{D}_2) Another aspect of “logicality” insisted upon by Carnap is that $c(H, E | K)$ should *generalize* the entailment relation.
 - This means (at least) that we need $c(H, E | K)$ to take a maximum (minimum) value when $E \& K \models H$ ($E \& K \models \sim H$).
 - Very few *relevance* measures c satisfy this “generalizing \models ” requirement. That’s another job for the inductive logician.
 - (\mathcal{D}_3) There must be *some* interesting “bridge principles” linking c and *some* relations of evidential support, in *some* contexts.
 - (\mathcal{D}_2) implies that *if* there are any such bridge principles linking *entailment* and *conclusive evidence*, these should be *inherited by c* . This brings us back to Harman’s problem!

Three Salient Quotes from Goodman [7]

👉 The “new riddle” is *about inductive logic (not epistemology)*.

Quote #1 (page 67): “Just as deductive logic is concerned primarily with a relation between statements — namely the consequence relation — that is independent of their truth or falsity, so inductive logic ... is concerned primarily with a comparable relation of confirmation between statements. Thus the problem is to define the relation that obtains between any statement S_1 and another S_2 if and only if S_1 may properly be said to confirm S_2 in any degree.”

Quote #2 (73): “Confirmation of a hypothesis by an instance depends ... upon features of the hypothesis other than its syntactical form”.

👉 But, Goodman’s *methodology* appeals to *epistemic* intuitions.

Quote #3 (page 73): “... the fact that a given man now in this room is a third son *does not increase the credibility of* statements asserting that other men now in this room are third sons, *and so does not confirm* the hypothesis that all men now in this room are third sons.”