Philosophy 148 — "Practice Final Exam"

[This is just a "practice exam", but it will be very similar in structure and content to the actual final exam.]

This exam contains five problems (worth 25 points each). You are to work **four** of these five problems. You also have the option of working a fifth problem for extra-credit (10 points worth). If you choose to try all five, then make sure you clearly mark which one is going to count as your extra-credit problem. (If you don't specify, we'll simply grade the last of the five problems in your blue book as your extra credit problem.) Read the instructions for each problem carefully, as some problems will involve a choice among multiple questions. Write all answers (legibly!) in your blue book(s). You have three hours to complete the exam. You are not to consult books or notes.

1 Proving a Theorem Both Axiomatically and Algebraically

Recall, the three Kolmogorov axioms for probability calculus (plus the definition of conditional probability) are:

1. For all propositions p, $\Pr(p) \geqslant 0$.
2. $\Pr(\top) = 1$, where $\top$ is any tautological proposition.
3. For all propositions p and q, if p and q are mutually exclusive (*i.e.*, if $p \& q \mathrel{=\!\!|\!\!\models} \bot$, where $\bot$ is any contradictory proposition), then $\Pr(p \vee q) = \Pr(p) + \Pr(q)$.
4. For all propositions p and q, $\Pr(p \mid q) =_{df} \dfrac{\Pr(p \,\&\, q)}{\Pr(q)}$.

This problem has two parts, and you are to do *both* of them:

(a) *Using these axioms* (plus definition), prove that *if* $Y \,\&\, Z \vDash X$, *then* $\Pr(X \mid Y) \geqslant \Pr(Z \mid Y)$. You may also use logic, Skyrms's rules, or any theorems proved during the semester. Be sure to justify each step in your proof.

(b) Prove the same theorem *algebraically*, using the following variables in your algebra.

X	Y	Z	Pr
T	T	T	a
T	T	F	b
T	F	T	c
T	F	F	d
F	T	T	e
F	T	F	f
F	F	T	g
F	F	F	h

2 Calculating Two Confirmation Measures on a Given Probability Model

Consider the probability model $\mathscr{M}$ characterized by the following stochastic truth-table:

E_1	E_2	H	$\Pr_{\mathscr{M}}$
T	T	T	0.1
T	T	F	0
T	F	T	0.2
T	F	F	0.2
F	T	T	0.1
F	T	F	0.1
F	F	T	0
F	F	F	0.3

This problem has four parts, and you are to do *all* of them:

(a) Calculate the two *likelihood ratios*: $l(H,E_1) = \dfrac{\Pr(E_1 \mid H)}{\Pr(E_1 \mid \sim H)}$, and $l(H,E_2) = \dfrac{\Pr(E_2 \mid H)}{\Pr(E_2 \mid \sim H)}$.

(b) Calculate the two s-values: $s(H,E_1) = \Pr(H \mid E_1) - \Pr(H \mid \sim E_1)$, and $s(H,E_2) = \Pr(H \mid E_2) - \Pr(H \mid \sim E_2)$.

(c) What do (a) and (b) imply about the *ordinal equivalence* of l and s? Explain. [Hint: recall that two measures $\mathfrak{c}_1$ and $\mathfrak{c}_2$ are *ordinally equivalent* iff for all H, E, H', and E', $\mathfrak{c}_1(H,E) \geqslant \mathfrak{c}_1(H',E')$ iff $\mathfrak{c}_2(H,E) \geqslant \mathfrak{c}_2(H',E')$.]

(d) Explain why (c) is an important issue for the foundations of (comparative) Bayesian confirmation theory.

3 Applying Bayes's Theorem and Bayesian Confirmation Theory

There are 3 coins in a box. One is a two-headed coin; another is a fair coin; and the third is a biased coin that comes up heads 75% of the time. A coin is sampled at random from the box (this means that each of the three has the same probability of being drawn), and then it is flipped. The coin comes up heads. Now, answer three questions:

(a) What is the probability that the coin flipped was the two-headed coin?

(b) Does the coin's having landed heads *confirm* or *disconfirm* (in the modern Bayesian sense) that the coin was the 75%-biased-in-favor-of-heads coin? Explain.

(c) Does the coin's having landed heads *confirm* or *disconfirm* that the coin was the fair coin? Explain.

Hints: Bayes's Theorem for a set of n mutually exclusive and exhaustive hypotheses $\{H_i\}$ has the following form:

$$\Pr(H_i \mid E) = \frac{\Pr(E \mid H_i)\Pr(H_i)}{\Pr(E)} = \frac{\Pr(E \mid H_i)\Pr(H_i)}{\sum_{i=1}^{n} \Pr(E \mid H_i)\Pr(H_i)}$$

Here, let H_1 = the coin is two-headed, H_2 = the coin is fair, H_3 = the coin is biased 75% in favor of heads, and E = the coin landed heads on the toss. And, remember, confirmation means *correlation* here.

4 Carnapian Logical Probabilities and Instantial Confirmation

Consider a language $\mathscr{L}$ with two predicates "R" and "B" and two individual constants "a" and "b". Such a language has 16 state descriptions. We have written these down for you in a truth-table format below. Now, think about Carnap's two logical measure functions $\mathfrak{m}^*$ and $\mathfrak{m}^\dagger$, defined over $\mathscr{L}$. Write down the probabilities assigned to each state description of $\mathscr{L}$ by $\mathfrak{m}^*$ and $\mathfrak{m}^\dagger$. That is, fill-in the two blank columns at the end of the truth-table for $\mathscr{L}$ below, thus converting it into two side-by-side stochastic truth-tables (one for $\mathfrak{m}^*$ and one for $\mathfrak{m}^\dagger$). [Hint: recall that $\mathfrak{m}^*$ operates on the structure descriptions of $\mathscr{L}$, whereas $\mathfrak{m}^\dagger$ operates on the state descriptions of $\mathscr{L}$.]

