Williamson's Argument Against KK

10/11/06 (B.F.)

Before moving on to this week's material, I want to begin by returning to Conee's argument against $E = K$. We decided last week that his argument, as stated, is not compelling. But, his example already contains a clear counterexample to $E = K$, just not the one he focused on. In his example, the expert testifies that (E') there is a distinction between how things appear (to S) to be experienced by S, and how they really are experienced by S. And, E' served to *defeat* (E) the evidence provided by S's "being appeared to ϕ-ly" for the claim that S is having a ϕ-ish experience. We agreed last time that it was controversial as to whether E was still evidence for S even though it had been defeated by E'. No problem. Just take E' to be the evidence in question. And, assume that E' is false (Conee explicitly allows this). Then, E' will be something that is evidence for S, but is not known by S. □

OK, now onto the argument against KK presented by Williamson in chapter 5. It seems to me that the strategy here is largely driven by formal arguments in modal logic that Williamson gave in a 1992 paper.[1] In that paper, Williamson was concerned not with the operator (K) "S knows that", but with the operator "it is clearly (or determinately) the case that". [That paper is largely about the logic of vague predicates.] These operators (both being read as a sort of necessity operator $\Box$ in this context) are significantly different (intuitively). And, I think some of the things Williamson assumes about the K operator in this chapter are pretty implausible (although, they might be more plausible for the "determinately true" operator). This threatens to undermine his argument against KK.

The argument itself is actually very simple (the presentation in the chapter is unnecessarily complex, I think). In a nutshell, it can be condensed down to the following line of reasoning. S looks at a tree but cannot tell how tall it is. The idea is to represent S's self-klnowledge of their own ignorance by the following claim (in schematic form, where i is a positive natural number):

(I_i) S knows that if the tree is $i + 1$ inches tall then S doesn't now that it isn't i inches tall.

Here, the conditional is allowed to be a material implication. Let $K\alpha$ stand for the proposition that S knows that the tree is α inches tall. Then, (I_i) can be written more perspicuously as follows:

(I_i) $K[(i + 1) \supset {\sim}K{\sim}i]$

This principle is just one of the premises of the argument. But, I don't find it at all plausible. Think about it in terms of the contraposition of the embedded conditional:

(I_i) $K[K{\sim}i \supset {\sim}(i + 1)]$

This says that S knows that if he knows that the tree is not i inches tall, then it is not $i + 1$ inches tall. That seems like an awful lot of knowledge for someone so shortsighted! But, putting these intuitions (of mine) to one side, we can already see that (I_i) is problematic by making a few simple inferences. First, let's assume that our operator K satisfies the T axiom (which everyone assumes):

(T) $K\alpha \supset \alpha$

[1] 'An Alternative Rule of Disjunction in Modal Logic', *Notre Dame Journal of Formal Logic*, 33 (1992) 89–100.

Then, by (I_i) and T, we can deduce $K{\sim}i \supset {\sim}(i+1)$, the contrapositive of which is $(i+1) \supset {\sim}K{\sim}i$. Moreover, the T axiom also implies that $K(i+1) \supset (i+1)$. Hence, we can infer $K(i+1) \supset {\sim}K{\sim}i$. But, this means (*e.g.*) that if S knows that the tree is 666 inches tall then he does not know that it isn't 665 inches tall. This just sounds false to me. So, I'm already worried about using (I_i) as a premise in a *reductio* of KK (or any other principle for that matter).[2] Let's come back to the argument now:

(KK) For any pertinent proposition p, if S knows p, then he knows that he knows p. That is, for pertinent propositions p, $Kp \supset KKp$.

(C) If p and all members of the set X are pertinent propositions, p is a logical consequence of X, and S knows each member of X, then he knows p.

(KK) and (C) are the other two premises of Williamson's *reductio*. Here's how it goes:

(1) S knows that the tree is not 0 inches tall [$K{\sim}0$].

- This is an (uncontroversial) assumption about S in Williamson's example.

(2) If the tree is 1 inch tall, then S does not know that the tree is not 0 inches tall [$1 \supset {\sim}K{\sim}0$].

- This follows from (I_i) and the T axiom (factivity of knowledge).

