Philosophy 148 — In-Class Quiz Answer Key

02/14/08

(1) Write the three (Kolmogorov) probability axioms we are using in this class:
A: If $\Pr(\bullet)$ is defined over a language $\mathcal{L}$, then for all $p, q \in \mathcal{L}$:

1. $\Pr(p) \geq 0$.
2. If $p \dashv\vDash \top$, then $\Pr(p) = 1$.
3. If $p \,\&\, q \dashv\vDash \bot$, then $\Pr(p \vee q) = \Pr(p) + \Pr(q)$.

(2) Write our definition of the conditional probability $\Pr(X \mid Y)$:
A: $\Pr(X \mid Y) \overset{\text{def}}{=} \dfrac{\Pr(X \,\&\, Y)}{\Pr(Y)}$

(3) $\sim(X \to Y) \dashv\vDash X \,\&\, {\sim}Y$ T/ F
A: T. $X \to Y$ is true on every interpretation but the one in which X is true and Y is false. So $\sim(X \to Y)$ is true just in case $X \,\&\, {\sim}Y$.

(4) Consider these two statements: $p \equiv q$ and $p \,\&\, {\sim}q$.

(*a*) These two statements are inconsistent (mutually exclusive). T/ F

(*b*) These two statements are contradictory. T/ F

A: T, F. Consider the truth-table below. There is no line on which both statements are true; therefore, they are inconsistent. However, they do not always have opposite truth-values — see line 3. So they are not contradictory.

p	q	$p \equiv q$	$p \,\&\, {\sim}q$
T	T	T	F
T	F	F	T
F	T	F	F
F	F	T	F

(5) Consider a monadic predicate-logical language $\mathcal{L}$ with two constants a and b (think: a universe of discourse containing two objects) and two predicates F and G. Which of the following state descriptions of $\mathcal{L}$ is entailed by the universal claim $(\forall x)(Fx \,\&\, {\sim}Gx)$? Circle the correct answer. (Exactly one is correct.)

(*i*) $Fa \,\&\, Ga \,\&\, Fb \,\&\, Gb$ (*iii*) $Fa \,\&\, {\sim}Ga \,\&\, Fb \,\&\, {\sim}Gb$
(*ii*) $Fa \,\&\, Ga \,\&\, {\sim}Fb \,\&\, {\sim}Gb$ (*iv*) ${\sim}Fa \,\&\, {\sim}Ga \,\&\, {\sim}Fb \,\&\, {\sim}Gb$

A: (*iii*). The universal quantification says that for any object in the universe of discourse, F is true of that object and G is false of it.

(6) Consider the probability model $\mathcal{M}$ described in this stochastic truth-table:

X	Y	State	$\Pr(s_i)$
T	T	s_1	0.1
T	F	s_2	0.2
F	T	s_3	0.3
F	F	s_4	0.4

Solve the following problems, concerning this model:

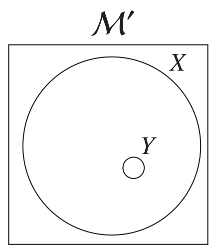
(*a*) Calculate the value of $\Pr(X)$ in $\mathcal{M}$.
A: 0.3. $\Pr(X) = \Pr(X \,\&\, Y) + \Pr(X \,\&\, {\sim}Y) = 0.1 + 0.2 = 0.3$

(b) Calculate the value of $\Pr(X \mid Y)$ in $\mathcal{M}$. **A:** $\frac{1}{4}$.

$$\Pr(X \mid Y) = \frac{\Pr(X \,\&\, Y)}{\Pr(Y)} = \frac{\Pr(X \,\&\, Y)}{\Pr(X \,\&\, Y) + \Pr(\sim X \,\&\, Y)} = \frac{0.1}{0.1 + 0.3} = \frac{1}{4}$$

(c) $\mathcal{M}$ provides a counter-example to $\Pr[\sim(X \,\&\, Y)] > \Pr(\sim X \to Y)$. T/ F
A: F. $\Pr(X \,\&\, Y) = 0.1$, so $\Pr[\sim(X \,\&\, Y)] = 0.9$.
$\sim X \to Y$ is false just in case $\sim X \,\&\, \sim Y$. $\Pr(\sim X \,\&\, \sim Y) = 0.4$, so $\Pr(\sim X \to Y) = 0.6$.
Thus *for this model* $\Pr[\sim(X \,\&\, Y)] > \Pr(\sim X \to Y)$, so *this model* does *not* provide a counter-example to the inequality. Note that this does not show the inequality is true for *every* model.

(7) Consider the probability model $\mathcal{M}'$ depicted by the following Stochastic Venn Diagram (note: the diagram IS drawn to scale, with areas of regions proportional to probabilities of corresponding propositions in $\mathcal{M}'$):



Circle true or false for each of the following, as they pertain to $\mathcal{M}'$:

(a) $X \vDash Y$ **A:** F. There are points in the X region where Y is not true.

(b) $Y \vDash X$ **A:** T. At every point in the Y region, X is true.

(c) $\Pr(X \mid Y) > 0.5$ **A:** T. Because $Y \vDash X$, $\Pr(X \mid Y) = 1$.

(d) $\Pr(Y \mid X) > 0.5$ **A:** F. The Y region occupies less than half of the X region.

(e) X and Y are correlated. **A:** T. As we just saw, $\Pr(X \mid Y) = 1$. But $\Pr(X) < 1$ (because X does not occupy the entire rectangle). So $\Pr(X \mid Y) > \Pr(X)$; X and Y are correlated.

(8) Suppose I roll a fair six-sided die (equal chance of any face coming up) and then flip a fair coin (equal chance of each side coming up), with the outcome of the die roll independent of the outcome of the coin flip. Define statements A, B, and C as follows:

A = 'The coin came up heads'
B = 'The die roll came up 3'
C = 'The die roll came up with an odd number'

Assuming Pr is a probability function in some probability model compatible with the above description of the situation, answer the following questions:

(a) What is $\Pr(A \,\&\, B)$? **A:** $\frac{1}{12}$.
A and B are independent, so $\Pr(A \,\&\, B) = \Pr(A) \cdot \Pr(B) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$.

(b) What is $\Pr(A \vee C)$? **A:** $\frac{3}{4}$. By the general additivity theorem, $\Pr(A \vee C) = \Pr(A) + \Pr(C) - \Pr(A \,\&\, C)$.
By independence, $\Pr(A \,\&\, C) = \Pr(A) \cdot \Pr(C)$. So

$$\Pr(A \vee C) = \Pr(A) + \Pr(C) - \Pr(A) \cdot \Pr(C) = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4}$$

(c) What is $\Pr(B \mid C)$? **A:** $\frac{1}{3}$. $\Pr(B \mid C) = \frac{\Pr(B \& C)}{\Pr(C)} = \frac{1/6}{1/2} = \frac{1}{3}$

(d) What is $\Pr(C \mid A)$? **A:** $\frac{1}{2}$. By independence, $\Pr(C \mid A) = \Pr(C) = \frac{1}{2}$.