Ra	Ba	Rb	Bb	State Descriptions (s_i)	$\mathfrak{m}^\dagger(s_i)$	$\mathfrak{m}^*(s_i)$
T	T	T	T	$Ra \& Ba \& Rb \& Bb$		
T	T	T	F	$Ra \& Ba \& Rb \& \sim Bb$		
T	T	F	T	$Ra \& Ba \& \sim Rb \& Bb$		
T	T	F	F	$Ra \& Ba \& \sim Rb \& \sim Bb$		
T	F	T	T	$Ra \& \sim Ba \& Rb \& Bb$		
T	F	T	F	$Ra \& \sim Ba \& Rb \& \sim Bb$		
T	F	F	T	$Ra \& \sim Ba \& \sim Rb \& Bb$		
T	F	F	F	$Ra \& \sim Ba \& \sim Rb \& \sim Bb$		
F	T	T	T	$\sim Ra \& Ba \& Rb \& Bb$		
F	T	T	F	$\sim Ra \& Ba \& Rb \& \sim Bb$		
F	T	F	T	$\sim Ra \& Ba \& \sim Rb \& Bb$		
F	T	F	F	$\sim Ra \& Ba \& \sim Rb \& \sim Bb$		
F	F	T	T	$\sim Ra \& \sim Ba \& Rb \& Bb$		
F	F	T	F	$\sim Ra \& \sim Ba \& Rb \& \sim Bb$		
F	F	F	T	$\sim Ra \& \sim Ba \& \sim Rb \& Bb$		
F	F	F	F	$\sim Ra \& \sim Ba \& \sim Rb \& \sim Bb$		

Now, assume that each of these two Carnapian models comes along with its own notion of "relevance confirmation" (in the spirit of contemporary Bayesian confirmation theory), defined (over $\mathscr{L}$) as follows:

- E $*$-confirms H iff $\mathfrak{m}^*(H \,\&\, E) > \mathfrak{m}^*(H) \times \mathfrak{m}^*(E)$.
- E $\dagger$-confirms H iff $\mathfrak{m}^\dagger(H \,\&\, E) > \mathfrak{m}^\dagger(H) \times \mathfrak{m}^\dagger(E)$.

Armed with your two stochastic truth-tables and these two definitions of Carnapian "relevance confirmation," answer all three of these questions about Carnapian logical probability and instantial confirmation (over $\mathscr{L}$):

(a) Does "$Ra \,\&\, Ba$" $\dagger$-confirm "$(\forall x)(Rx \supset Bx)$" (in $\mathscr{L}$)? Note: in $\mathscr{L}$, "$(\forall x)(Rx \supset Bx)$" is equivalent to the conjunction "$(Ra \supset Ba) \,\&\, (Rb \supset Bb)$". So, this is equivalent to asking the following question:

$$\text{Is } \mathfrak{m}^\dagger[(Ra \supset Ba) \,\&\, (Rb \supset Bb) \,\&\, (Ra \,\&\, Ba)] > \mathfrak{m}^\dagger[(Ra \supset Ba) \,\&\, (Rb \supset Bb)] \times \mathfrak{m}^\dagger(Ra \,\&\, Ba)?$$

(b) Does "$Ra \,\&\, Ba$" $*$-confirm "$(\forall x)(Rx \supset Bx)$" (in $\mathscr{L}$)? Note: in $\mathscr{L}$, "$(\forall x)(Rx \supset Bx)$" is equivalent to the conjunction "$(Ra \supset Ba) \,\&\, (Rb \supset Bb)$". So, this is equivalent to asking the following question:

$$\text{Is } \mathfrak{m}^*[(Ra \supset Ba) \,\&\, (Rb \supset Bb) \,\&\, (Ra \,\&\, Ba)] > \mathfrak{m}^*[(Ra \supset Ba) \,\&\, (Rb \supset Bb)] \times \mathfrak{m}^*(Ra \,\&\, Ba)?$$

(c) Given your answers to (a) and (b), does $\dagger$-confirmation meet *Nicod's Condition*? Does $*$-confirmation? Explain how you know.

Bonus (**optional**, worth 5 points of extra-credit). Answer (a) and (b) for "$\sim Ba \,\&\, \sim Ra$" instead of "$Ra \,\&\, Ba$". Now, does "$Ra \,\&\, Ba$" confirm "$(\forall x)(Rx \supset Bx)$" *more strongly than* "$\sim Ba \,\&\, \sim Ra$" confirms "$(\forall x)(Rx \supset Bx)$" on either of these Carnapian theories? [Hint: In order to answer this question, all you need to look at are the *differences* between the left and right hand sides of the inequalities in the definitions of $*$ and $\dagger$ confirmation, above. *E.g.*, $\mathfrak{m}^*(H \,\&\, E) - \mathfrak{m}^*(H) \times \mathfrak{m}^*(E)$.] What does this imply about the prospects of implementing a comparative Bayesian-style approach to the Ravens Paradox in either of these Carnapian frameworks?

5 The Paradox of Confirmation (*i.e.*, the Paradox of the Ravens)

Explain the paradox of the ravens. Explain how the paradox arises, and what seems intuitively paradoxical about it. Then, explain **two** of the following approaches to the paradox. For each, say whether the approach takes the paradox to be paradoxical; if so, how the approach attempts to resolve the paradox; if not, why the approach thinks it's not paradoxical. Finally, of the two approaches you discuss, explain which one you think handles the paradox better, and why.

1. Hempel's response.
2. The Traditional Bayesian response.
3. The new Bayesian response I outlined in class.