(3) Therefore, the tree is not 1 inch tall [${\sim}1$].

- This follows from (1) and the contrapositive of (2).

(4) Therefore, S knows that the tree is not 1 inch tall [$K{\sim}1$].

- In other words, S knows (3). This follows from (C) because (3) is a logical consequence of pertinent propositions S knows to be true. By (KK), S knows (1), and by (I_0), S knows (2).

(5) Therefore, by repeating the same reasoning m times, we can infer $K{\sim}1, \ldots, K{\sim}m$.

(6) But, by assumption, the tree is (say) 666 inches tall. And, after 666 iterations of this reasoning, we will end-up concluding that $K{\sim}666$, which by T implies ${\sim}666$. Contradiction.

Williamson blames (KK) for this absurdity. And, he offers an explanation of why (KK) fails. He says that knowledge requires safety from error, and one can be safe from error without being safely safe from error. In other words, safety doesn't iterate, so (since safety is necessary for knowledge) knowledge doesn't iterate (123–125). But, I doubt that's the right diagnosis. It's not that I doubt that safety doesn't iterate (or even that safety is necessary for knowledge). We've already seen that (I_i) is far from obvious, and (C) is not totally obvious either (although, it seems more plausible to me than I_i does). To see why this diagnosis is not so clearly correct, consider a different line of reasoning leading to the same absurdity, but one which rests on (I_i) and the following two principles:[3]

($C^\star$) If S knows $p_1, \ldots, p_n$, and S competently deduces q from $p_1, \ldots, p_n$, then S knows q.

($K^\star$) S knows that ($C^\star$) is correct.

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[2] See Heathcote's paper "KT and the diamond of knowledge", which is linked from our website.

[3] See Ramachandran's paper "Williamson's argument against the KK principle", which is linked from our website.

(C*) is what Williamson calls "intuitive closure". He uses it to defend the previous closure condition (C). So, (C*) is no less plausible than (C), by Williamson's lights. Now, run a new *reductio* as follows:

(I_0) S knows that if the tree is 1 inch tall, then he does not know that the tree is not 0 inches tall.

- This is just an instance of (I_i). Formally, it's $K(1 \supset \sim K \sim 0)$.

(7) S knows that he knows that the tree is not 0 inches tall.

- *This instance of* (KK) seems fine here. Formally, its $KK\sim 0$.

Now, suppose that S competently deduces (8) from (the contrapositive of) $1 \supset \sim K \sim 0$ and $K\sim 0$.

(8) The tree is not 1 inch tall. [Formally, this is ~ 1.]

By intuitive closure, since (8) was competently deduced from two propositions S knows, we have:

(9) S knows that the tree is not 1 inch tall. [Formally, this is $K\sim 1$.]

Assume further that S goes through this reasoning himself, and assume ($K^\star$). Then, S competently deduces (9) from propositions he knows [namely, $1 \supset \sim K \sim 0$, $K\sim 0$, and (C*)]. Thus, by (C*), we have:

(10) S knows that he knows that the tree is not 1 inch tall. [Formally, this is $KK\sim 1$.]

Now, reiteration of this reasoning [using (I_1) and (9) this time rather than (I_0) and (7)], yields:

(11) S knows that he knows that the tree is not 2 inches tall. [Formally, this is $KK\sim 2$.]

And, 664 iterations later, we will be able to infer:

(12) S knows that the tree is not 666 inches tall. [Formally, this is $K\sim 666$.]

This is the same contradiction Williamson derived using (I_i), (KK), and (C). But, this time, we derived it using only (I_i), (K*), (C*), and an innocuous instance of (KK). It's even harder to believe that (KK) is responsible for *this* absurdity. And, (C*) is even more plausible (according to Williamson) than (C) is. So, (I_i) is looking suspicious here (again). We can push our analysis even further by talking only about true belief (B) instead of knowledge. This will cast doubt on the diagnostic relevance of appeals to safety in this context. Here's how an analogous "true belief *reductio*" might be run.

In the same sort of scenario, assume there are certain values of α where S is confident that the tree is above (and, hence, not) α inches tall. S is agnostic concerning values of α in the neighbourhood of the actual height of the tree: for a range of values of α he neither believes nor disbelieves that it is α inches tall. S, aware of his own cautiousness and indecision, realizes all of this. Thus:

(I*) S believes truly that if he believes truly that the tree is not i inches tall, the tree is not $i+1$ inches tall.

The other premises we will make use of are the following three:

(BC) If p and all members of the set X are pertinent propositions, p is a logical consequence of X, and S believes truly each member of X, then he believes truly that p.

(BB) For any pertinent proposition p, if S believes truly p, then he believes truly that he believes truly p. [Formally, for pertinent p, $Bp \supset BBp$.]

(13) S believes truly that the tree is not 0 inches tall. [Formally, $B{\sim}0$.]

Now, from (BB) and (13), we may infer:

(14) S believes truly that he believes truly that the tree is not 0 inches tall. [Formally, $BB{\sim}0$.]

And, (I*) and (14) ascribe (true) beliefs to S that jointly entail that the tree is not 1 inch tall. So, (I*), (14) and (BC) yield:

(15) S believes truly that the tree is not 1 inch tall. [Formally, $B{\sim}1$.]

Reiteration of this reasoning will allow us to infer $B{\sim}2, \ldots, B{\sim}666$. This leads to the same contradiction as before — that the tree both is and is not 666 inches tall. But, (KK) is nowhere to be found, so it *can't* be to blame this time. And, *even if* (BB) is to blame here, it can't have anything to do with *safety*, since safety is not necessary for true belief. Anyhow, (BB) isn't plausibly to blame here, since the conclusion that there is a proposition S believes truly but *is not in a position to believe truly that he believes truly* is far less plausible than the parallel conclusion about knowledge. [What would the *obstacle* to such a second-order true belief be?] If we maintain (BB), then we must reject (I*) or (BC). Let's think about (BC) first. The debate about the closure of *knowledge* under (known) logical consequence usually involves examples that do *not* involve violations of closure of *true belief* under (truly believed) logical consequence. For instance, consider Dretske's example about a zebra named Sam. You know that Sam is a zebra, but you don't know Sam isn't a cleverly painted horse, which you know follows from Sam's being a zebra. In Dretske's example, you still *truly believe* that Sam isn't a cleverly painted horse, so this won't be a counterexample to closure of your true beliefs under (truly believed) logical consequence. Some people have claimed that lottery and/or preface paradox cases are counterexamples to the closure of *rational* belief. I suppose these sorts of cases could also be counterexamples to the closure of true belief as well. That's worth thinking about and discussing. In any case, closure is not sacrosanct (in the literature) for either true belief or knowledge. And, if closure is called into question, then this just *undermines* Williamson's *reductio* against (KK). So, Williamson would want to maintain closure (at least, intuitive closure and even (C), which were used in the two *K-reductios*). My suspicion is that it is the I-principles [(I_i) and (I*)] that are primarily to blame in all of the three *reductios* we have seen above. I think even the simple considerations adduced at the outset of my discussion should give one pause about such principles.

Another point worth making here is that even contemporary epistemic internalists (like Conee) needn't accept (KK). What an internalist will be committed to is something weaker, like Conee's

(D_2) If S knows p, then it is not the case that S has a strong unoverridden reason to doubt p.

Indeed, we ought to be able to run a Conee-style argument against the luminosity of K itself, and such an argument needn't depend on any suspicious I-principle. I think it is very instructive to try to formulate such an argument. Can we do it? This may be tricky. Maybe we could have a case of expert testimony to the effect that there is a distinction between what you *believe* you believe *vs* what you *actually* believe (maybe some studies on self-deception about beliefs?). Then you could get a defeater for your evidence for your belief *that you believe* p. So, you could then have (strong unoverridden) reason to doubt Bp (hence, Kp), even though (say) you have no reason to doubt p. If (D_2) is the hallmark of epistemic internalism, the it seems that such internalist counterexamples to (KK) should be possible. And, if such examples are out there, then (*i*) it's unclear why we need any arguments inspired by modal logic to get the job done, and (*ii*) it's also unclear what's really at issue here between Williamson and the (contemporary, reasonable) epistemic internalists. Again, they both reject luminosity, even for knowledge. So, where's the *epistemological* dispute here?